Nonequilibrium phenomena in (homogeneous) quantum gases

All equilibrium systems are alike; each nonequilibrium one is out of equilibrium in its own way (Anna Karenina principle in many-body physics)

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Les Houches, Sep 2021

Leo Tolstoy, 1877
Outline

Part 1: Intro
1.1 Motivation, universality vs. stamp collecting
1.2 Experimental system(s) and tools

Part 2: Two unintentionally-nonequilibrium stories
2.1 Weak interactions + losses
2.2 Strong interactions + quench + losses (example of prethermalization)

Part 3: Three related intentionally-nonequilibrium stories
3.1 Critical dynamics
3.2 Turbulence
3.3 Universality far from equilibrium
Part 2.1:
Weak interactions + losses
First surprise in a box... weird spontaneous cooling

Very low density:
(1) neglect interactions,
(2) only one-body losses!

energy: $E/N = \text{const}$
enthalpy: $H/N = (E + PV)/N = \text{const}$

isoenthalpic rarefaction = Joule-Thomson effect

Wikipedia: ... temperature change of a real gas (as opposed to an ideal gas) ...
Quantum Joule-Thomson effect

**More general:**

- **Classical:**
  - Cooling for **affinity** between particles
  - Heating for **aversion** between particles

**Quantum:**
- **Bosons cool**
- **Fermions heat**
Joule-Thomson Effect and Quantum Statistics

In view of the numerous physical and astrophysical applications of the new quantum statistics it may be worth while to investigate the Joule-Thomson effect for a gas obeying Fermi-Dirac or Bose-Einstein statistics. The calculation is simple and runs on the usual lines. The results obtained are quite interesting.

\[
\left( \frac{\partial T'}{\partial p} \right)_i = 0.076^\circ/\text{atmos}.
\]

for helium at $5^\circ \text{K}$. The Van der Waals effect is much the larger, but the statistical effect is still 10 per cent of it. It therefore seems possible that the Joule-Thomson effect under suitable conditions may provide an experimental test of the statistics obeyed by gases, say, helium.

D. S. Kothari.

B. N. Srivasava.
Quantum Joule-Thomson effect

Even simpler in 2014 (for a partially condensed gas):

\[ N', N_0 \]

constant \( T \),

vary atom number \( N \)

\( N' \) – thermal atoms

\( N_0 \) – condensed

“... one part condenses, the rest remains a saturated ideal gas.”

(Einstein, 1925)

Exp. confirmation: T.F. Schmidutz et al., PRL 112, 040403 (2014)
Quantum JT in a saturated Bose gas
Quantum JT in a saturated Bose gas
Quantum JT in a saturated Bose gas

CHEZ BOSE

LIMITED SEATING
Quantum Joule-Thomson effect

\[ \frac{E}{N} \propto \frac{N'T}{N} = \frac{N_cT}{N} \propto \frac{T^{5/2}}{N} = \text{const.} \]

Is this really cooling? (only) Sort of... temperature does drop... but you lose the BEC

QJT coefficient:

\[ \mu_{JT} = \left( \frac{\partial T}{\partial P} \right)_h \propto \frac{h^3}{T^{3/2}} \]

1937 hope: \( 0.076^\circ/\text{atmos.} \)

2014: \( > 10^9 \text{K/atmos.} \)
Outlook: can do even better with 3-body recombination losses
(just theory so far)

\[ \mathcal{P} = \frac{\dot{N}_{\text{th}}/N_{\text{th}}}{\dot{N}/N} = \frac{3}{5} \frac{\Gamma_{\text{th}}}{\Gamma} \]

\[ \Gamma = -\dot{N}/N \]

would love \( \mathcal{P} > 1 \)

1-body loss (Quantum J-T):
\[ \Gamma_{\text{th}} = \Gamma_0 = \Gamma \]
\[ \mathcal{P} = \frac{3}{5} < 1 \]

3-body recombination:
\[ \frac{\dot{n}}{n} = -g_3 K_3 n^2 \]
\[ g_3 = \frac{3!}{n^3} \left( \frac{1}{3!} n_0^3 + \frac{1}{2!} 3n_0^2 n_{\text{th}} + 3n_0 n_{\text{th}}^2 + n_{\text{th}}^3 \right) \]

"purification coefficient"

"no boson bunching in a BEC" favours loss of thermal atoms!
3-body cooling and purification – the maths (some of it)

“purification coefficient”

\[ P = \frac{\dot{N}_{\text{th}}/N}{N/N} = \frac{3 \Gamma_{\text{th}}}{5 \Gamma} \quad \Gamma = -\dot{N}/N \]

\[ \Gamma_{0}N_{0} = K_{3}\left(N_{0}^{3} + 6N_{0}N_{\text{th}} + 6N_{0}N_{\text{th}}^{2} + 0\right)/V^{2} \]

\[ \Gamma_{\text{th}}N_{\text{th}} = K_{3}\left(0 + 3N_{0}^{2}N_{\text{th}} + 12N_{0}N_{\text{th}}^{2} + 6N_{\text{th}}^{3}\right)/V^{2} \]

\[ \Gamma N = \Gamma_{0}N_{0} + \Gamma_{\text{th}}N \]

condensed fraction

\[ \eta = \frac{N_{0}}{N} = \frac{n_{0}}{n} \]

\[ P = \frac{9}{5} \frac{2 - \eta^{2}}{6 - 9\eta^{2} + 4\eta^{3}} \]

L. H. Dogra et al., PRL 123, 020405 (2019)

(more complicated with mean-field interactions – see paper)
Part 2.2:
Strong interactions + non-equilibrium (quench + losses)

Dynamics of a Bose gas quenched to “unitarity”
Interactions as strong as possible (allowed by quantum mechanics)
Atomic Bose gases

Relevant lengthscales: $d, \lambda$

Unitarity: $a \rightarrow \infty$

Experiments: ENS, Cambridge, JILA, Chicago...

Magnetic field $a$

$\lambda \sim \frac{1}{\sqrt{T}}$

density $d = n^{-1/3}$

Relevant lengthscales:

- $d, \lambda$
- $d/a$, $d/\lambda$
- temperature, interaction strength

hard (+ losses)

trivial

beyond mean field

mean-field BEC

Bogoliubov, Lee-Huang-Yang…

Feshbach
Part 2.2.1: Universal degenerate unitary Bose gas

Only one lengthscale (not really true... see later)

\[ k_n = (6\pi^2 n)^{1/3} \quad E_n = \frac{\hbar^2}{2m} k_n^2 \quad t_n = \frac{\hbar}{E_n} \]

(“Fermi” momentum, energy, time)

**Expect:**

Self-similar gases of different densities

\[ a \to \infty \quad \text{really means} \quad a_{\text{eff}} \sim d = n^{-1/3} \]

Universal energy per particle \( E/E_n \sim 1 \), condensed/superfluid fraction...

**Problem:**

Does this state even exist?

Universality arguments also for \( m \)-body losses

\[ \dot{N}/N \sim -1/t_n \quad \text{“lifetime”} \sim \hbar/E_n \quad \text{“}Q\sim 1\text{”} \]

Well defined equilibrium properties?
Universality in decay (and heating) dynamics

Universality hypothesis:
\[ \frac{\dot{N}}{N} \propto -N^\gamma \quad \gamma = \frac{2}{3} \]
(in a box trap)

Crossover
degenerate $\Rightarrow$ thermal gas

Quench
$\alpha \to \infty$
and back

C. Eigen, J.A.P. Glidden,
R. Lopes, N. Navon, ZH,
R.P. Smith, PRL 2017

2/3 also @ JILA:
Klauss et al., PRL 2017

Thermal-gas theory @ ENS:
Rem et al., PRL 2013
An aside: 26/9? 

\[ \frac{\dot{N}}{N} \propto -N^\gamma \]

\[ \frac{\dot{N}}{N} \propto -N^2 a^4 \quad \rightarrow \quad \frac{\dot{N}}{N} \propto -N^2 \lambda^4 \propto -\frac{N^2}{T^2} \]

Kinetic-energy heating (ENS theory):

\[ \frac{dT}{T} = -(4/9) \frac{dN}{N} \quad \rightarrow \quad T \propto N^{-4/9} \]

\[ \gamma = 2 + 2 \times \frac{4}{9} = 26/9 \]
"Universal crossover"
degenerate $\rightarrow$ thermal gas

Different initial densities:
Always $N_c = 0.43(4)N_0$

Crossover time: $t_c = 4.0(4)t_{n0}$

"Q $\sim 4$"

Can collapse all loss curves

Dimensionless $t_n \times \dot{N}/N = f(E/E_n)$

$E_c = 1.7(2)E_n$

$\frac{E^2}{E_n^2}$

C. Eigen, J.A.P. Glidden, R. Lopes, N. Navon, ZH, R.P. Smith, PRL 2017
Momentum- and time-resolved dynamics

\[ n(\text{fixed } k, t) \]

\[ n_k \times k_n^3 \]

Get:

“equilibration time” \( \tau(k) \)

“steady state” \( \bar{n}_k(k) \)

Universal “prethermal steady state”

(Half-)Time to steady state

\[
\frac{\tau}{t_n} \propto \frac{k_n^2}{k_n^3} \quad \text{and} \quad \propto \left(\frac{k_n^2}{k_n^3}\right)^2
\]

3 densities

\begin{align*}
\bar{n}_k k_n^3 & \sim 10^{-4} \\
& \sim 10^{-3} \\
& \sim 10^{-2}
\end{align*}

\begin{align*}
\frac{n (\mu m^{-3})}{k_n} & = 1.6, 2.7, 5.1
\end{align*}

Consistent with “prethermalization”

Bogoliubov-like excitations

\[
\mu \sim E_n
\]

\[
v_s \sim \hbar k_n / m
\]

\[
\tau \sim 1 / \omega
\]

Steady-state momentum distribution

Exponential?

Energy \[0.7E_n\]

Condensed fraction 20% (not 0%)

Coherent 2-body processes slower than in a degenerate gas

3-body loss/heating \textbf{much} slower

\textbf{nice timescale separation}

W. Li and T.-L. Ho, PRL 2012
Part 2.2.2: “Universality” in a thermal unitary gas?

Coherent 2-body processes slower than in a degenerate gas

3-body loss/heating much slower  

W. Li and T.-L. Ho, PRL 2012

nice timescale separation

\[
\begin{align*}
E/k_B (\mu K) & \quad t_{\text{hold}} (\mu s) \\
0.6 & \quad 10 \quad 100 \quad 1000 \\
0.2 & \quad \text{understood heating} \\
0.4 & \quad \text{interesting early times}
\end{align*}
\]

\[
\begin{align*}
n_k (\mu m^3) & \quad k (\mu m^{-1}) \\
10^{-3} & \quad 0 \quad 5 \quad 10 \quad 15 \\
10^{-5} & \quad \text{distribution}
\end{align*}
\]

\[
\text{BUT... two lengthscales:} \quad d = n^{-1/3}, \quad \lambda \sim 1/\sqrt{T}
\]
Momentum-space redistribution at early times

Some $n$ and $T$

Two different $n$ and $T$

characteristic times

Are these curves “universal”?

\[
\begin{align*}
    k_n &= (6\pi^2 n)^{1/3} \\
    E_n &= \frac{\hbar^2}{2m} k_n^2 \\
    t_n &= \frac{\hbar}{E_n}
\end{align*}
\]

\[
\begin{align*}
    k_\lambda &= 1/\lambda \\
    E_\lambda &= k_B T \\
    t_\lambda &= \frac{\hbar}{k_B T}
\end{align*}
\]

infinitely many scale candidates:
Unitary Thermal Gas Scaling Laws

<table>
<thead>
<tr>
<th>$n_i , (\mu m^{-3})$</th>
<th>0.7</th>
<th>1.1</th>
<th>1.2</th>
<th>1.2</th>
<th>1.7</th>
<th>2.0</th>
<th>2.2</th>
<th>2.7</th>
<th>3.1</th>
<th>3.8</th>
<th>4.2</th>
<th>5.1</th>
<th>5.2</th>
<th>5.6</th>
<th>6.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_i , (nK)$</td>
<td>200</td>
<td>160</td>
<td>100</td>
<td>70</td>
<td>80</td>
<td>190</td>
<td>100</td>
<td>170</td>
<td>100</td>
<td>190</td>
<td>150</td>
<td>190</td>
<td>190</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

$\tau / \sqrt{t_n t_\lambda}$

$\bar{\delta \varepsilon} / (\sqrt{E_n})$

### Table

<table>
<thead>
<tr>
<th>Momentum</th>
<th>Just</th>
<th>$k_\lambda \sim 1/\lambda \sim \sqrt{T}$</th>
<th>Kind of obvious</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td></td>
<td>$\sqrt{t_n t_\lambda} \sim \hbar / \sqrt{E_n T}$</td>
<td>Makes some sense</td>
</tr>
<tr>
<td>Energy</td>
<td>Just</td>
<td>$E_n$</td>
<td>Weird</td>
</tr>
</tbody>
</table>
More on unitary Bose gases...
(not in this course)

It’s not actually all universal...

Efimov 1970: Borromean 3-body bound states
First observation: enhanced 3-body loss [Kraemer et al., Nature 2006]
Coherent effects (including dynamics): “3-body clock shift” [Fletcher et al., Science 2017]
Efimov molecules: [Klauss et al., PRL 2017]