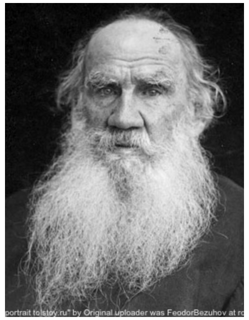
Nonequilibrium phenomena in (homogeneous) quantum gases

All equilibrium systems are alike;
each nonequilibrium one is out of equilibrium in its own way
(Anna Karenina principle in many-body physics)



Leo Tolstoy, 1877

Zoran Hadzibabic
University of Cambridge

EPSRC
Engineering and Physical Sciences Research Council

DesOEQ

THE ROYAL SOCIETY

Les Houches, Sep 2021

MARIE CURIE ACTIONS

QUANTERA
ERA-NET Cofund in Quantum Technologies

NAQUAS

erc

Outline

Part 1: Intro

1.1 Motivation, universality vs. stamp collecting

1.2 Experimental system(s) and tools

Part 2: Two unintentionally-nonequilibrium stories

2.1 Weak interactions + losses

2.2 Strong interactions + quench + losses (example of prethermalization)

Part 3: Three related intentionally-nonequilibrium stories

3.1 Critical dynamics

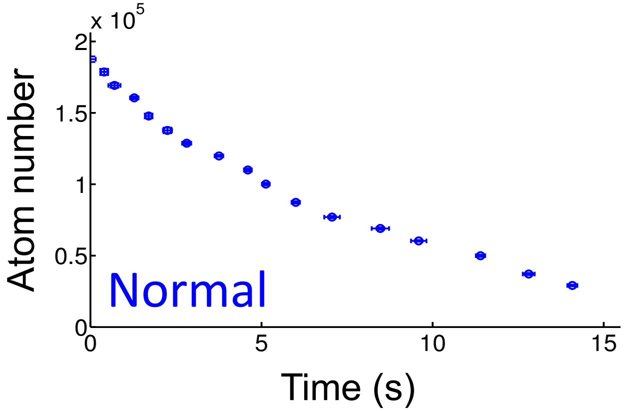
3.2 Turbulence

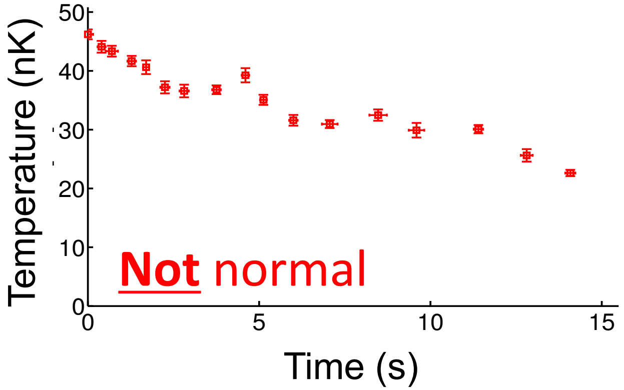
3.3 Universality far from equilibrium

Part 2.1:
Weak interactions + losses

First surprise in a box… weird spontaneous cooling

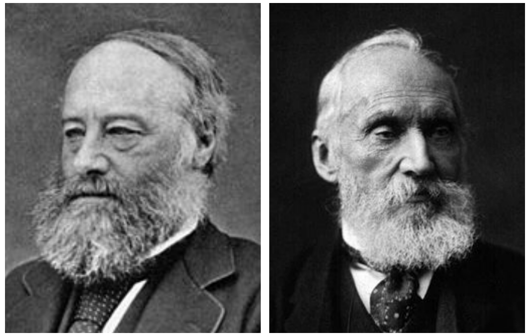
ultra-high vacuum





isoenthalpic rarefaction

= Joule-Thomson effect



Very low density:

(1) neglect interactions,

(2) only one-body losses!

energy: $E/N = \text{const}$

enthalpy: $H/N = (E + PV)/N = \text{const}$

Wikipedia: … temperature change of a *real* gas (as opposed to an ideal gas) …

Quantum Joule-Thomson effect

More general:
~~Classical:~~

affinity
Cooling for ~~attractive forces~~ between particles

aversion
Heating for ~~repulsive forces~~ between particles

Quantum:

Bosons cool
Fermions heat

nature

DECEMBER 4, 1937

Joule-Thomson Effect and Quantum Statistics

IN view of the numerous physical and astrophysical applications of the new quantum statistics it may be worth while to investigate the Joule-Thomson effect for a gas obeying Fermi-Dirac or Bose-Einstein statistics. The calculation is simple and runs on the usual lines. The results obtained are quite interesting.

$$\left(\frac{\partial T}{\partial p}\right)_i = 0{\cdot}076^\circ/\text{atmos.}$$

for helium at 5° K. The Van der Waals effect is much the larger, but the statistical effect is still 10 per cent of it. It therefore seems possible that the Joule-Thomson effect under suitable conditions may provide an experimental test of the statistics obeyed by gases, say, helium.

D. S. KOTHARI.
B. N. SRIVASAVA.

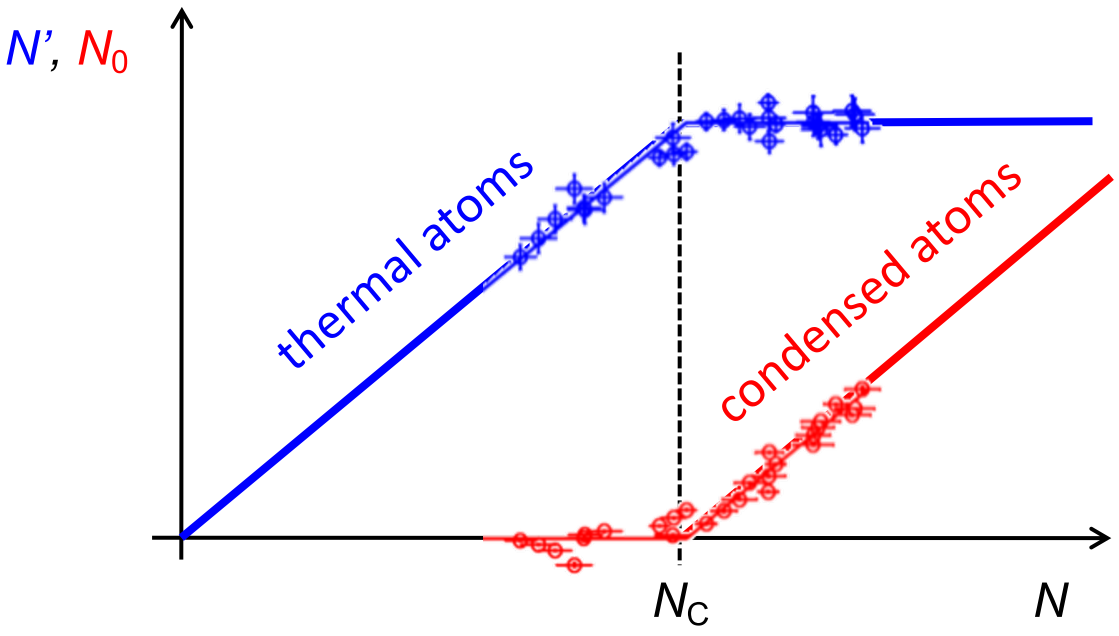
Quantum Joule-Thomson effect

Even simpler in 2014 (for a partially condensed gas):

"… one part condenses,
the rest remains a **saturated** ideal gas."
(Einstein, 1925)

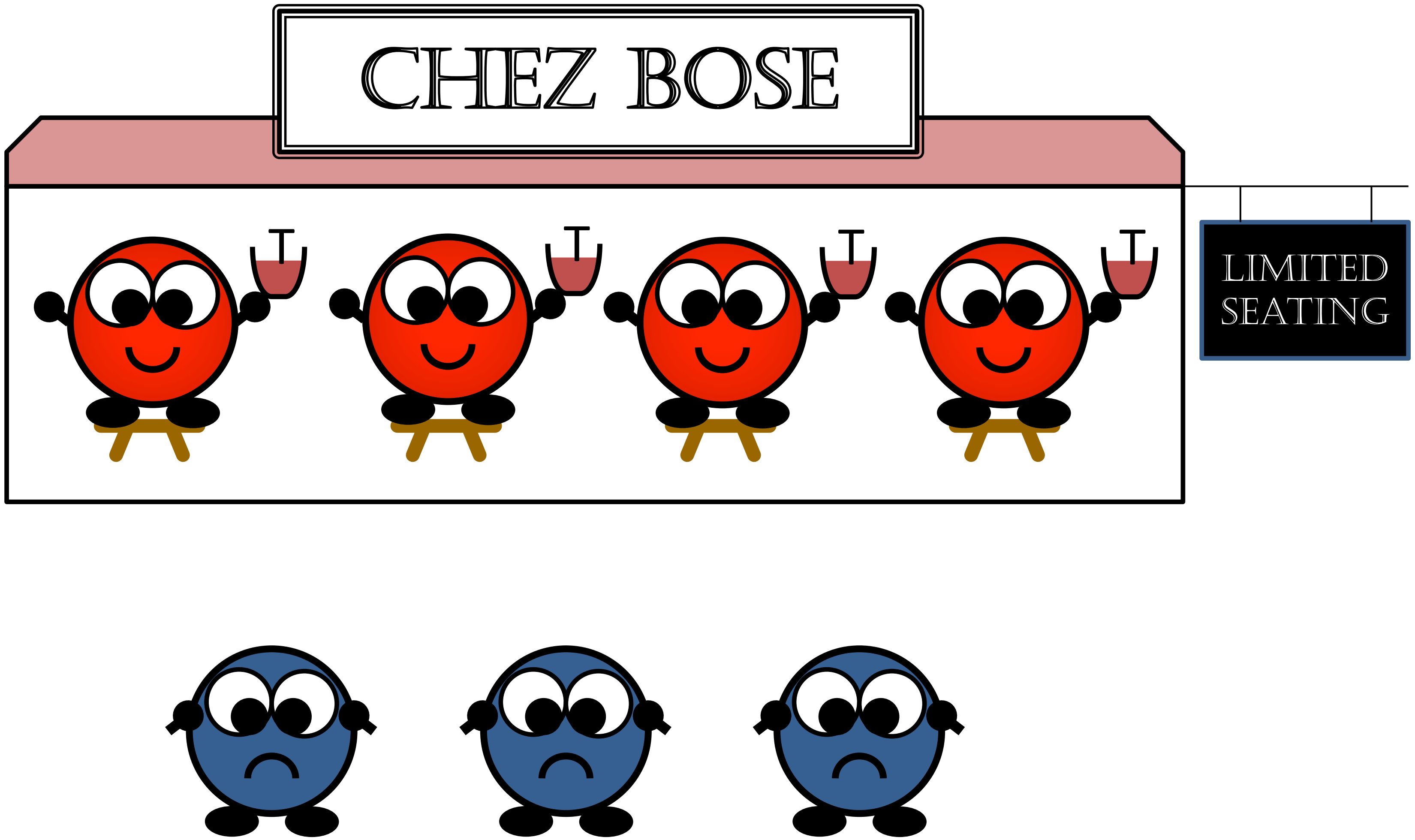
constant T,
vary atom number N

N' – thermal atoms
N_0 – condensed

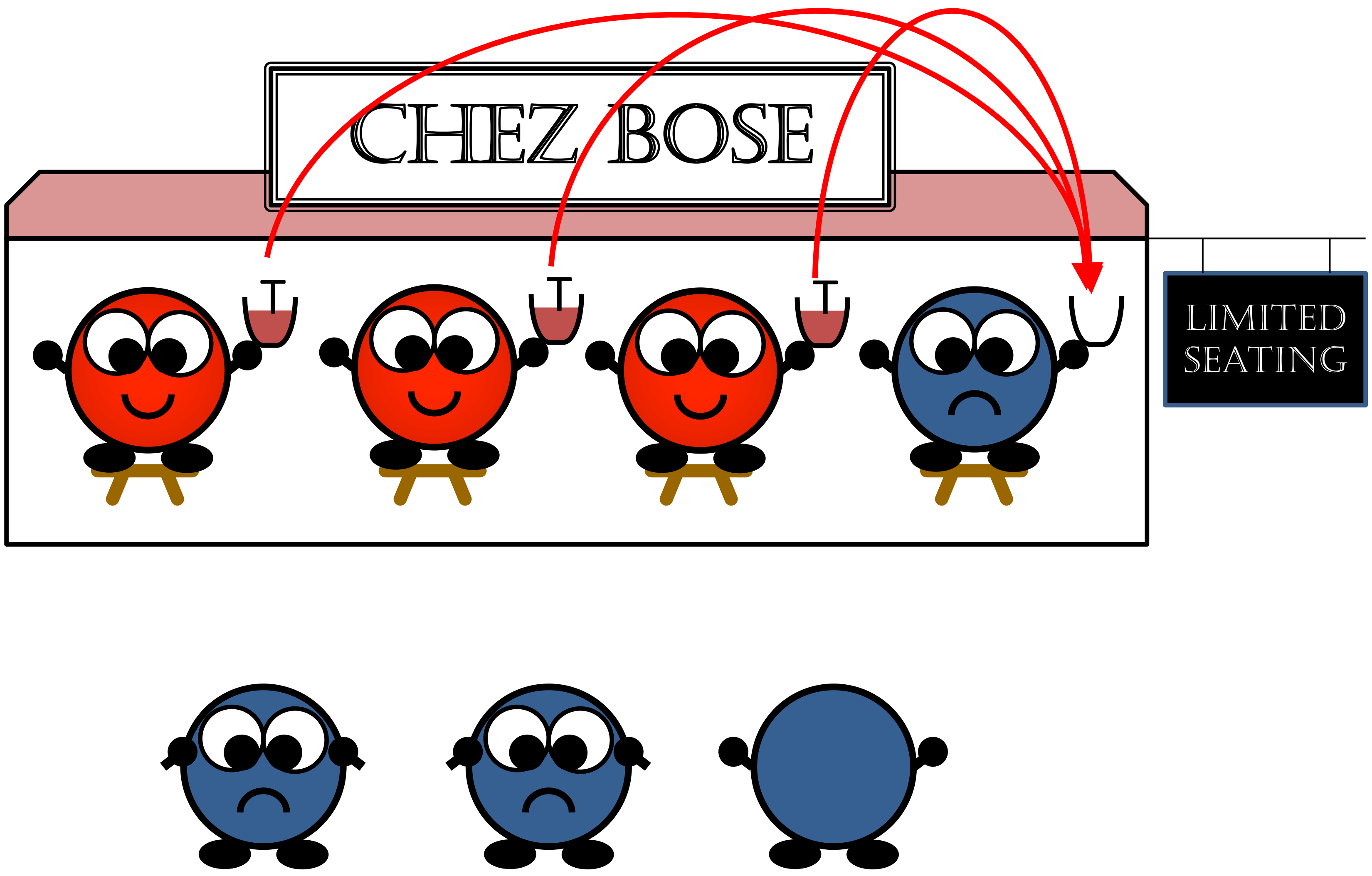


Exp. confirmation: T.F. Schmidutz *et al.*, PRL **112**, 040403 (2014)

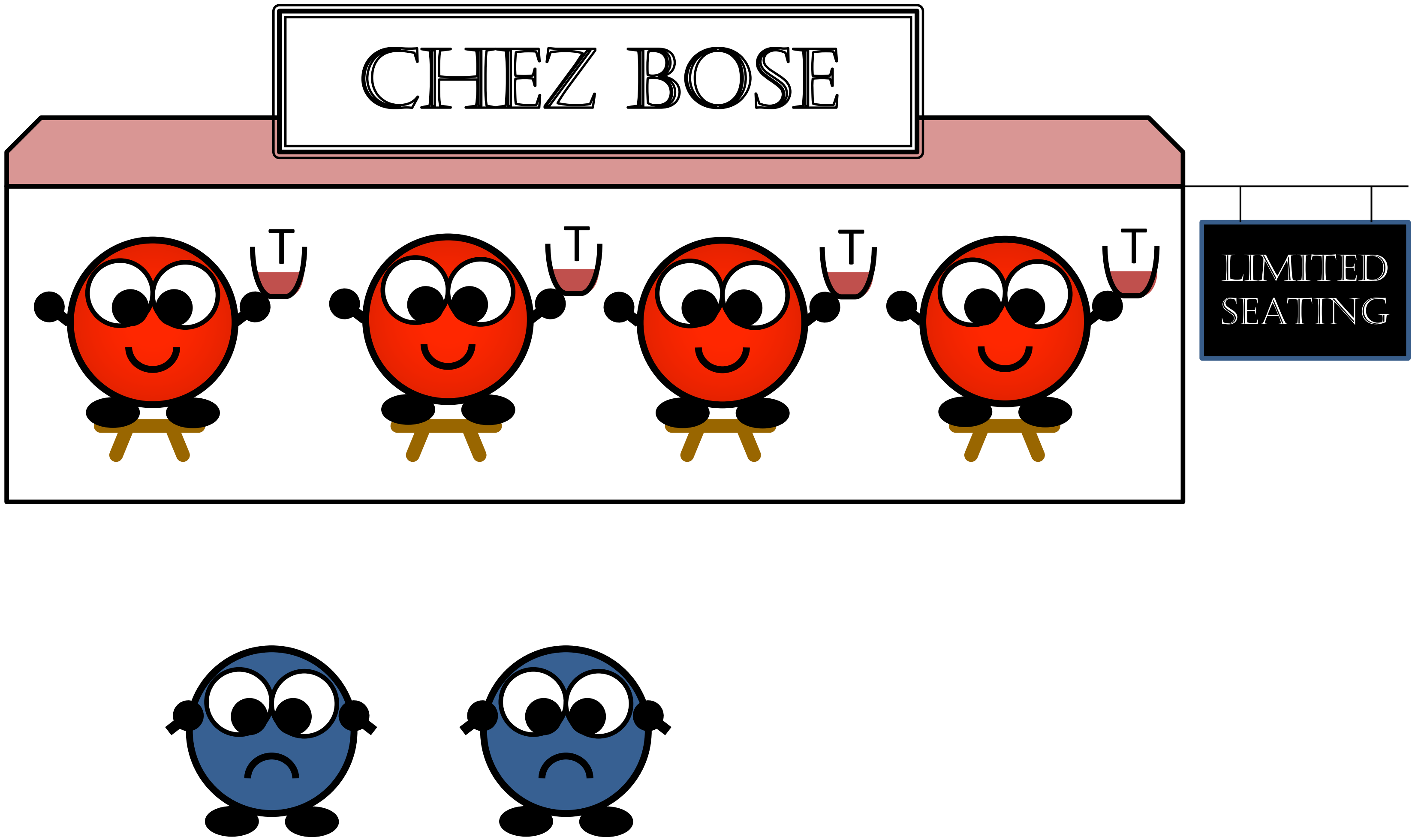
Quantum JT in a saturated Bose gas



Quantum JT in a saturated Bose gas



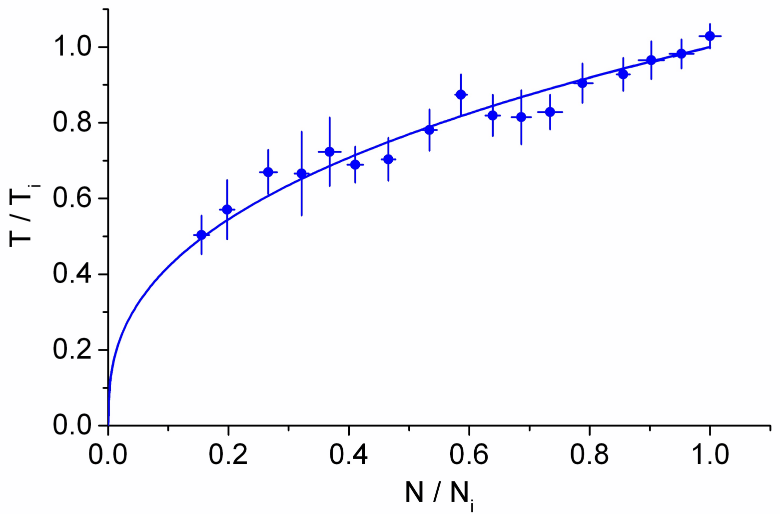
Quantum JT in a saturated Bose gas



Quantum Joule-Thomson effect

$$\frac{E}{N} \propto \frac{N'T}{N} = \frac{N_c T}{N} \propto \frac{T^{5/2}}{N} = \text{const.}$$

T. F. Schmidutz *et al.*, PRL **112**, 040403 (2014)



QJT coefficient:

$$\mu_{\mathrm{JT}} = \left(\frac{\partial T}{\partial P}\right)_h \propto \frac{\hbar^3}{T^{3/2}}$$

1937 hope: $0{\cdot}076°/\mathrm{atmos.}$

2014: $> 10^9\ \mathrm{K/atmos.}$

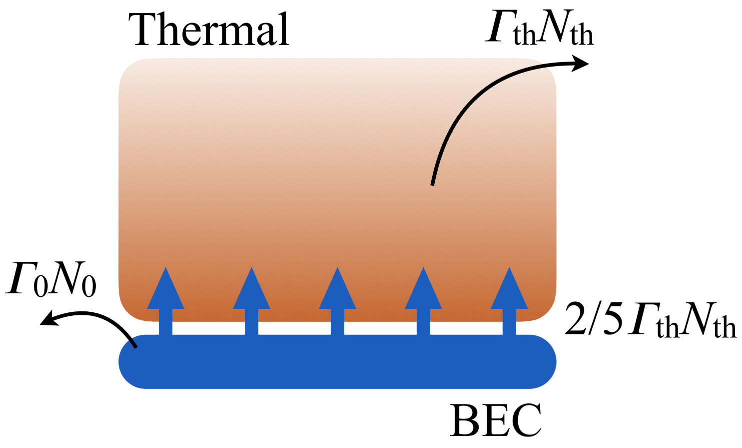
Is this really cooling?

(only) Sort of… temperature does drop… but you lose the BEC

Outlook: can do even better with 3-body recombination losses

(just theory so far)

L. H. Dogra *et al.*, PRL **123**, 020405 (2019)



"purification coefficient"

$$\mathcal{P} = \frac{\dot{N}_{\rm th}/N_{\rm th}}{\dot{N}/N} = \frac{3}{5}\frac{\Gamma_{\rm th}}{\Gamma}$$

$$\Gamma = -\dot{N}/N$$

would love $\mathcal{P} > 1$

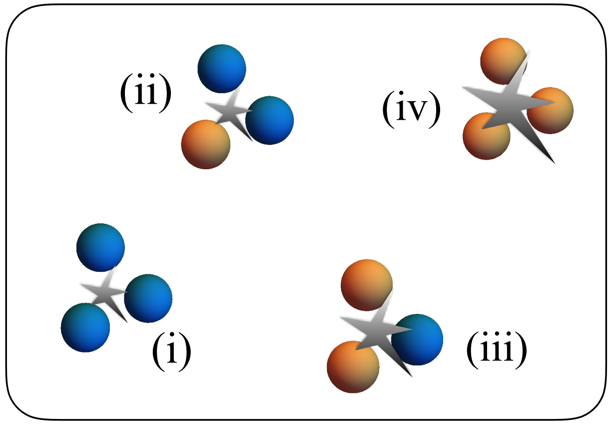
1-body loss (Quantum J-T): $\Gamma_{\rm th} = \Gamma_0 = \Gamma$ $\quad \mathcal{P} = 3/5 < 1$

3-body recombination:

$$\dot{n}/n = -g_3 K_3 n^2$$

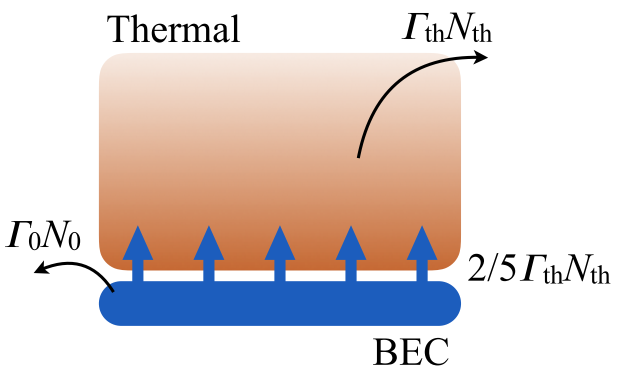
$$g_3 = \frac{3!}{n^3}\left(\frac{1}{3!}n_0^3 + \frac{1}{2!}3n_0^2 n_{\rm th} + 3n_0 n_{\rm th}^2 + n_{\rm th}^3\right)$$

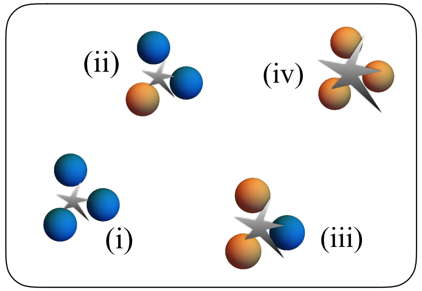
"no boson bunching in a BEC"
favours loss of thermal atoms!



3-body cooling and purification – the maths (some of it)

"purification coefficient" $\mathcal{P} = \frac{\dot{N}_{\rm th}/N_{\rm th}}{\dot{N}/N} = \frac{3}{5}\frac{\Gamma_{\rm th}}{\Gamma}$ $\qquad \Gamma = -\dot{N}/N$





$$\Gamma_0 N_0 = K_3\left(N_0^3 + 6N_0^2 N_{\rm th} + 6N_0 N_{\rm th}^2 + 0\right)/V^2$$

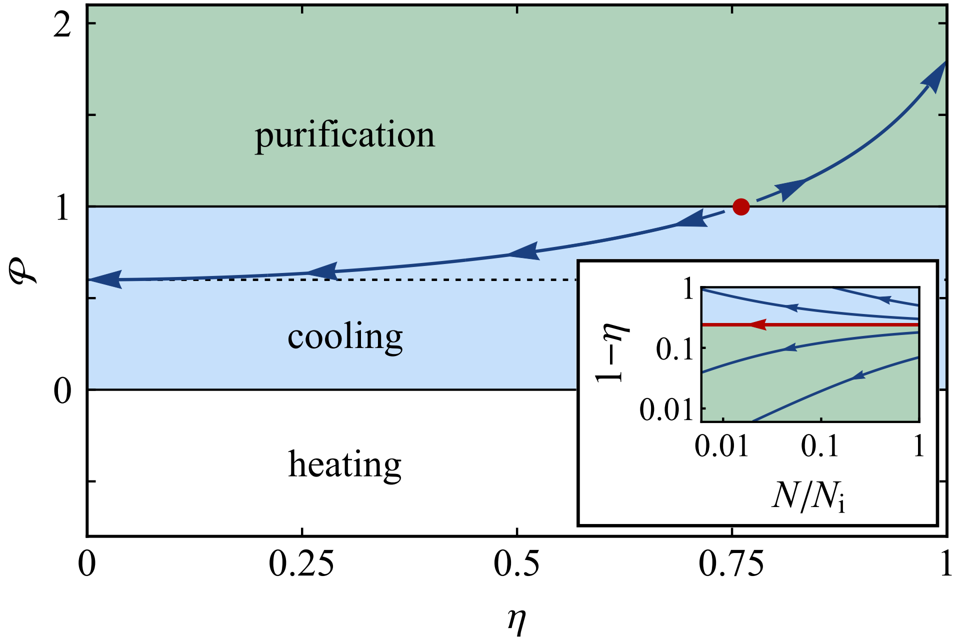
$$\Gamma_{\rm th} N_{\rm th} = K_3\left(0 + 3N_0^2 N_{\rm th} + 12N_0 N_{\rm th}^2 + 6N_{\rm th}^3\right)/V^2$$

$$\Gamma N = \Gamma_0 N_0 + \Gamma_{\rm th} N$$

condensed fraction

$$\eta = \frac{N_0}{N} = \frac{n_0}{n}$$

$$\mathcal{P} = \frac{9}{5}\,\frac{2-\eta^2}{6-9\eta^2+4\eta^3}$$



L. H. Dogra *et al.*, PRL **123**, 020405 (2019)

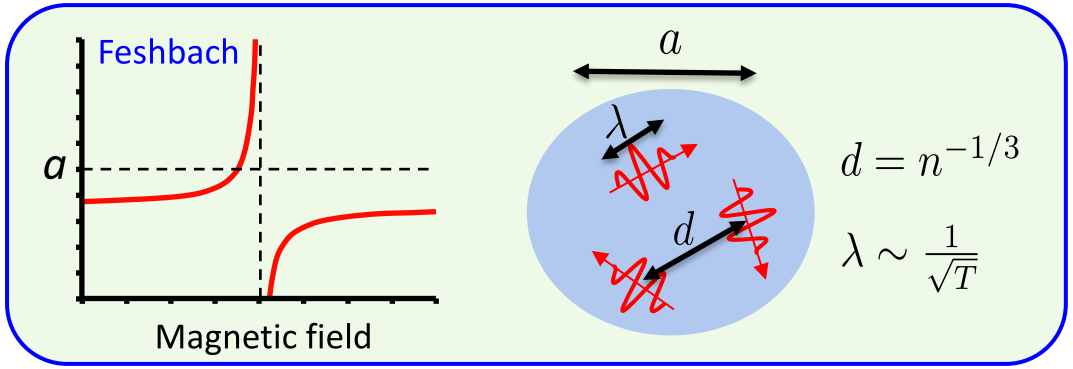
(more complicated with mean-field interactions – see paper)

Part 2.2:
Strong interactions + non-equilibrium (quench + losses)

Dynamics of a Bose gas quenched to “unitarity”

Interactions as strong as possible (allowed by quantum mechanics)

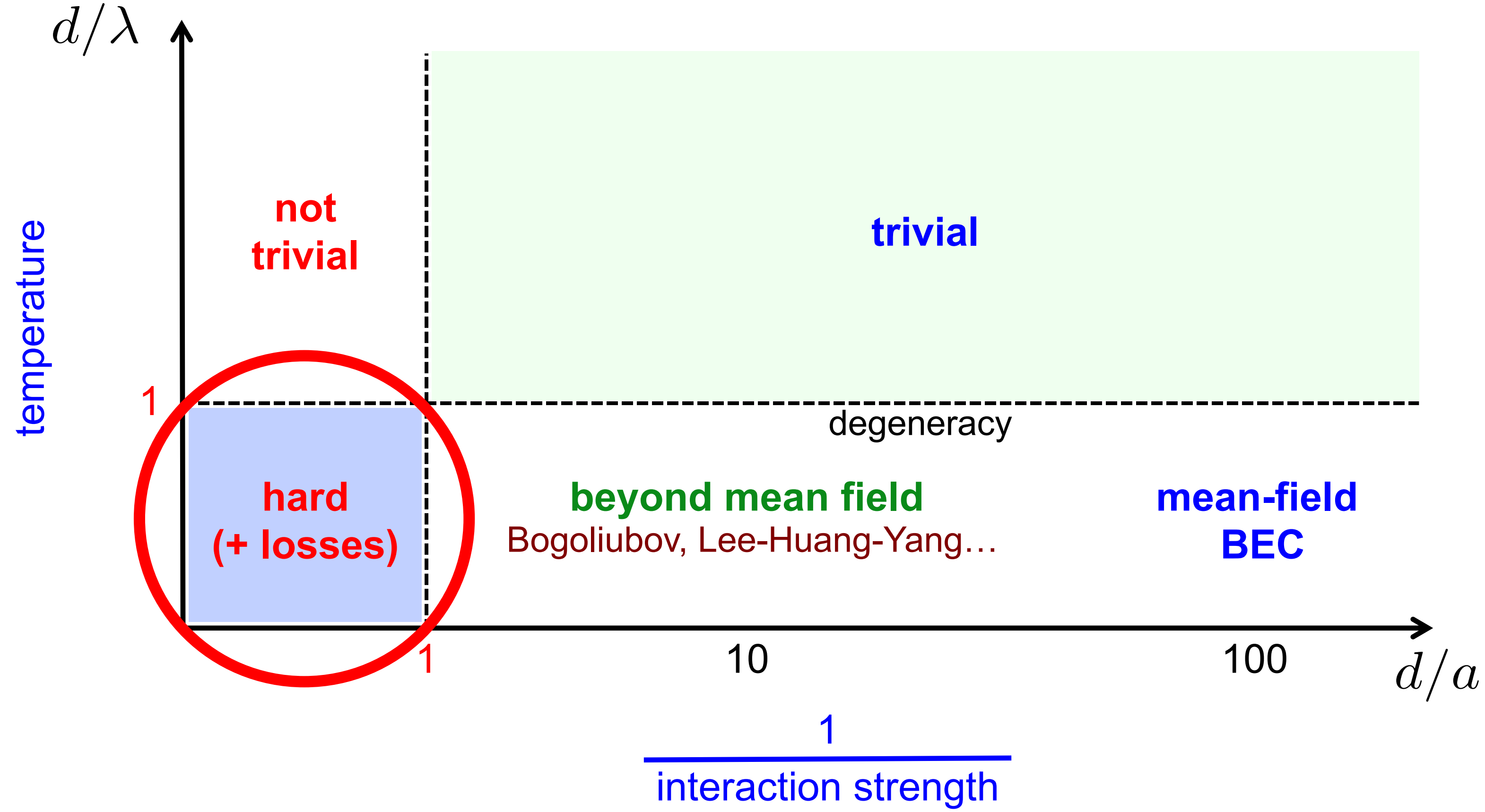
Atomic Bose gases



Relevant lengthscales: d, ~~λ~~, ~~a~~

Unitarity: $a \to \infty$

Experiments:
ENS, Cambridge, JILA, Chicago...



Part 2.2.1: Universal degenerate unitary Bose gas

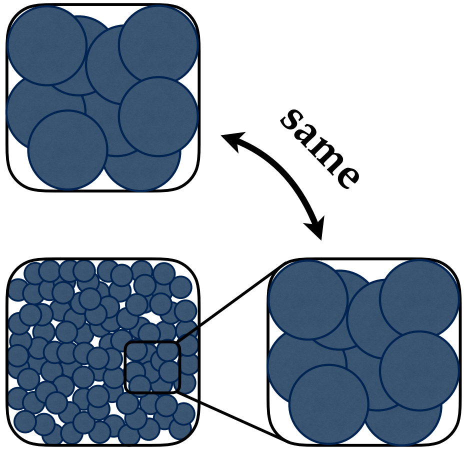
Only one lengthscale
(not really true… see later)

$$k_n = (6\pi^2 n)^{1/3} \qquad E_n = \frac{\hbar^2}{2m} k_n^2 \qquad t_n = \frac{\hbar}{E_n}$$

("Fermi" momentum, energy, time)

Expect: Self-similar gases of different densities

$a \to \infty$ really means $a_{\text{eff}} \sim d = n^{-1/3}$



Universal energy per particle $E/E_n \sim 1$,
condensed/superfluid fraction
…

Problem: Does this state even exist?

Universality arguments also for *m*-body losses

$\dot{N}/N \sim -1/t_n$ "lifetime" $\sim \hbar/E_n$ "$Q \sim 1$"

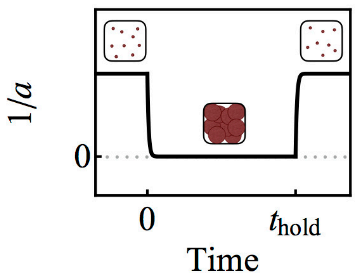
Well defined equilibrium properties?

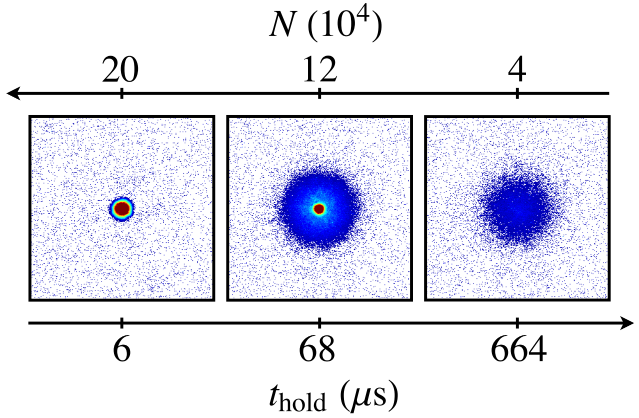
Universality in decay (and heating) dynamics

Quench

$a \to \infty$

and back





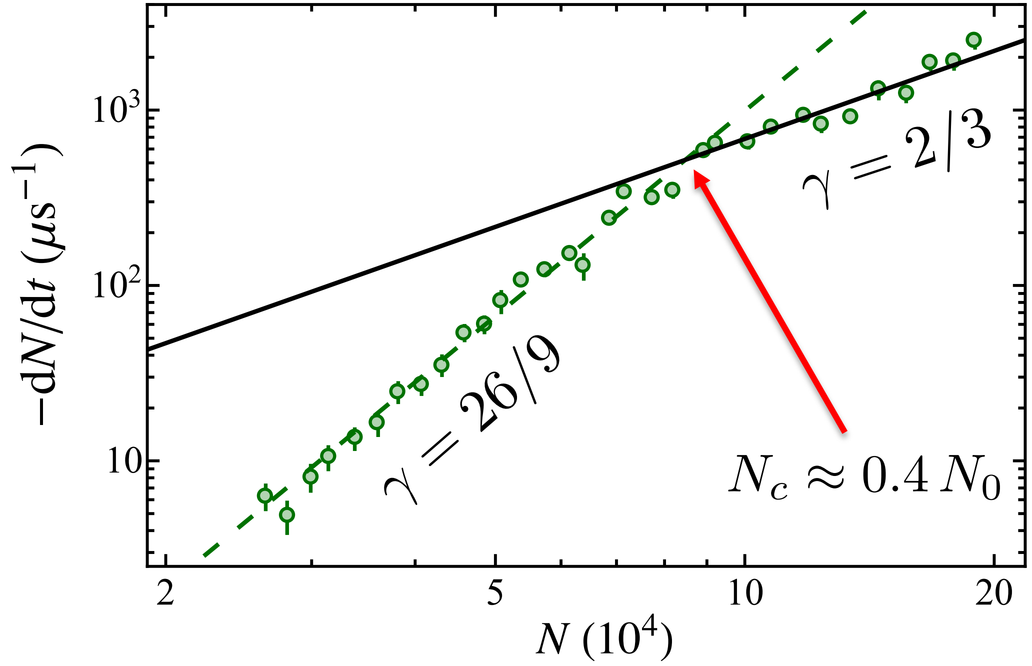
Universality hypothesis:

$$\dot{N}/N \propto -N^{\gamma} \qquad \gamma = 2/3$$

(in a box trap)

Crossover

degenerate ➔ thermal gas



C. Eigen, J.A.P. Glidden, R. Lopes, N. Navon, ZH, R.P. Smith, PRL 2017

2/3 also @ JILA: Klauss *et al.*, PRL 2017

Thermal-gas theory @ ENS: Rem *et al.*, PRL 2013

An aside: 26/9?

$$\dot{N}/N \propto -N^{\gamma}$$

$$\dot{N}/N \propto -N^2 a^4 \qquad \rightarrow \qquad \dot{N}/N \propto -N^2 \lambda^4 \propto -N^2/T^2$$

Kinetic-energy heating (ENS theory):

$$\mathrm{d}T/T = -(4/9)\,\mathrm{d}N/N \qquad \rightarrow \qquad T \propto N^{-4/9}$$

$$\gamma = 2 + 2 \times 4/9 = 26/9$$

"Universal crossover"

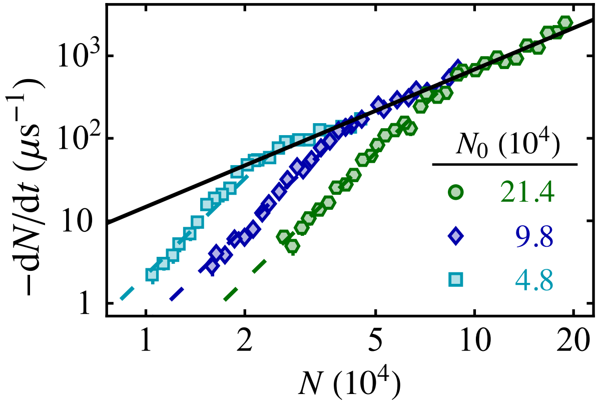
degenerate ➔ thermal gas

C. Eigen, J.A.P. Glidden,
R. Lopes, N. Navon, ZH,
R.P. Smith, PRL 2017

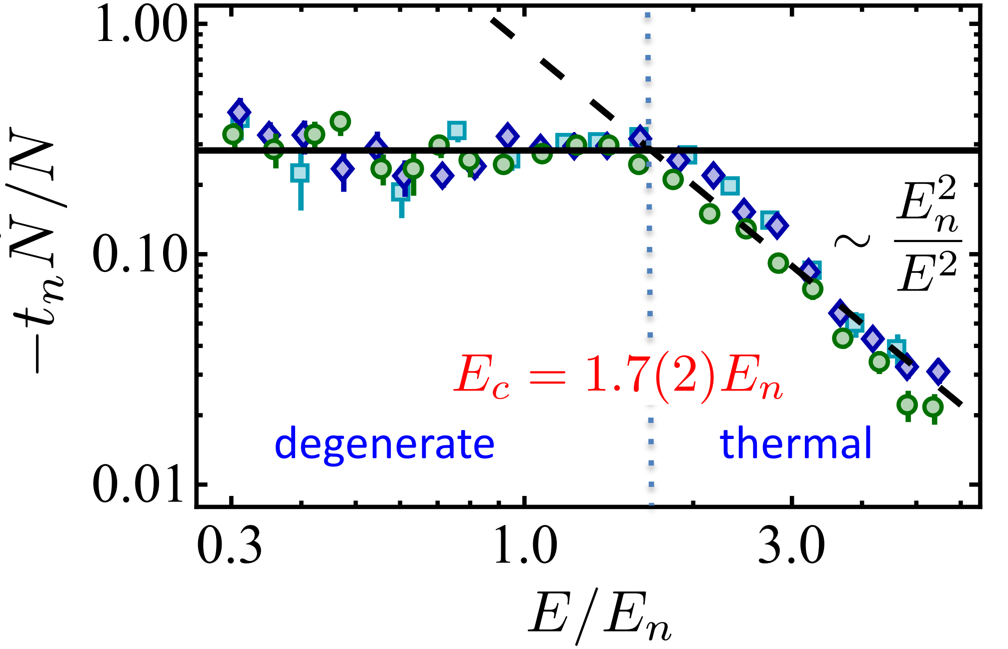
Can collapse all loss curves

Different initial densities:

Always $N_c = 0.43(4) N_0$

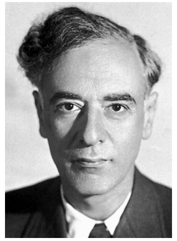


Dimensionless $t_n \times \dot{N}/N = f(E/E_n)$



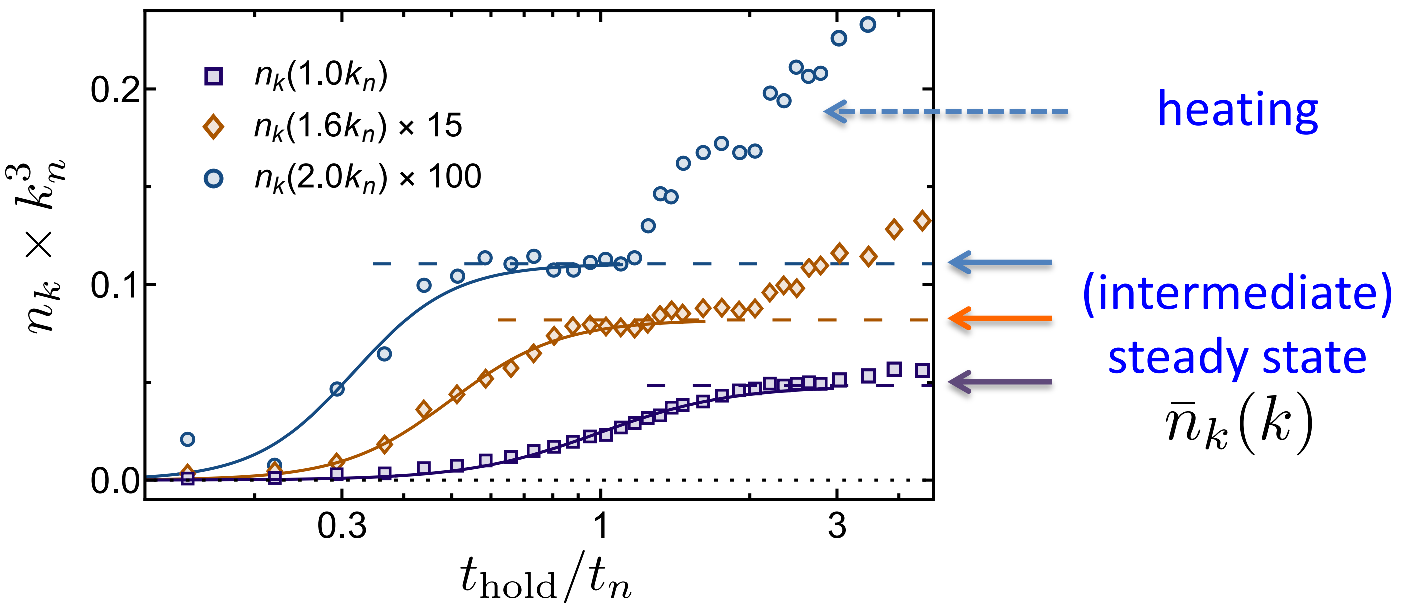
Crossover time: $t_c = 4.0(4)\, t_{n_0}$ ☺

"$Q \sim 4$"



Momentum- and time-resolved dynamics

$n(\text{fixed } k, t)$



Get: "equilibration time" $\tau(k)$

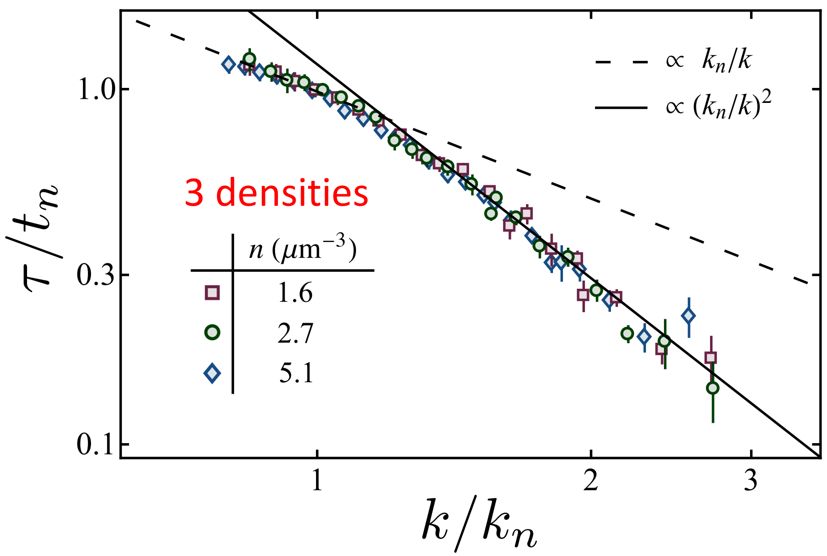
"steady state" $\bar{n}_k(k)$

C. Eigen, J.A.P. Glidden, R. Lopes, E.A. Cornell, R.P. Smith, and ZH, Nature **563**, 221 (2018)

Universal “prethermal steady state”

C. Eigen *et al.*,
Nature **563**, 221 (2018)

(Half-)Time to steady state



Consistent with “prethermalization”

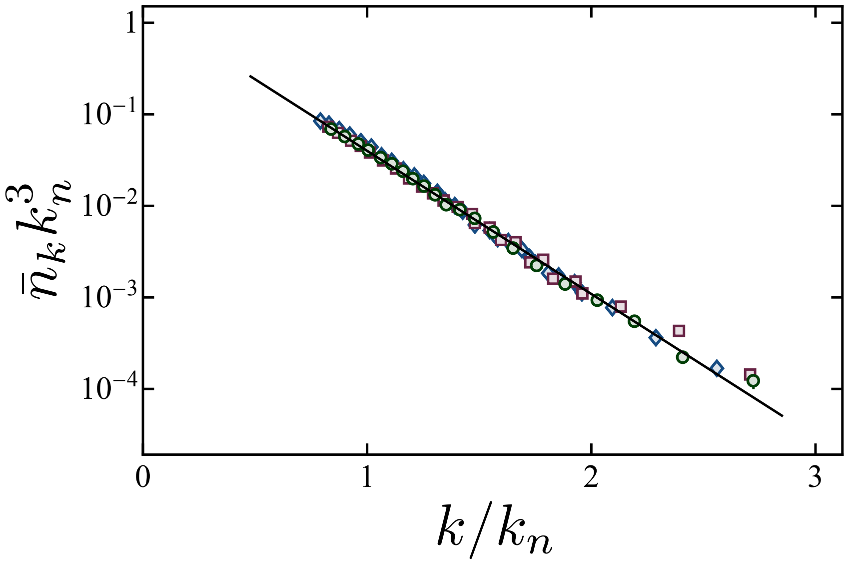
Bogoliubov-like excitations

$$\mu \sim E_n$$

w/ $\quad v_s \sim \hbar k_n/m$

$$\tau \sim 1/\omega$$

Steady-state momentum distribution



Exponential?

Energy $\quad 0.7E_n$

Condensed fraction 20% (not 0%)

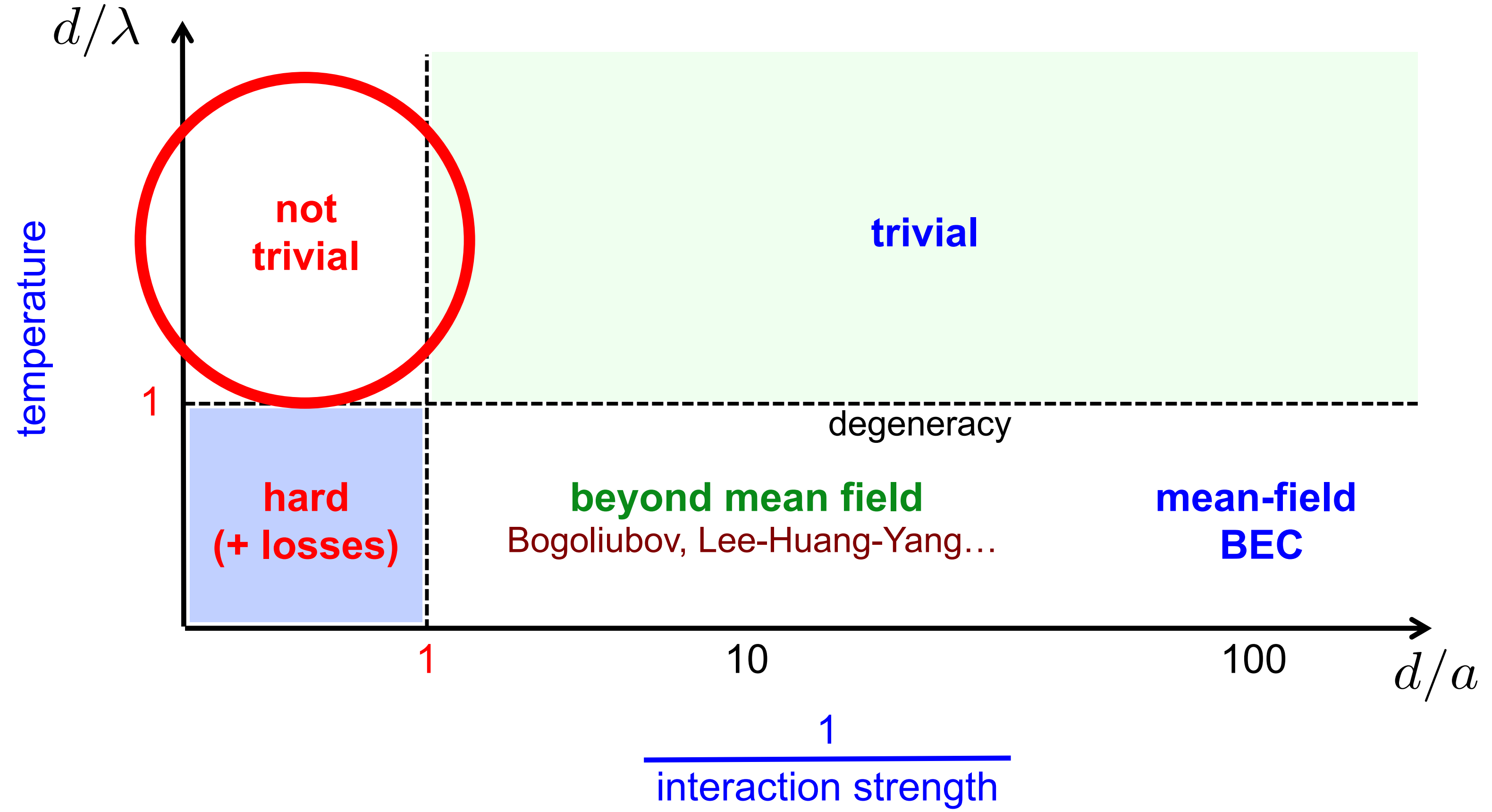
Part 2.2.2: "Universality" in a thermal unitary gas?

Coherent 2-body processes slower than in a degenerate gas

3-body loss/heating ***much*** slower

W. Li and T.-L. Ho, PRL 2012

nice timescale separation



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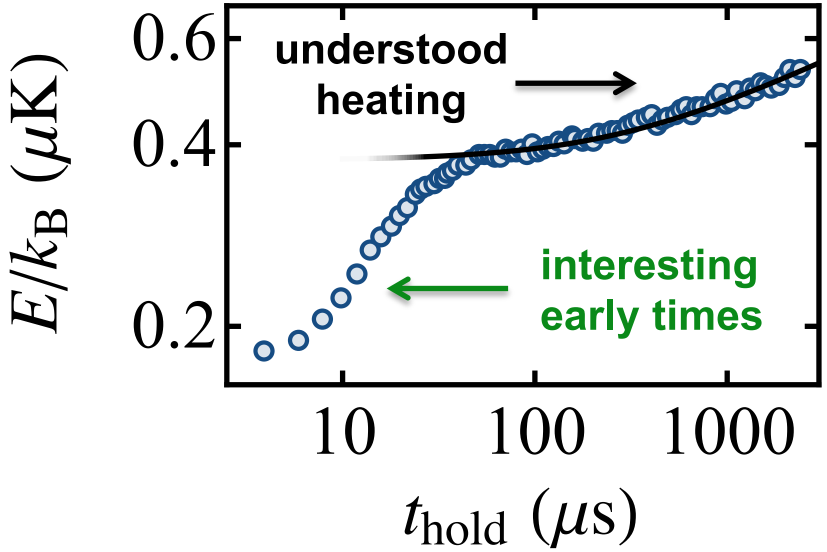
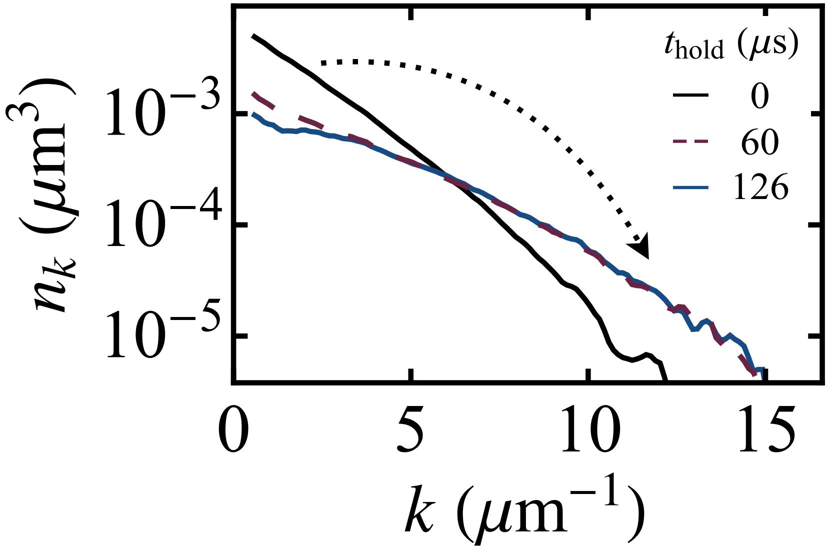
Energy



Momentum (re-)distribution

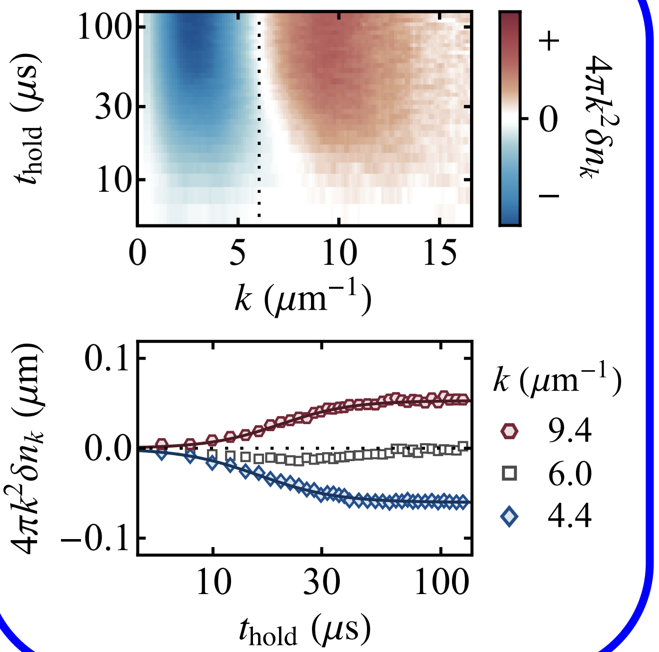


BUT… two lengthscales:

$$d = n^{-1/3}, \qquad \lambda \sim 1/\sqrt{T}$$

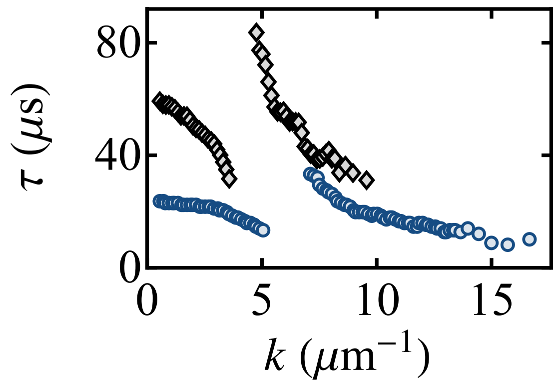
Momentum-space redistribution at early times

Some n and T

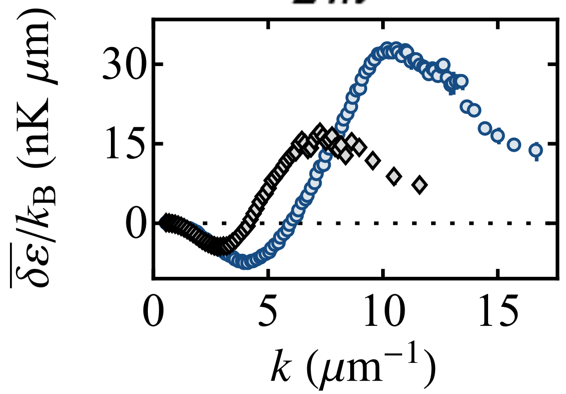


Two different n and T

characteristic times



$$\overline{\delta\varepsilon} = \frac{\hbar^2 k^2}{2\,m} 4\pi k^2 \overline{\delta n_k}$$



Are these curves "universal"?

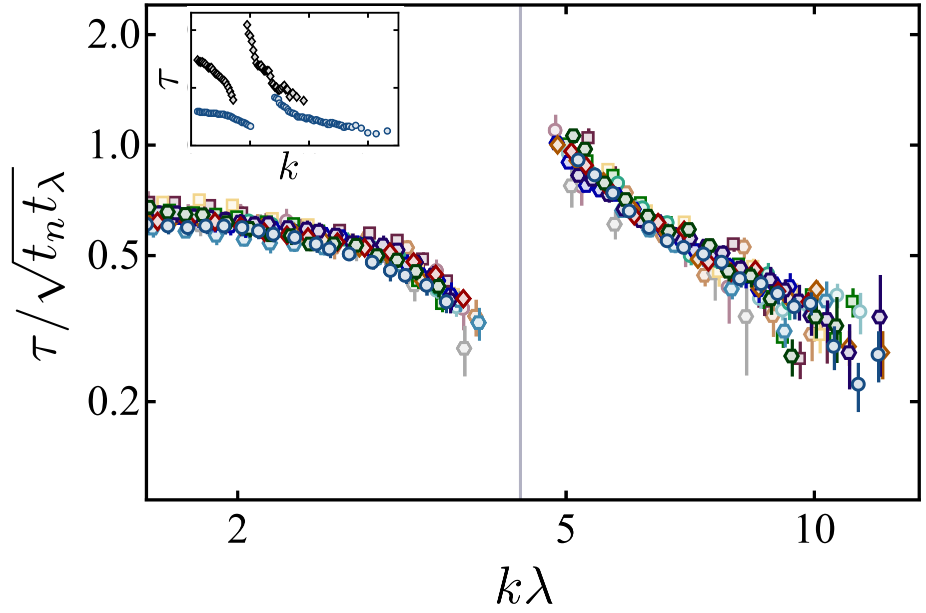
infinitely many scale candidates:

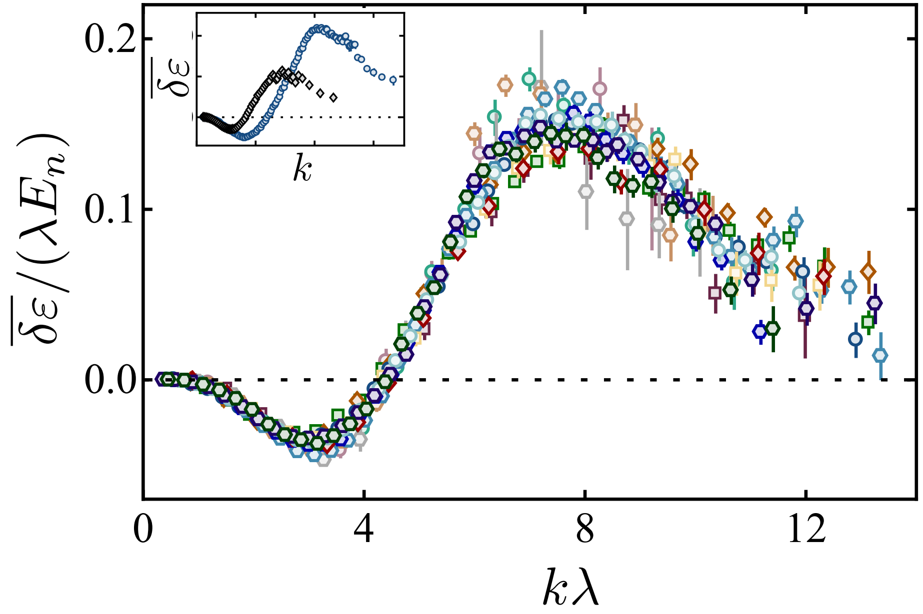
$$k_n = (6\pi^2 n)^{1/3} \qquad E_n = \frac{\hbar^2}{2m} k_n^2 \qquad t_n = \frac{\hbar}{E_n}$$

$$k_\lambda = 1/\lambda \qquad E_\lambda = k_{\mathrm{B}} T \qquad t_\lambda = \frac{\hbar}{k_{\mathrm{B}} T}$$

Unitary Thermal Gas Scaling Laws

n_i (μm^{-3})	0.7	1.1	1.2	1.2	1.7	2.0	2.2	2.7	3.1	3.8	4.2	5.1	5.2	5.6	6.8
T_i (nK)	200	160	100	70	80	190	100	170	100	190	150	190	190	150	200





momentum	just $k_\lambda \sim 1/\lambda \sim \sqrt{T}$	kind of obvious
time	$\sqrt{t_n t_\lambda} \sim \hbar/\sqrt{E_n T}$	makes some sense
energy	just E_n	weird

More on unitary Bose gases…
(not in this course)

It's not actually all universal…

Efimov 1970: Borromean 3-body bound states

First observation: enhanced 3-body loss [Kraemer *et al*., Nature 2006]
Coherent effects (including dynamics): "3-body clock shift" [Fletcher *et al*., Science 2017]
Efimov molecules: [Klauss *et al*., PRL 2017]