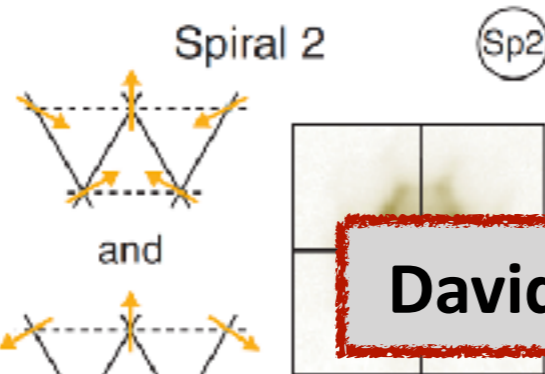
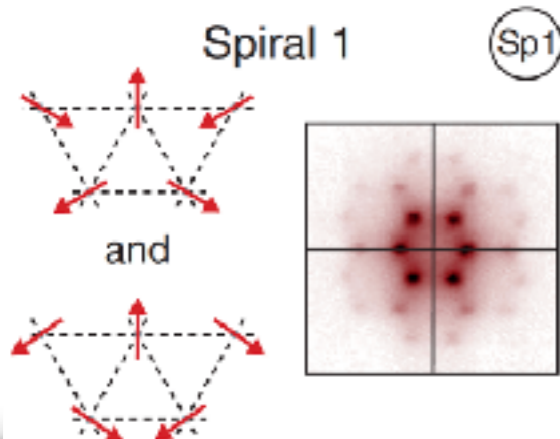
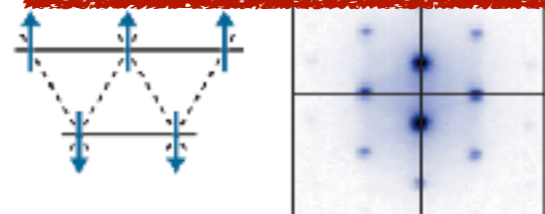
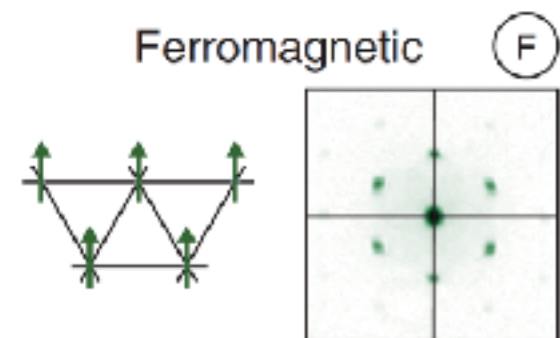
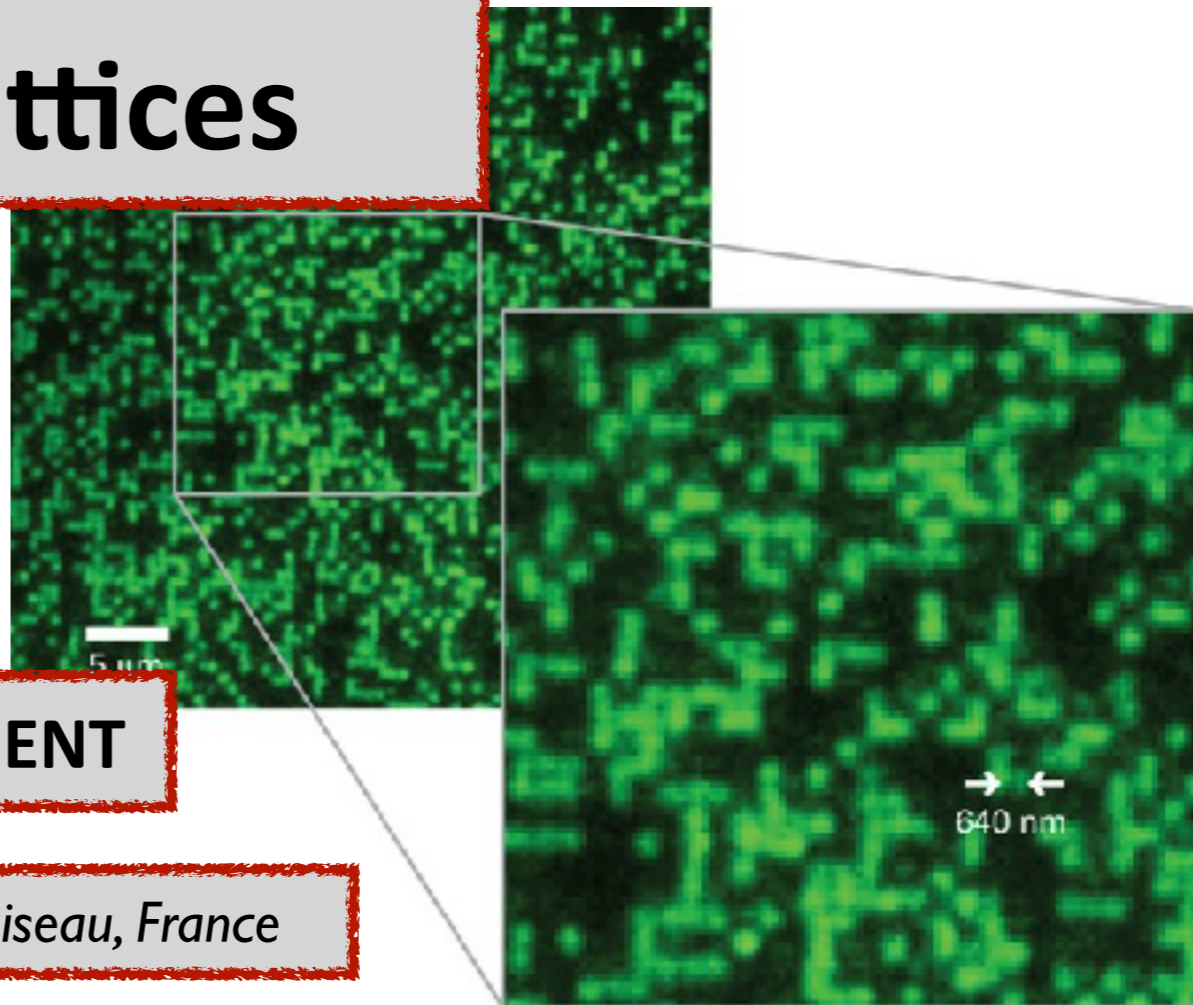


Quantum gases in Optical Lattices



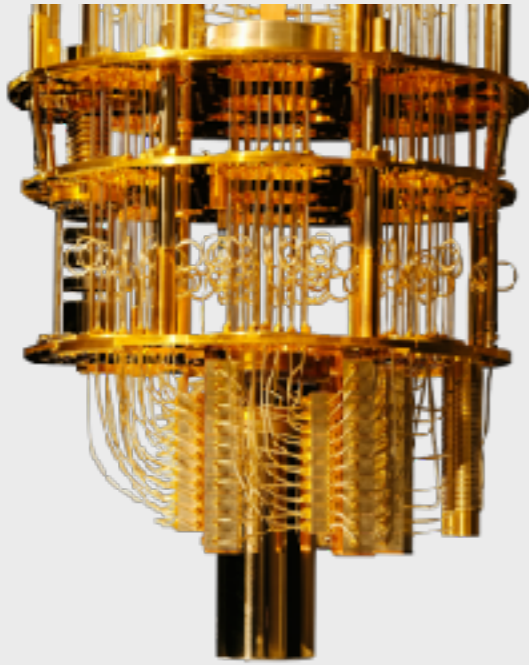
David CLEMENT

Institut d'Optique - Palaiseau, France

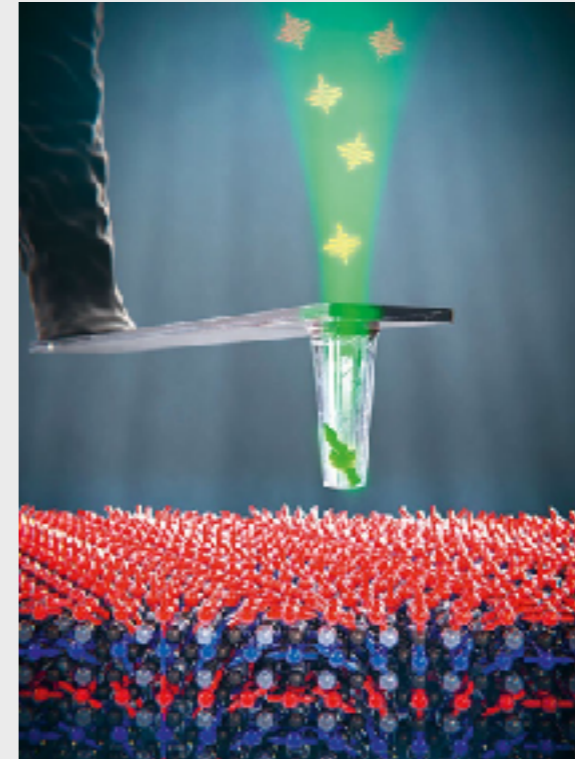


Quantum technologies - 4 pillars

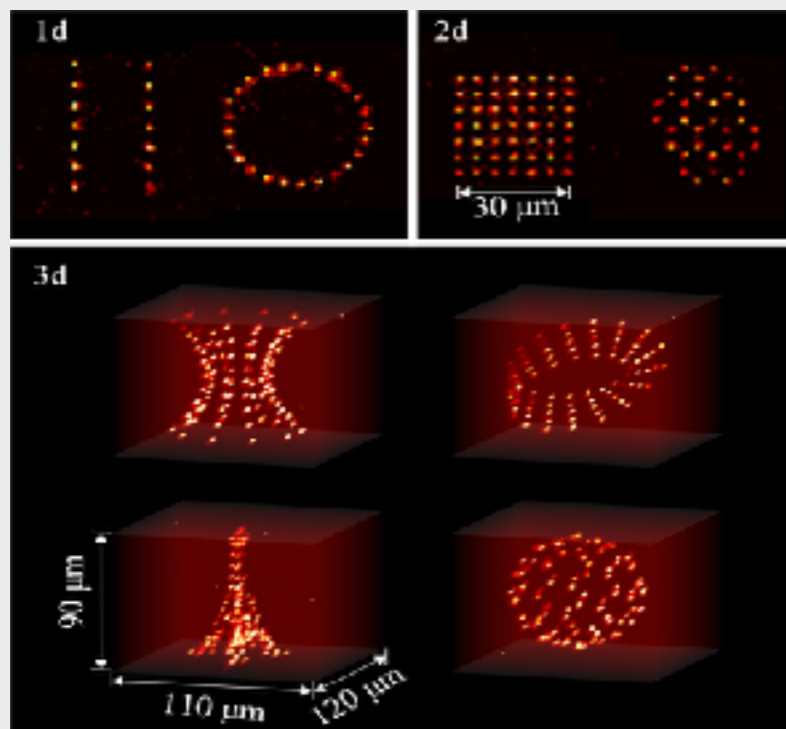
Quantum computation



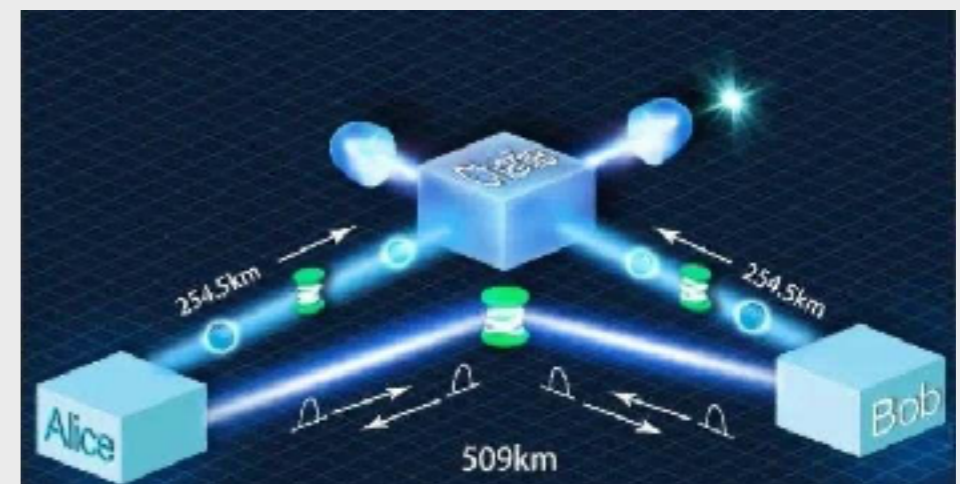
Quantum sensing



Quantum simulation

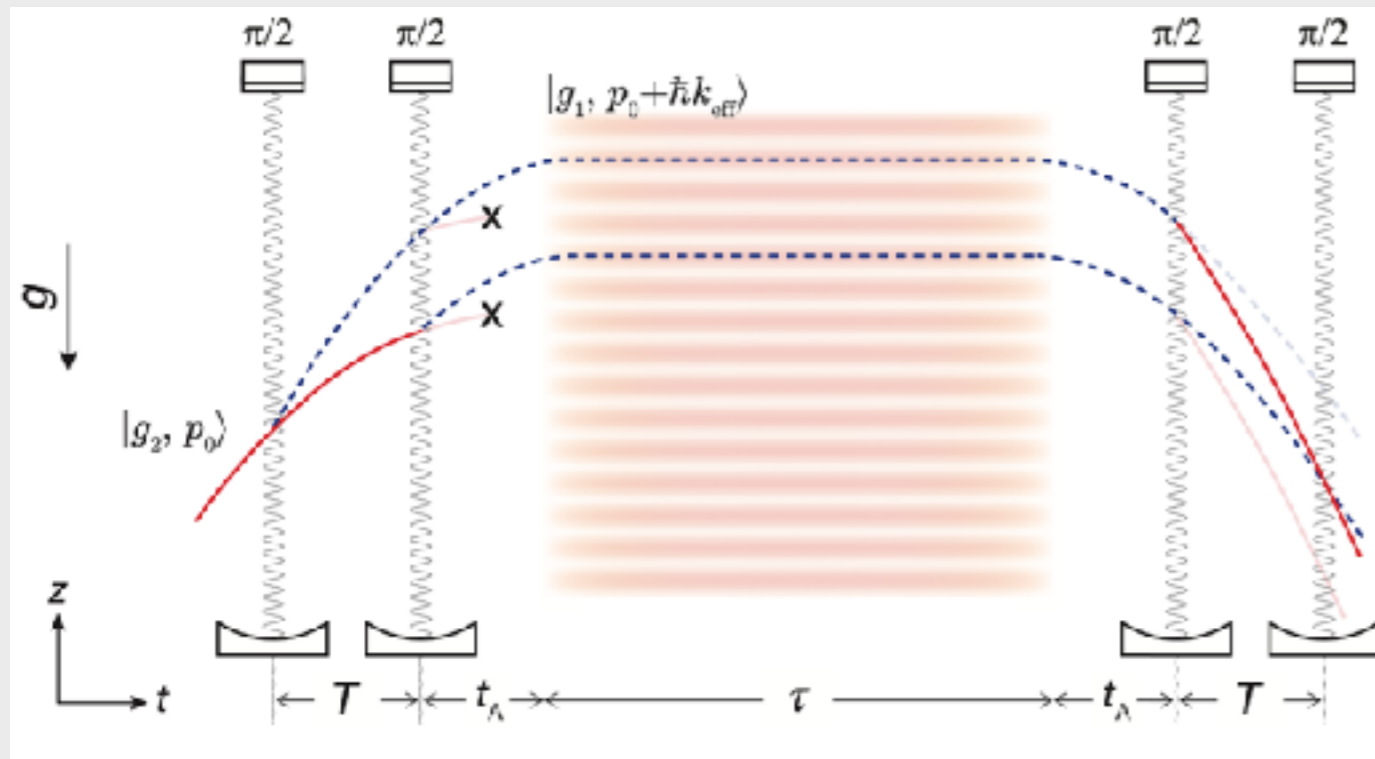


Quantum communication



Quantum sensing - metrology

Atom interferometers



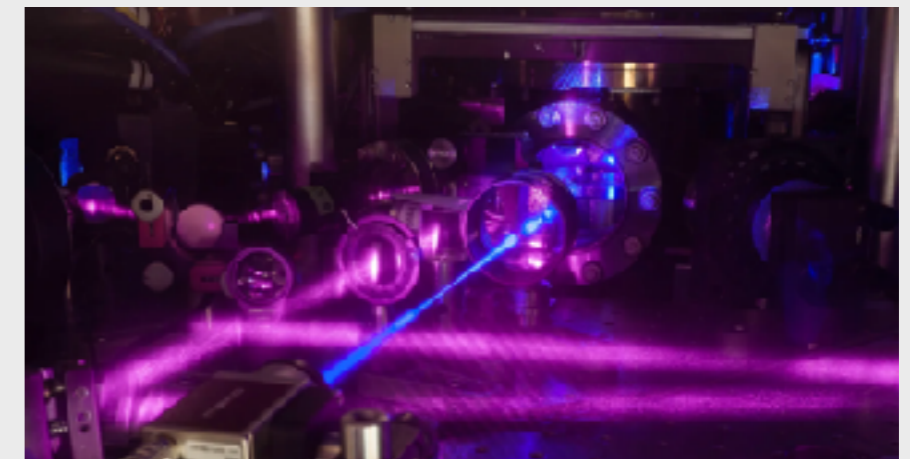
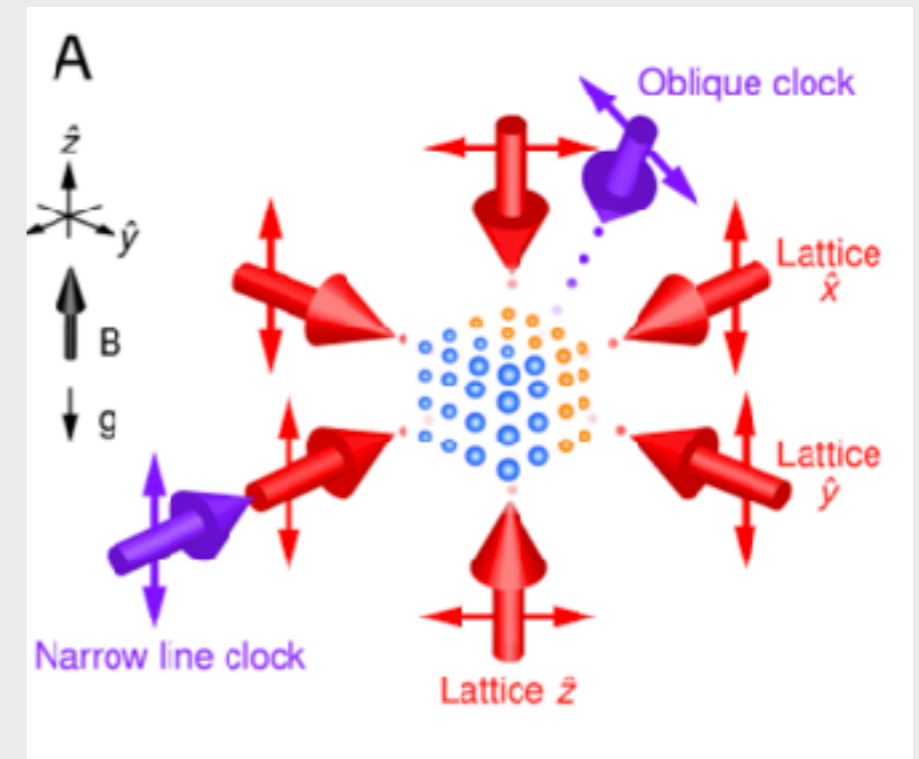
Probing gravity by holding atoms for 20s
Science **366**, 745 (2019)

Optical lattices to manipulate momentum states:
beamsplitters and mirrors

(lecture #1)

...also Bloch oscillations to measure forces

Optical clocks in lattices



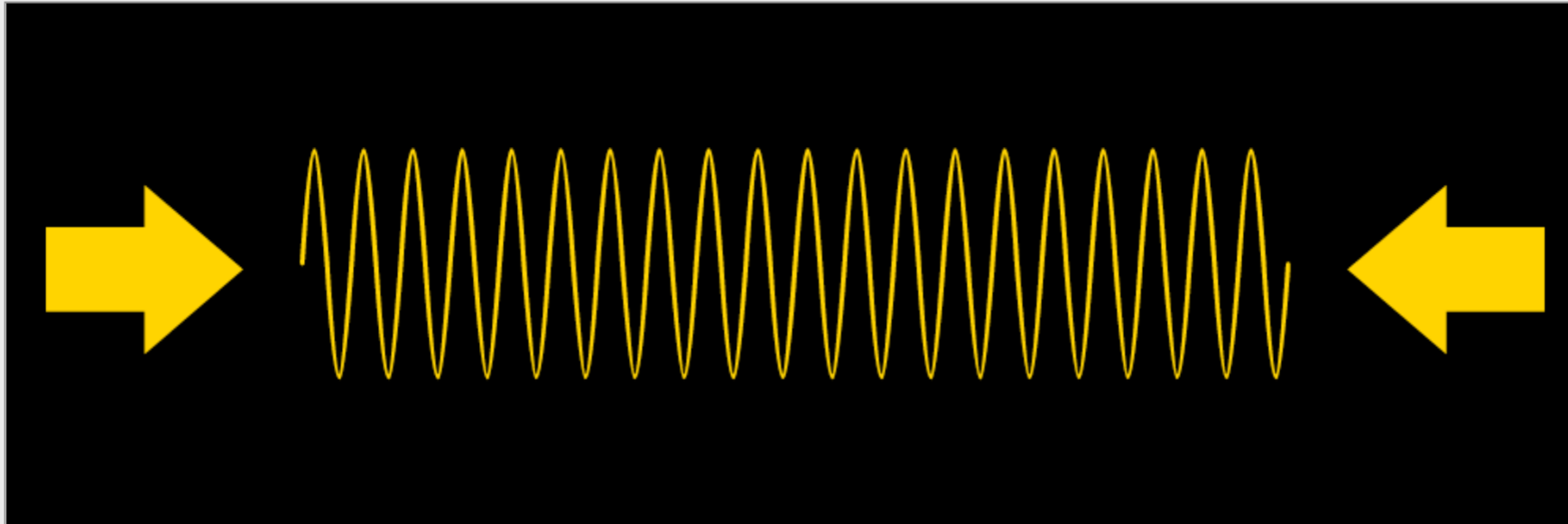
Precision $<10^{-18}$ over 1 hour
Campbell et al. Science **358**, 90 (2017)

Atoms in deep optical lattices

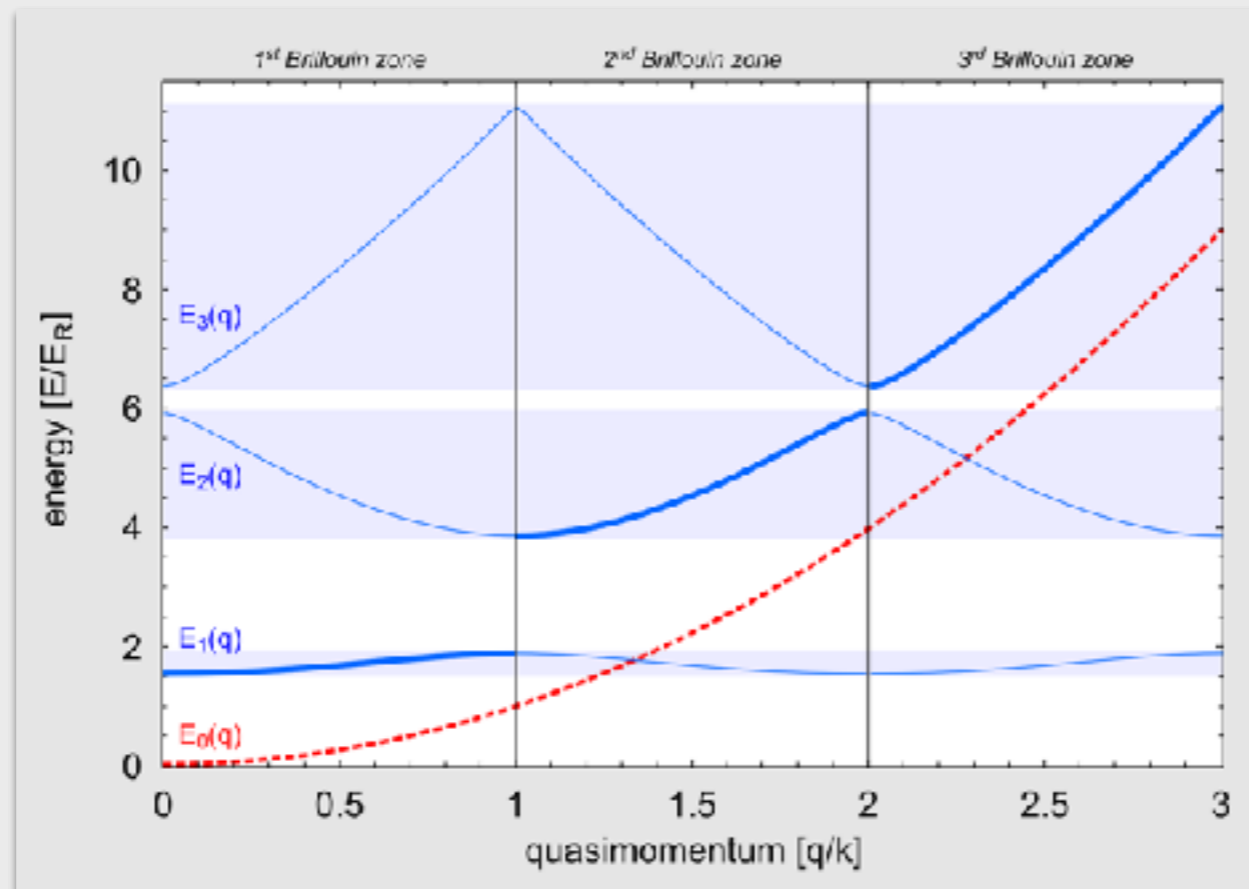
(lecture #3)

Quantum simulation

Realisation cristalline structures to simulate correlated matter



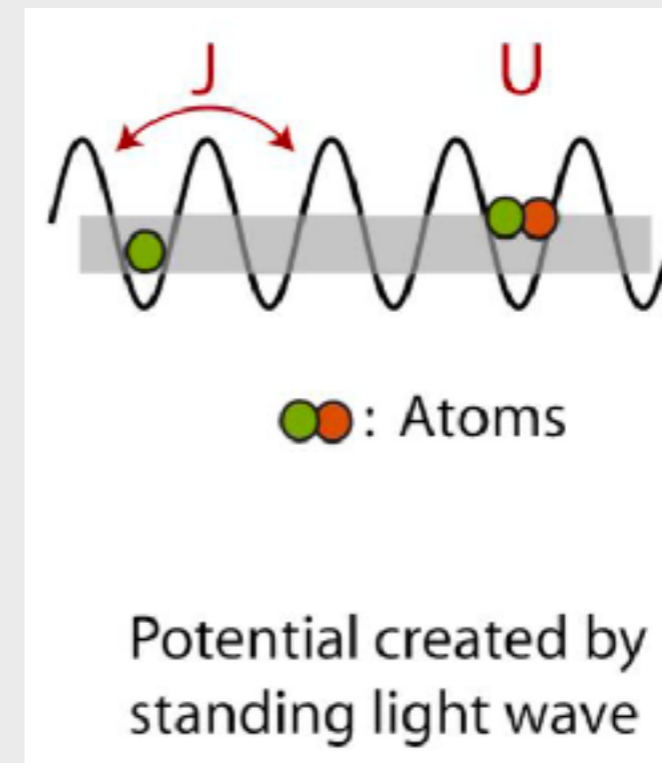
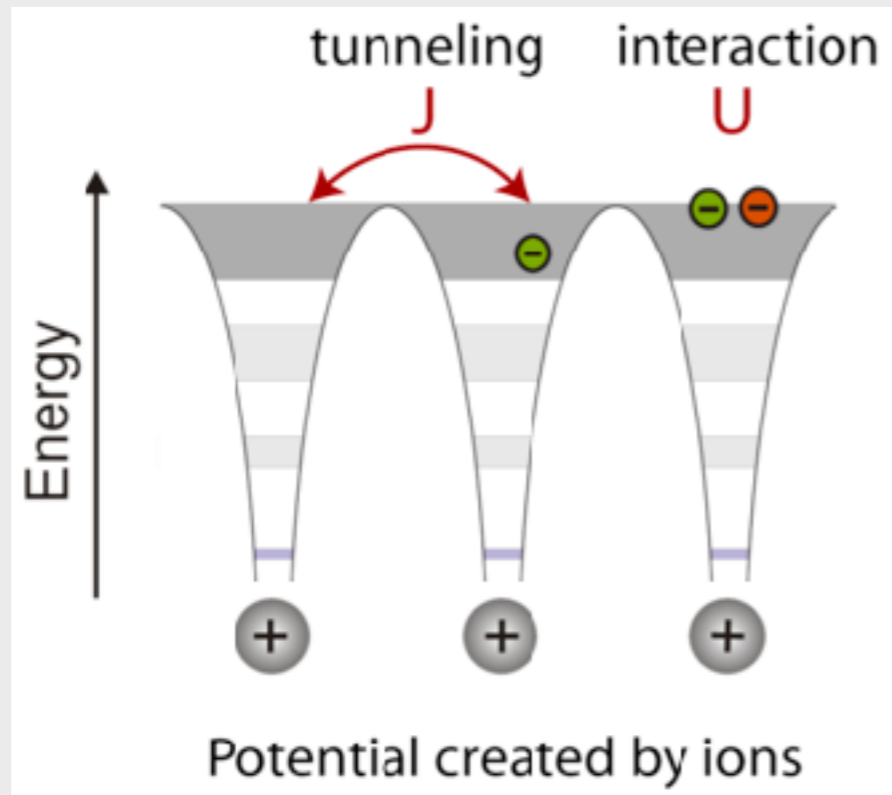
Optical lattices to create band structures as in solids *(lecture #2)*



Quantum simulation

Realisation cristalline structures to simulate correlated matter

Implementation of Hubbard hamiltonians ([lectures #3 et #4](#))



Understand and design strongly-correlated quantum matter

- interplay quantum fluctuations/interactions
- quantum phase transitions
- quantum magnetism
- ...

Lectures on optical lattice: *(tentative)* program

Lecture #1 - introduction to optical lattices

- short reminder on light shifts
- description of optical lattices
- pulsed lattices: coherent coupling of momentum states

Lecture #2 - band structure for non-interacting gases

- Bloch theorem and wave-functions
- band structure
- non-interacting lattice bosons and fermions

Lecture #3 - interacting bosons

- tight-binding regime
- Bose-Hubbard hamiltonian
- superfluid-to-Mott transition

Lecture #4 - interacting fermions

- Fermi-Hubbard hamiltonian
- Mott transition
- super-exchange and magnetism

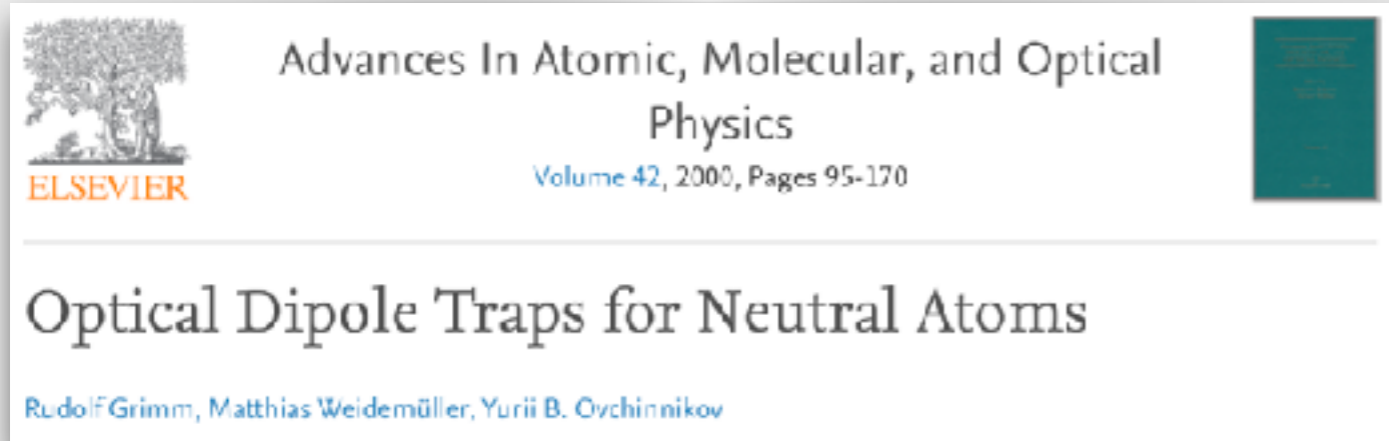
Quantum gases in Optical Lattices

Lecture #1

David CLEMENT

Institut d'Optique - Palaiseau, France

Short reminder on light atom interactions

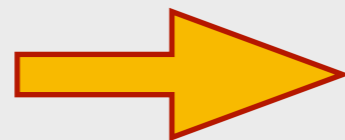


$$U_{\text{dip}}(\mathbf{r}) = -\frac{3\pi c^2}{2\omega_0^3} \left(\frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(\mathbf{r}),$$

$$\Gamma_{\text{sc}}(\mathbf{r}) = \frac{3\pi c^2}{2\hbar\omega_0^3} \left(\frac{\omega}{\omega_0} \right)^3 \left(\frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right)^2 I(\mathbf{r})$$

For most practical purposes $|\Delta| \ll \omega_0$

$$\hbar\Gamma_{\text{sc}} = \frac{\Gamma}{\Delta} U_{\text{dip}}$$

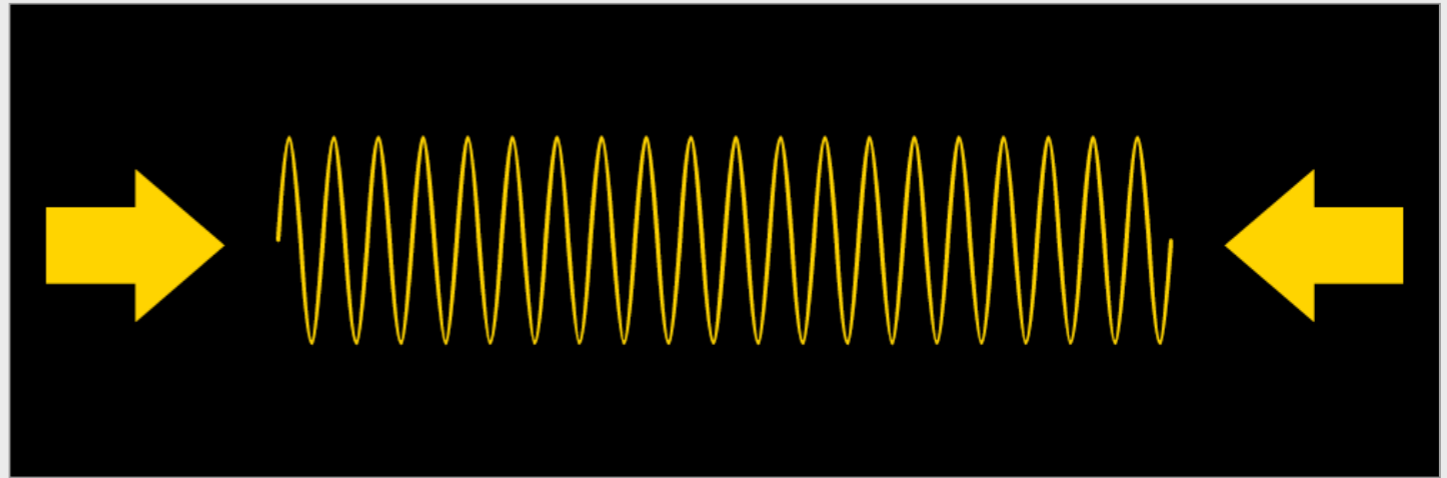


neglected spontaneous emission at large detuning!

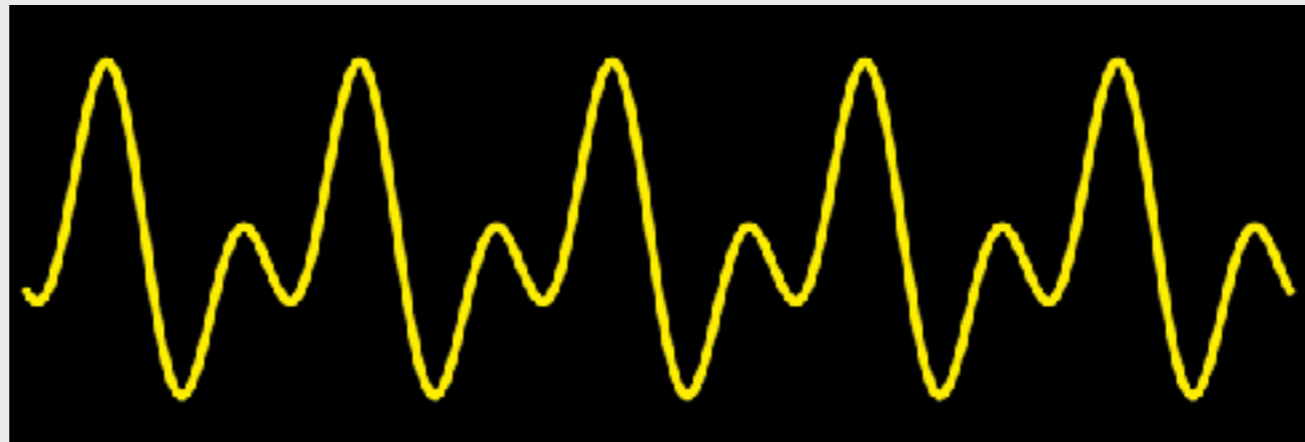
Optical lattice

Simple counter-propagating 1D lattice
(no defects, no phonons)

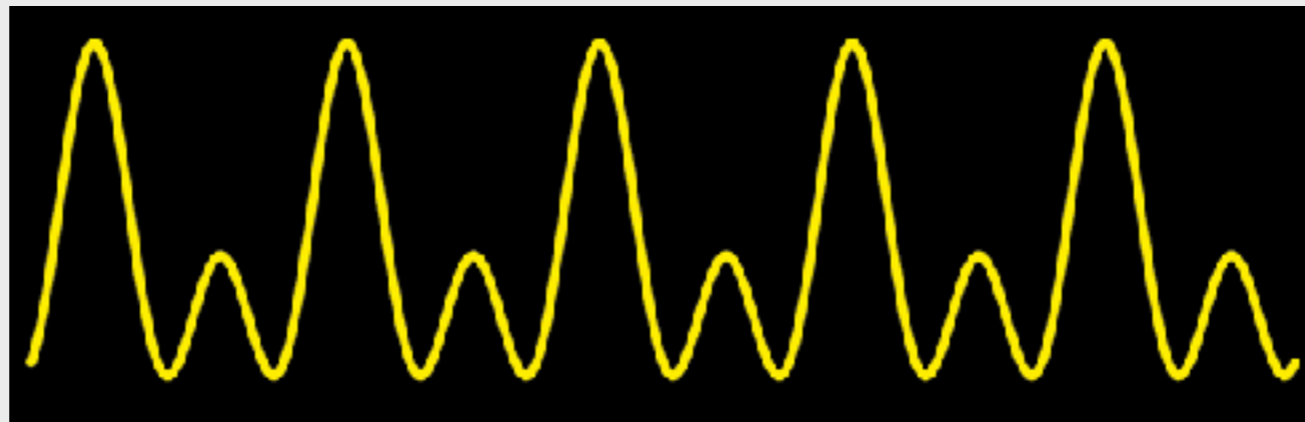
$$V(x) = V_0 \sin(kx)^2$$



Superlattices with two periods λ , 2λ

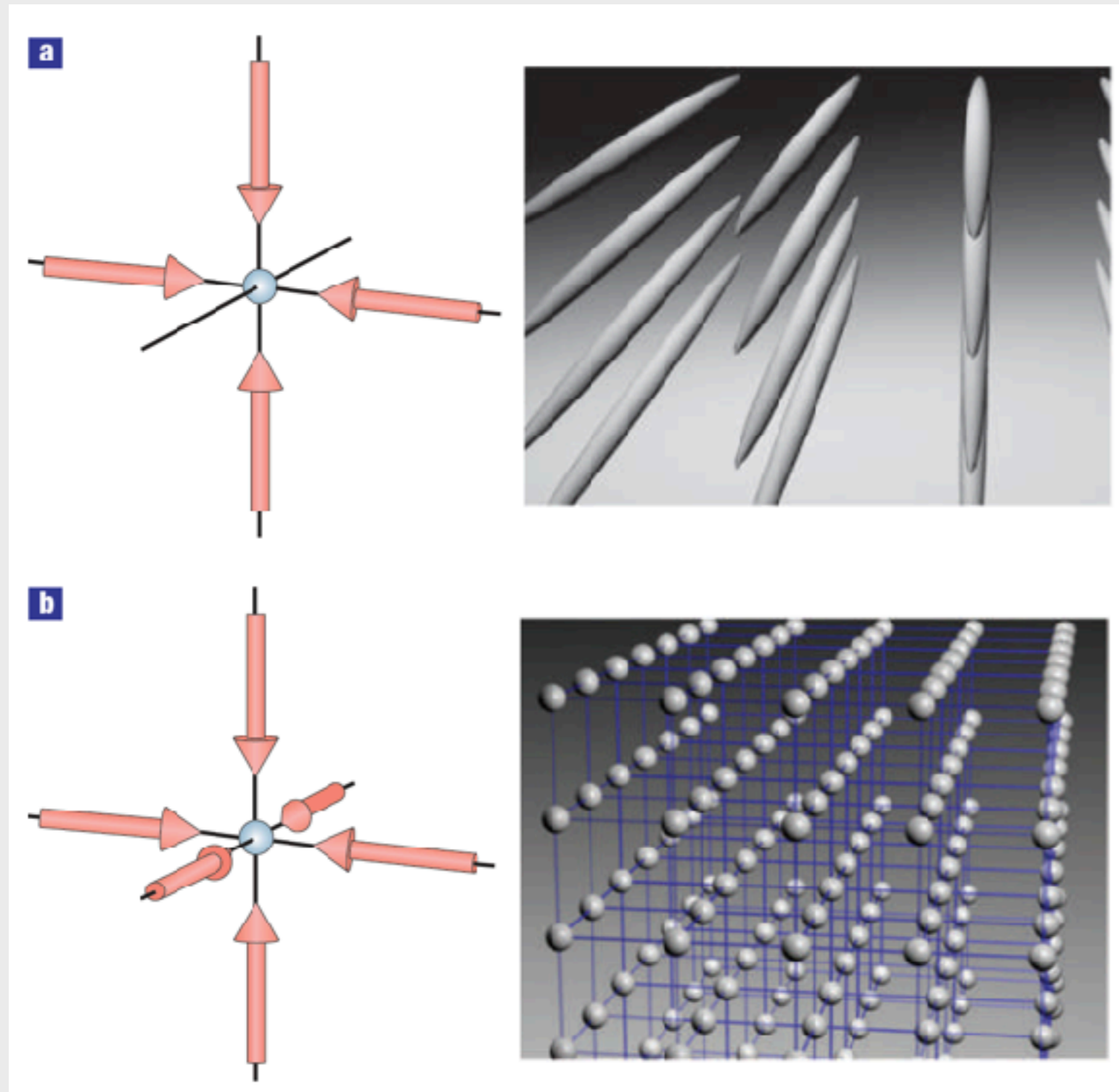


tilted double-wells



balanced double-wells

Optical lattice in higher-dimensions



Detuning of beams in the different directions to average interference effects between different axis to zero

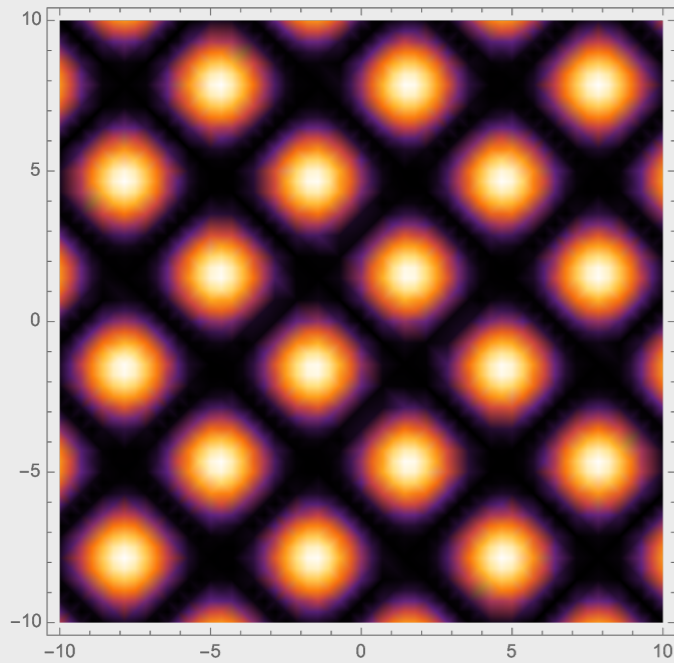
$$\omega_x - \omega_y \sim 10 - 100 \text{ MHz}$$

I. Bloch - Review Optical lattices - Nature Physics (2005)

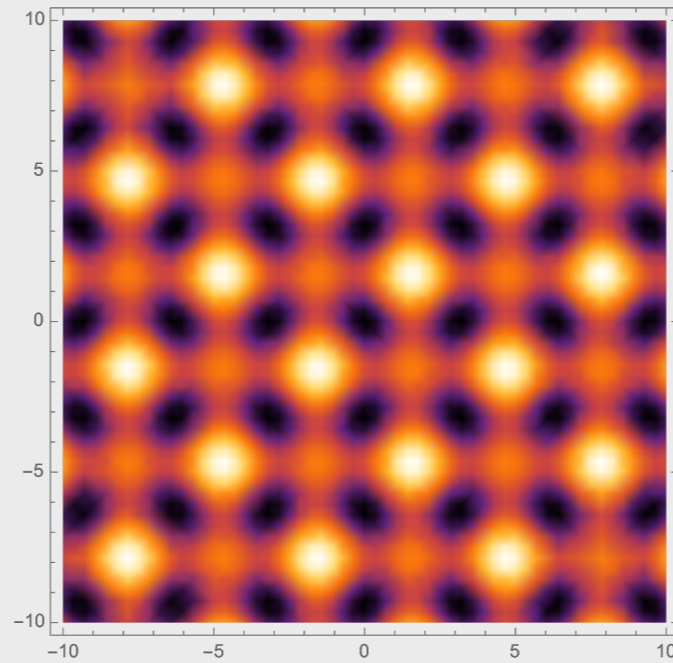
More exotic optical lattices

lattices with varying spacing, orientation and symmetry

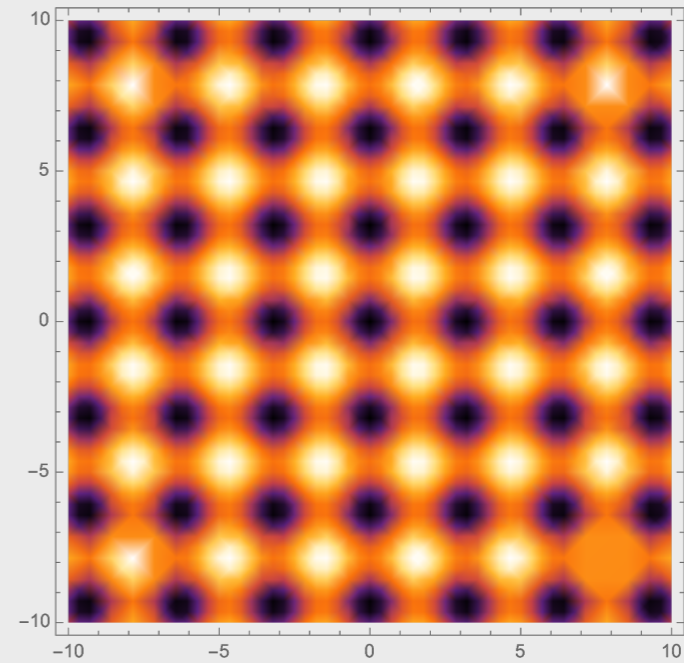
$$I(x, y) \propto [\sin(kx) + \sin(ky)e^{i\phi}]^2$$



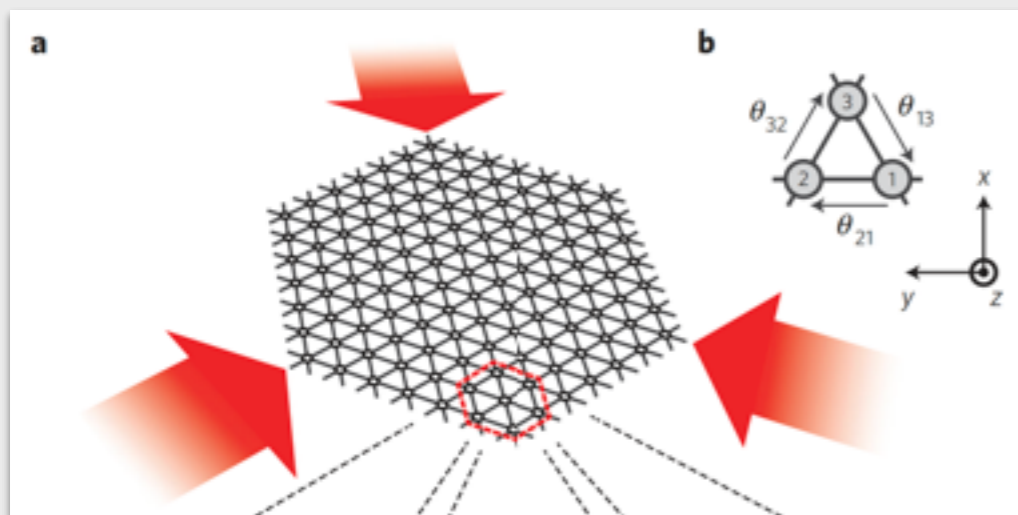
$\phi = 0$



$\phi = 2\pi/5$

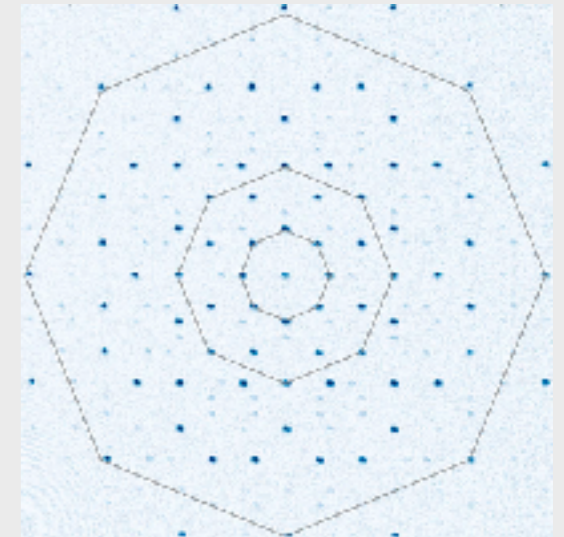
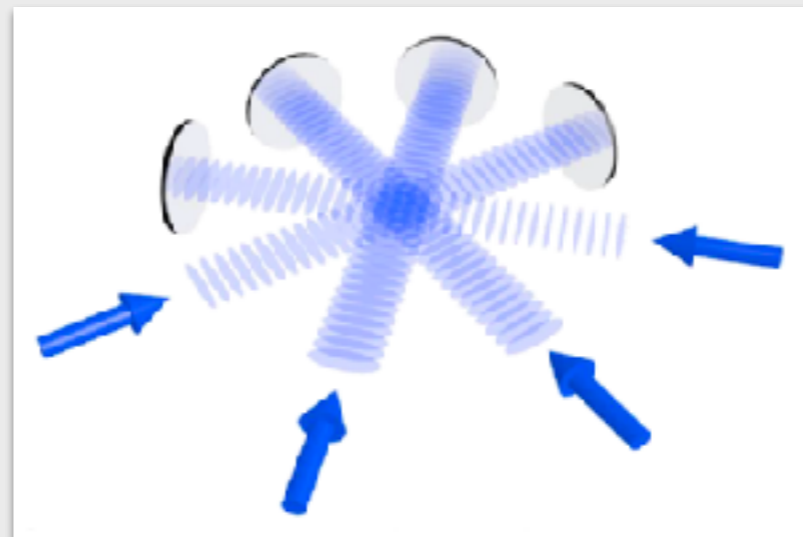


$\phi = \pi/2$



honeycomb and triangular
Struck et al., Nat. Phys. (2013)

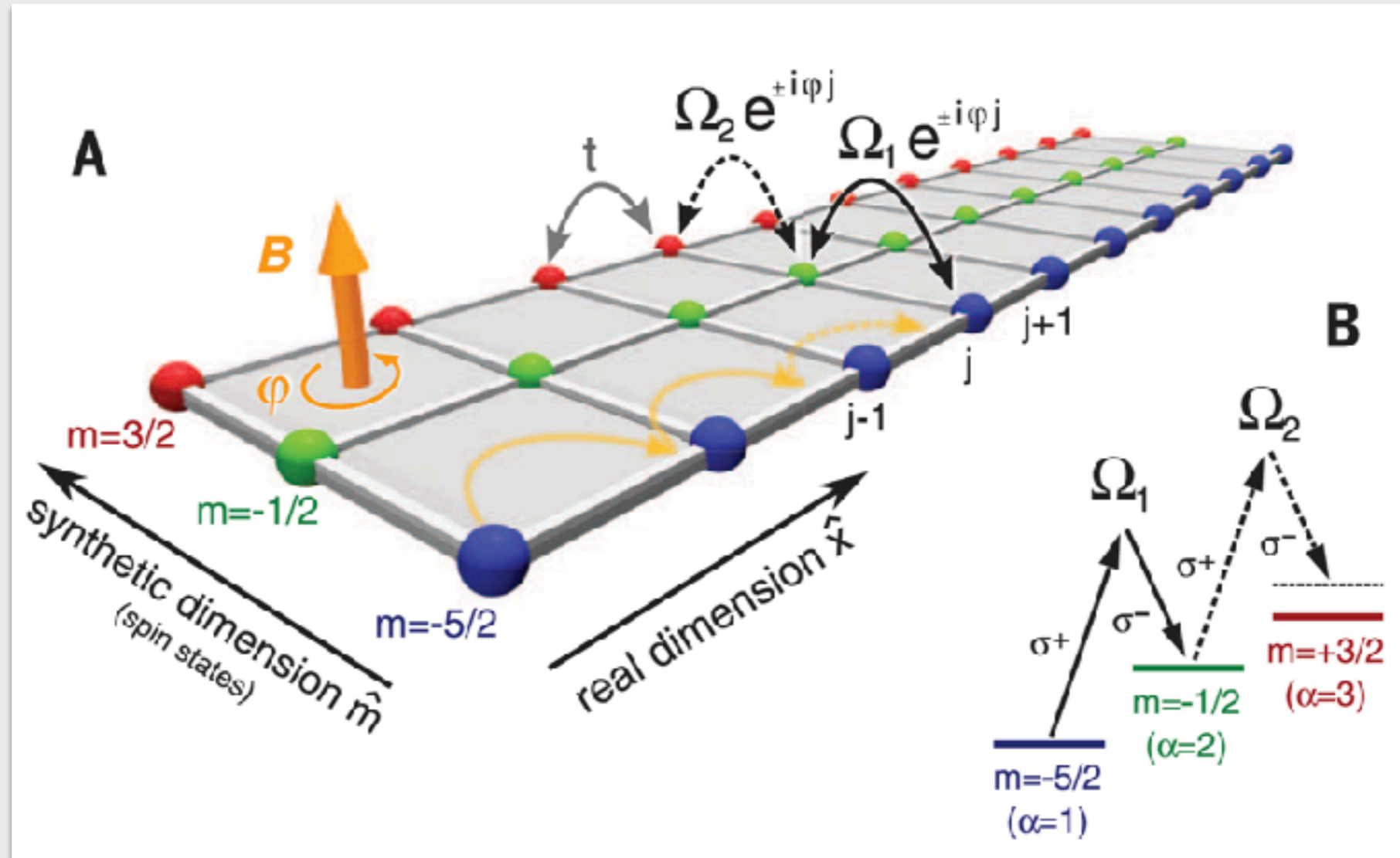
quasi-crystal (self similarity in k -space)



Viebahn et al., PRL 122, 110404 (2019)

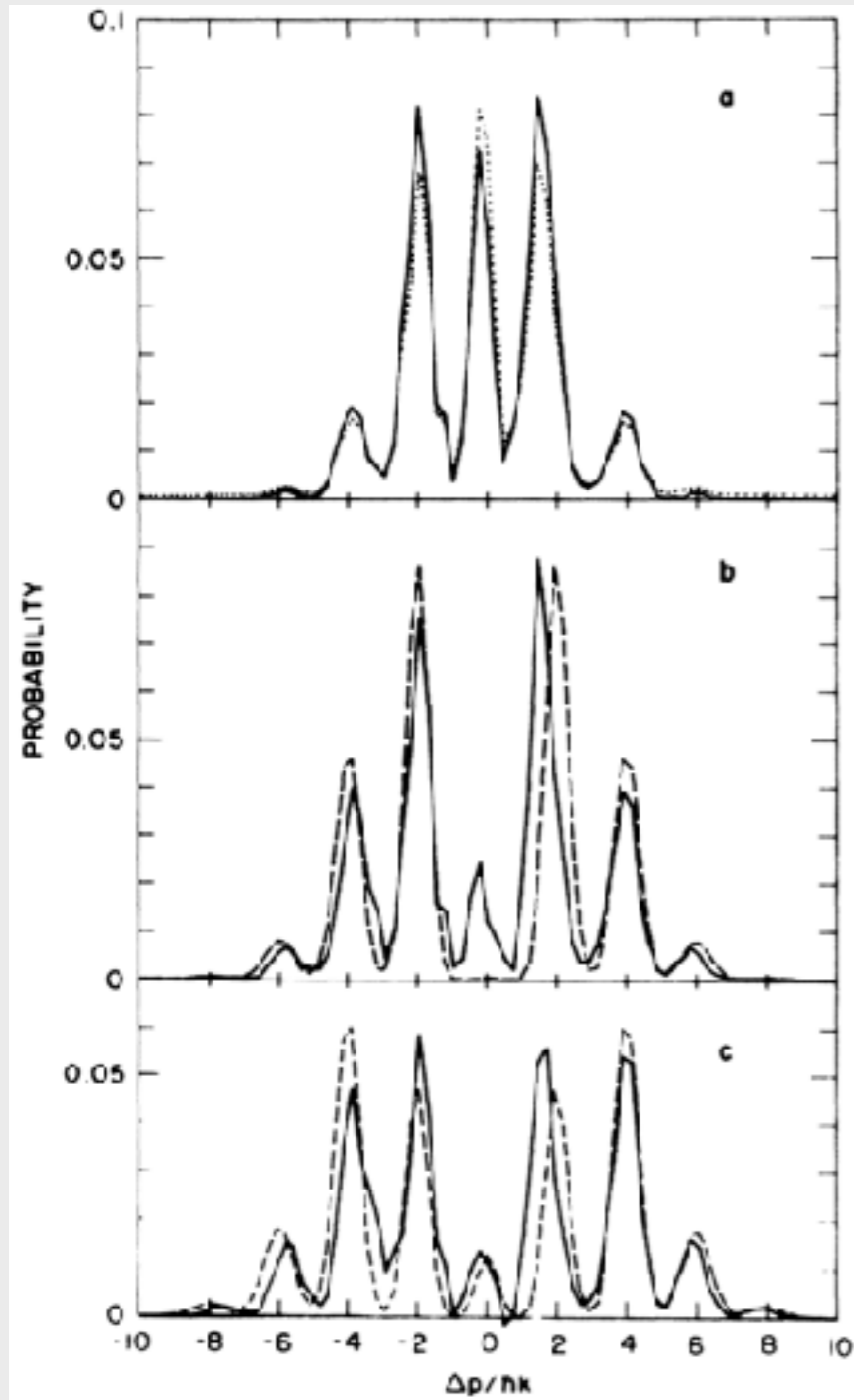
(even) more exotic optical lattices

Lattices with synthetic dimensions: internal degrees of freedom of the atoms



Mancini et al., Science (2015)

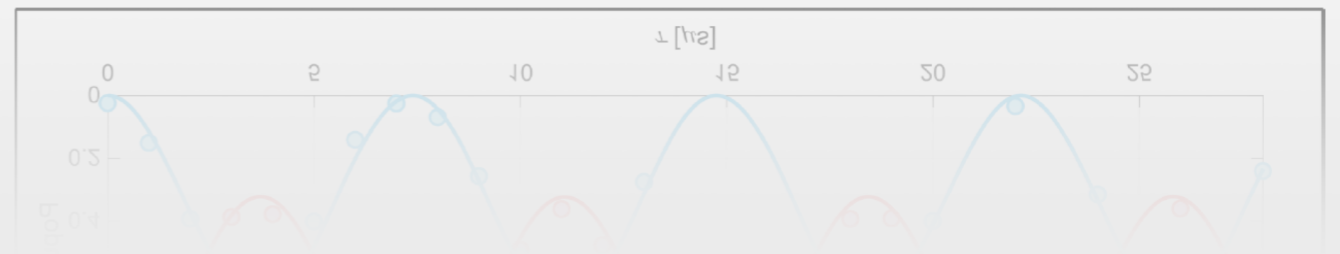
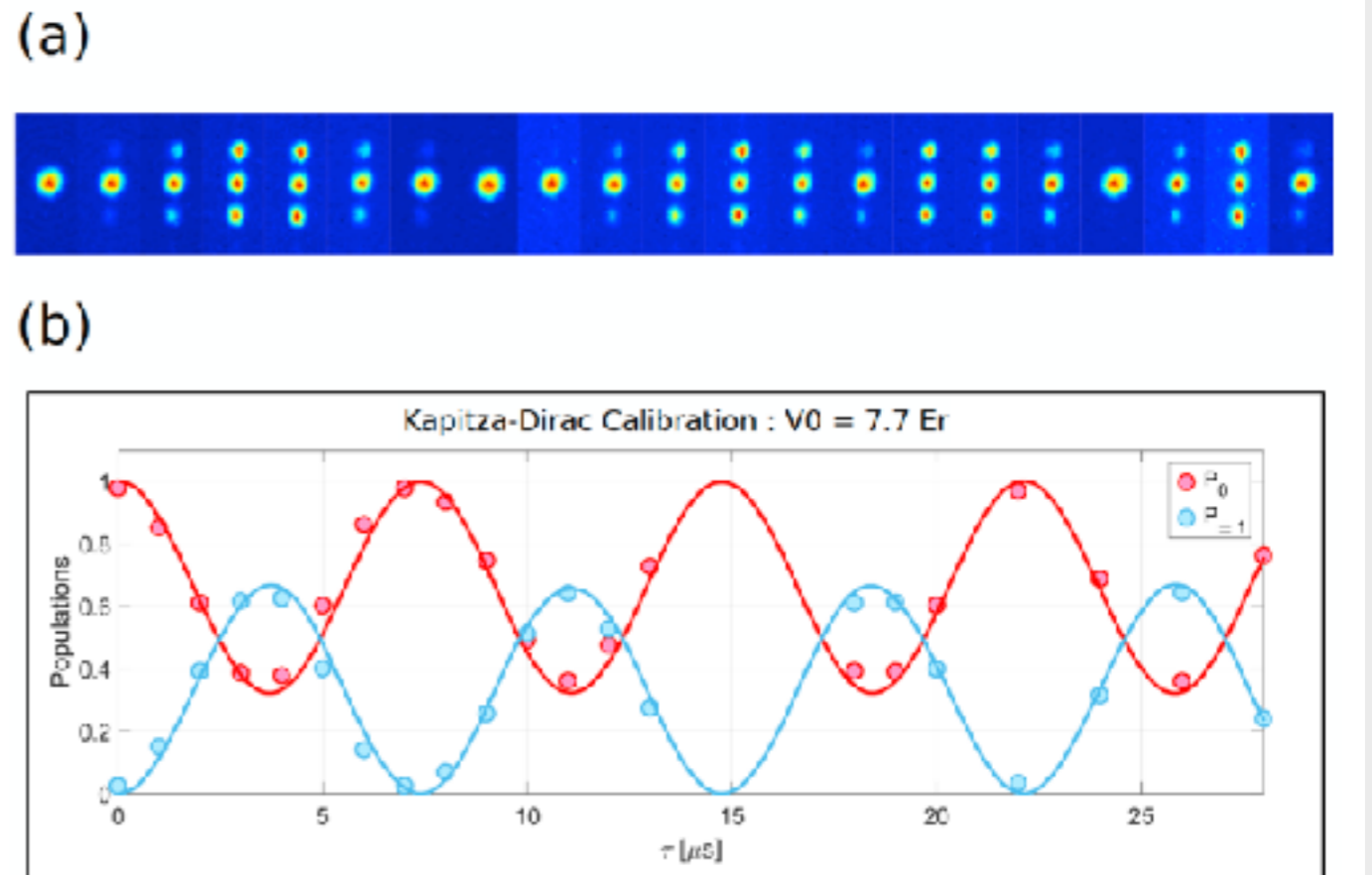
Pulsed 1D lattice



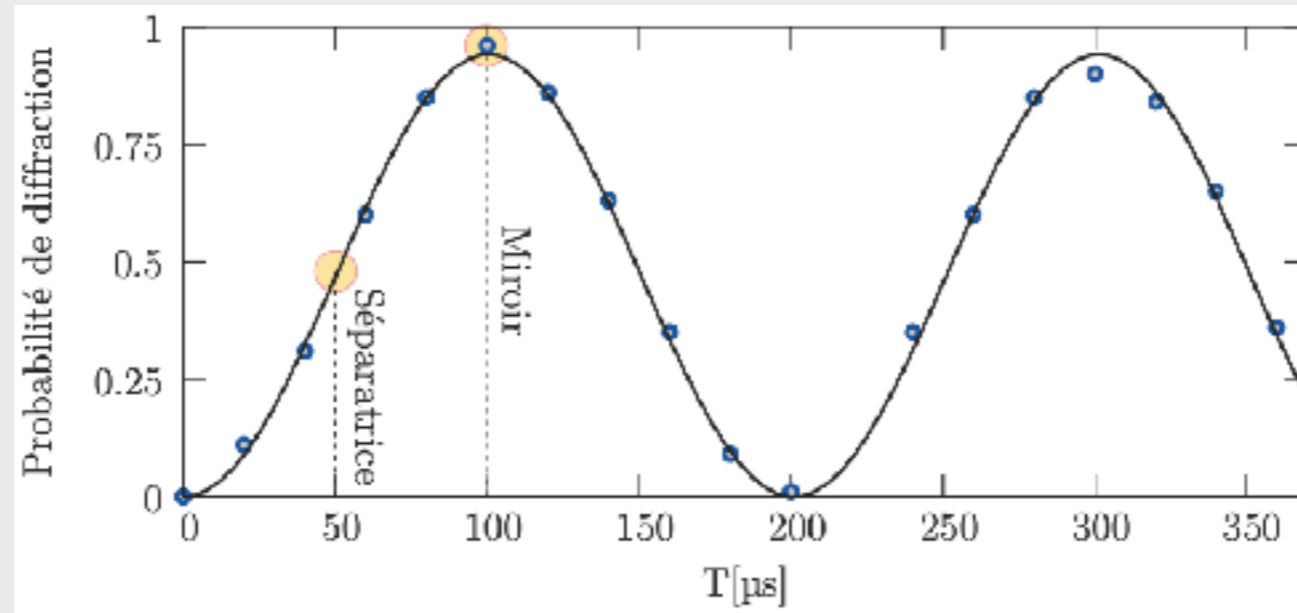
Gould et al. PRL 56, 827 (1986)

Calibrate lattice amplitudes

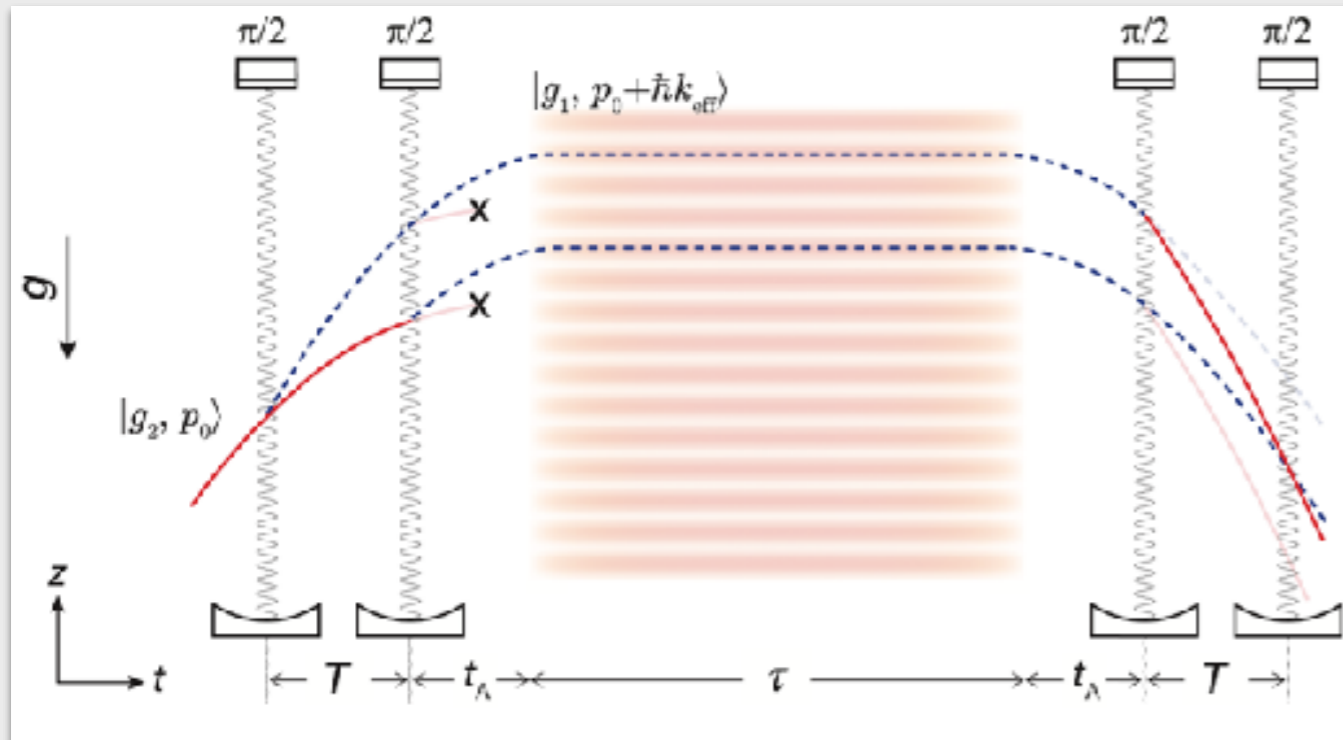
Institut d'Optique (2021)



Beamsplitters and mirrors

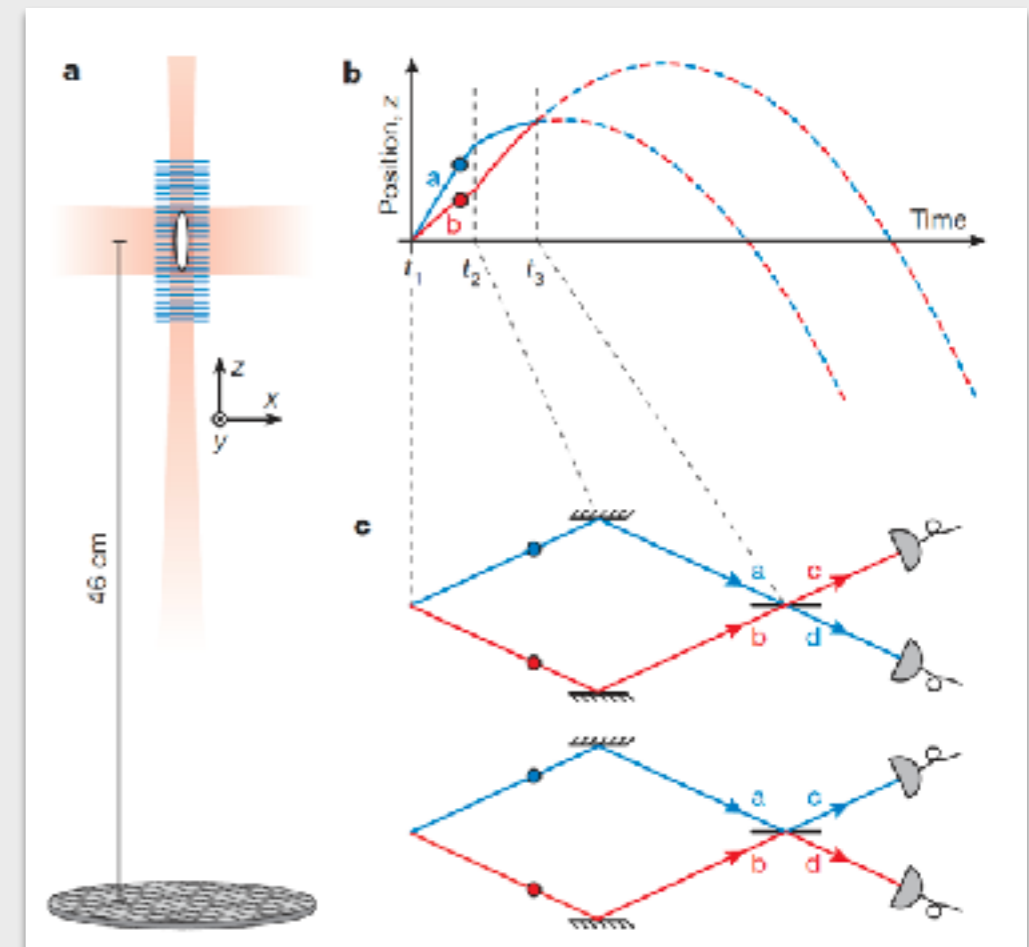


Atom interferometry



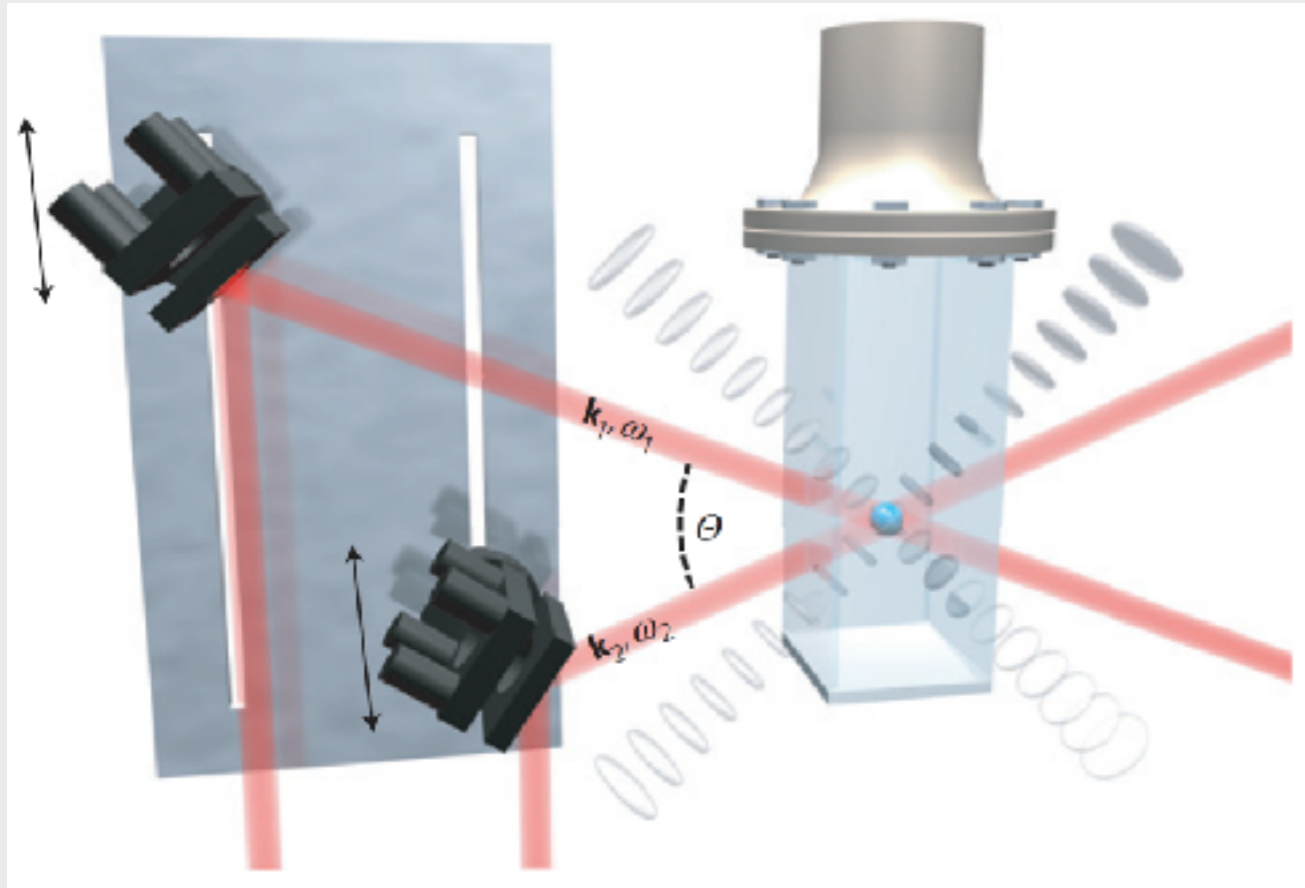
Science **366**, 745 (2019)

Atom optics: Hong-Ou-Mandel exp.



Lopes et al. Nature (2015)

Bragg diffraction: dynamical structure factor



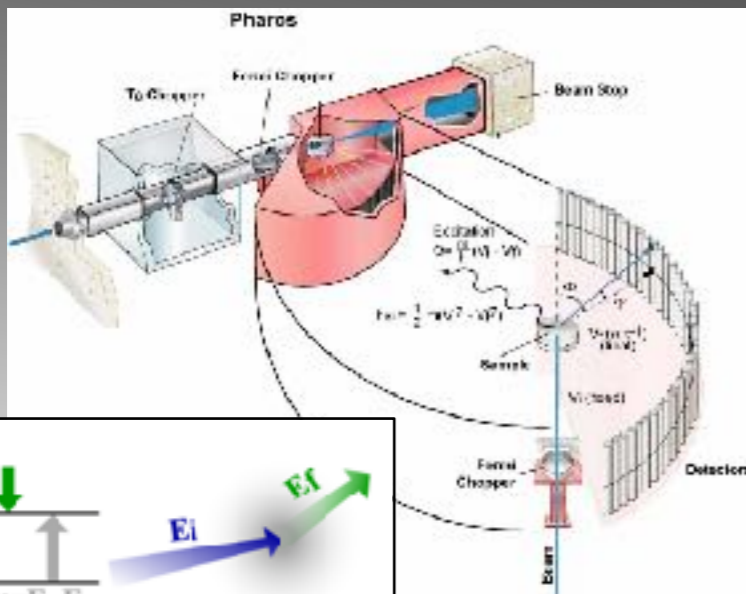
Ernst et al. Nat Phys. (2010)

Momentum transfer: $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$

Energy transfer: $\hbar\omega = \hbar(\omega_1 - \omega_2)$

If the perturbation is small (linear response),
measure of the **dynamical structure factor** $S(\mathbf{q}, \omega)$

Neutron scattering in Superfluid Helium:
exchange momentum & energy



$$S(\mathbf{q}, \omega) = \frac{1}{Z} \sum_{i,f} e^{-\beta E_i} |\langle \phi_f | \psi^\dagger(\mathbf{q} - \mathbf{k}) \psi(\mathbf{k}) | \phi_i \rangle|^2 \delta(\hbar\omega + E_f - E_i)$$

For a given momentum transfer, only one (or a few)
energy transfer would resonantly excite the system

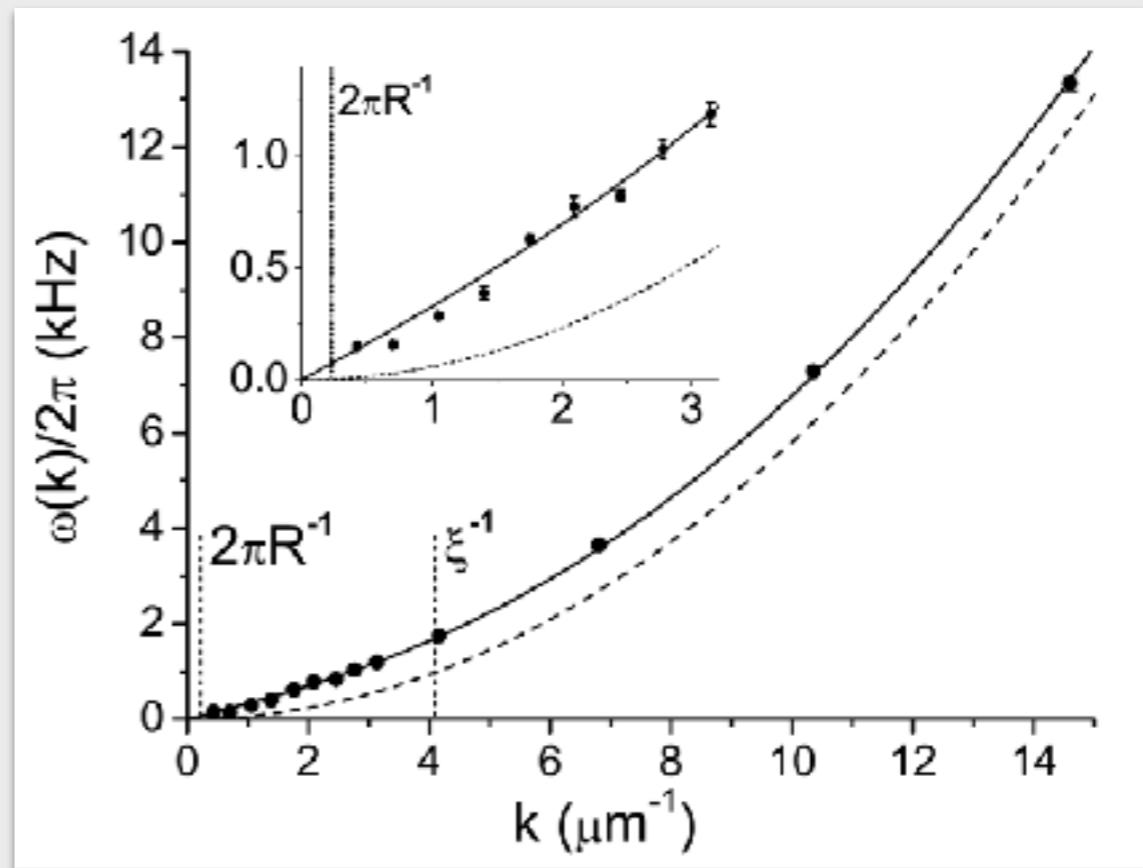
Dispersion relation of interacting BECs

$$S(\mathbf{q}, \omega) = \frac{1}{Z} \sum_{i,f} e^{-\beta E_i} |\langle \phi_f | \psi^\dagger(\mathbf{q} - \mathbf{k}) \psi(\mathbf{k}) | \phi_i \rangle|^2 \delta(\hbar\omega + E_f - E_i)$$

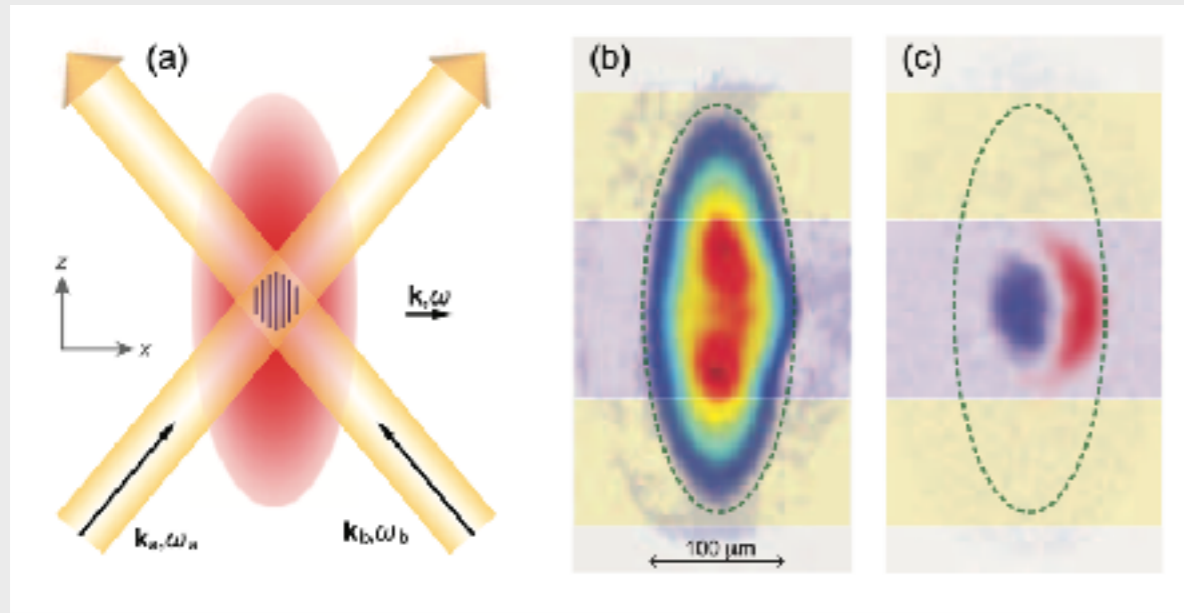
Start from a BEC with $q = 0$

$$\delta(\hbar\omega + \epsilon(\mathbf{q}) - \epsilon(\mathbf{0}))$$

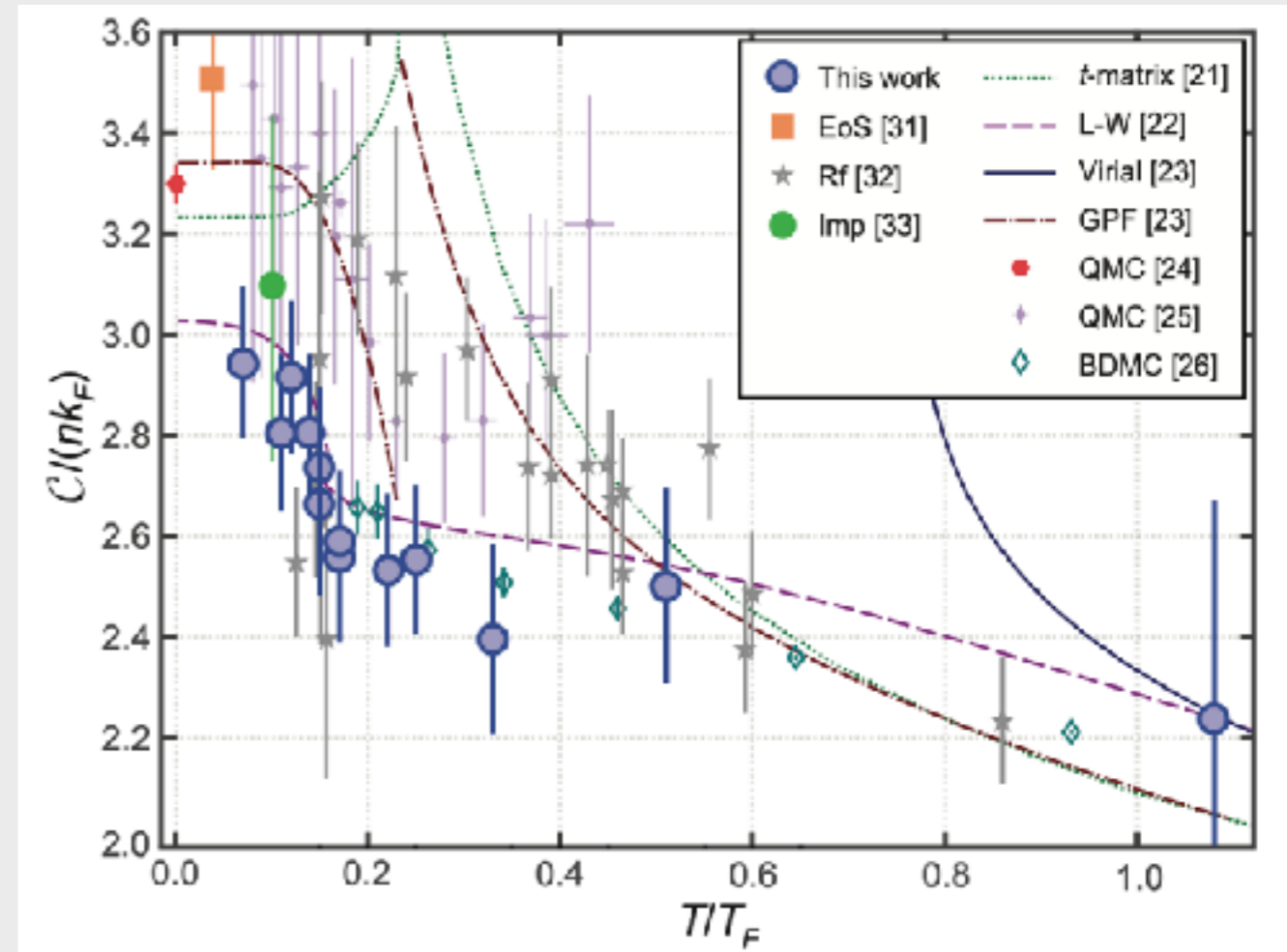
Observation of **Bogoliubov spectrum of excitations**, with phonons at low momenta



Probing strongly-correlated fermions

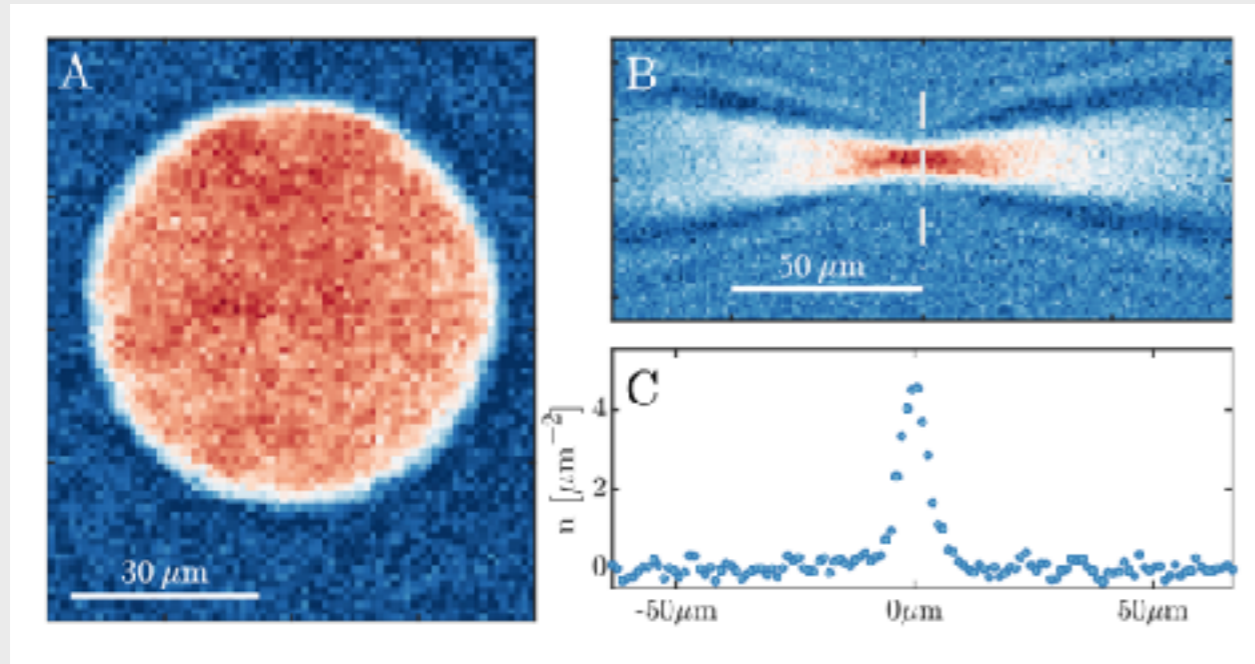


At large momentum transfer, dynamical structure factor contains information about short-distance two-body wave-function: **measure of Tan's contact**



Carcy et al. PRL **122**, 203401 (2019)

Superfluidity in 2D correlated fermions



Superfluidity observed in correlated 2D Fermi gas: no excitations (either phonons or pairs) below critical velocity

