Quantum gases in Optical Lattices
Lecture #3

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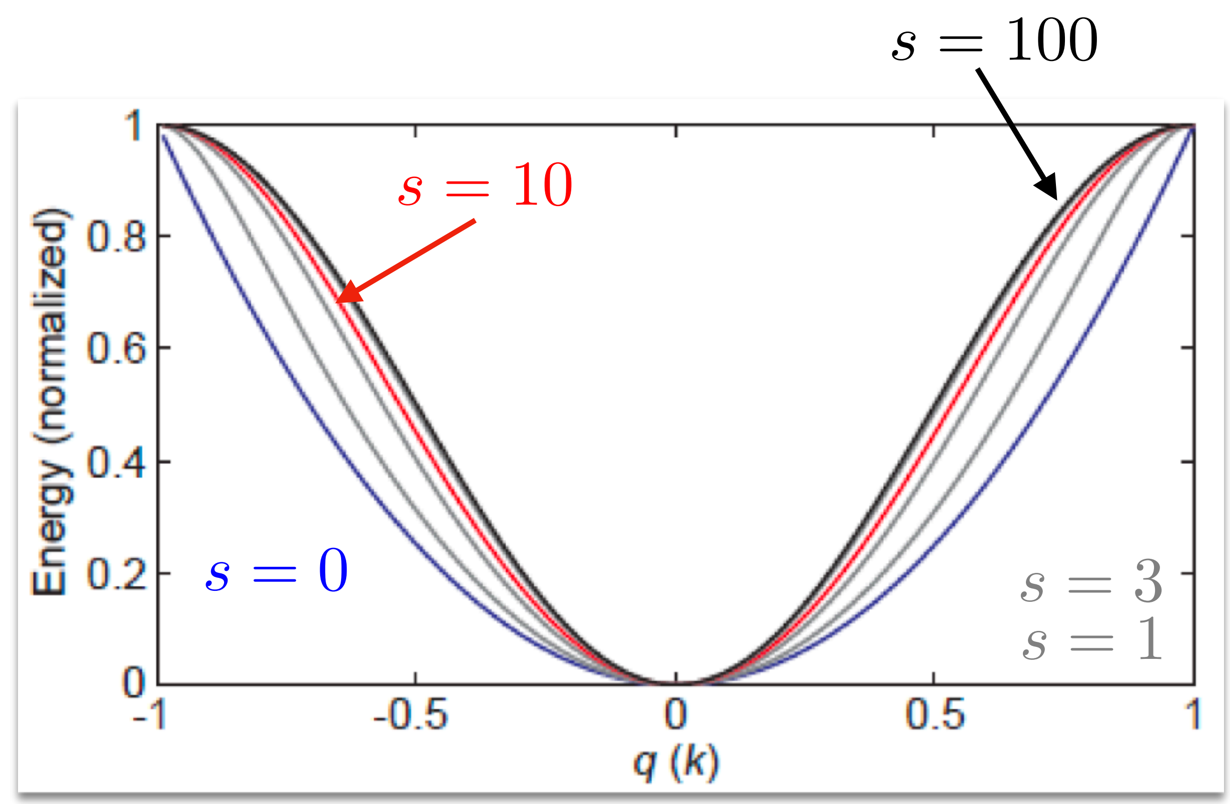
Lowest-energy band

Normalized and shifted lowest-energy bands.

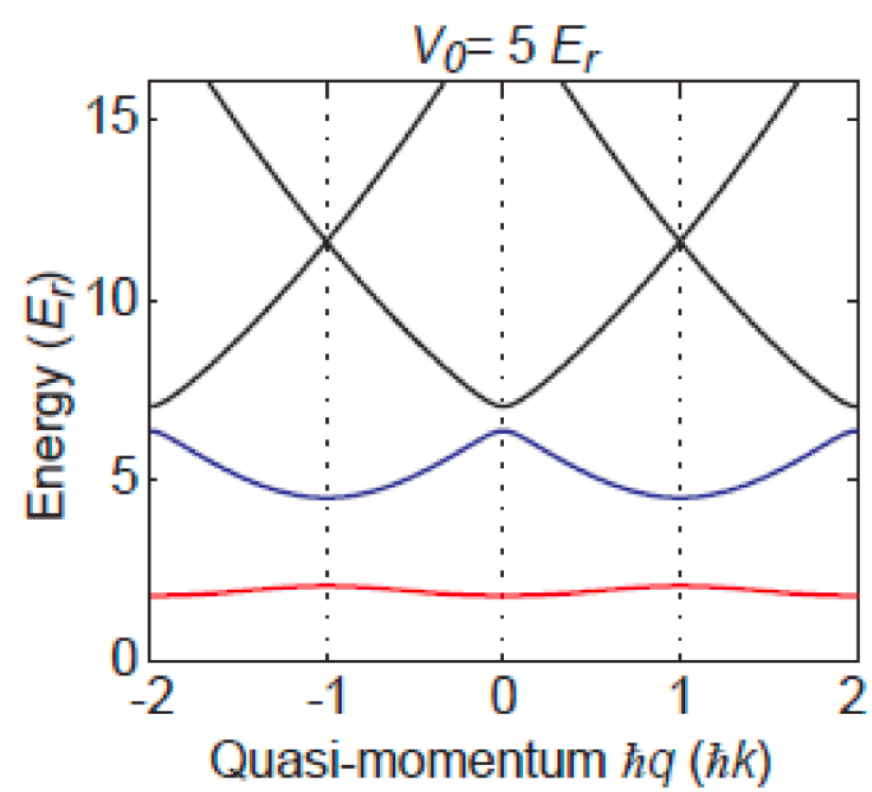


lowest-energy band dispersion smoothly changes from parabolic to sinusoidal:

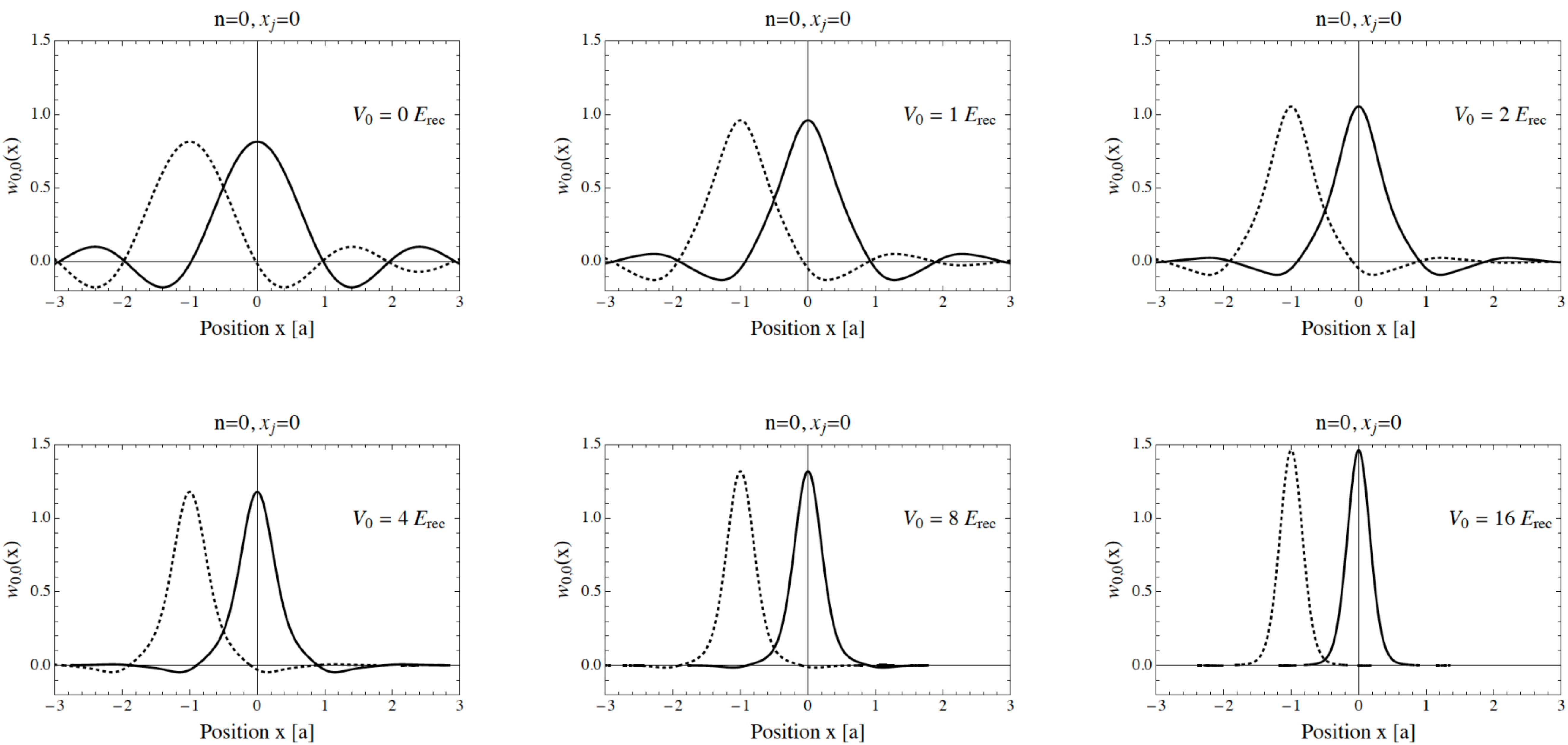
$$E(q) - E_0 \simeq 4J \sin(qa/2)^2$$

Tight-binding regime (*lectures #3, #4*)

$s \gtrsim 5$: energy bandwidth << energy bandgap

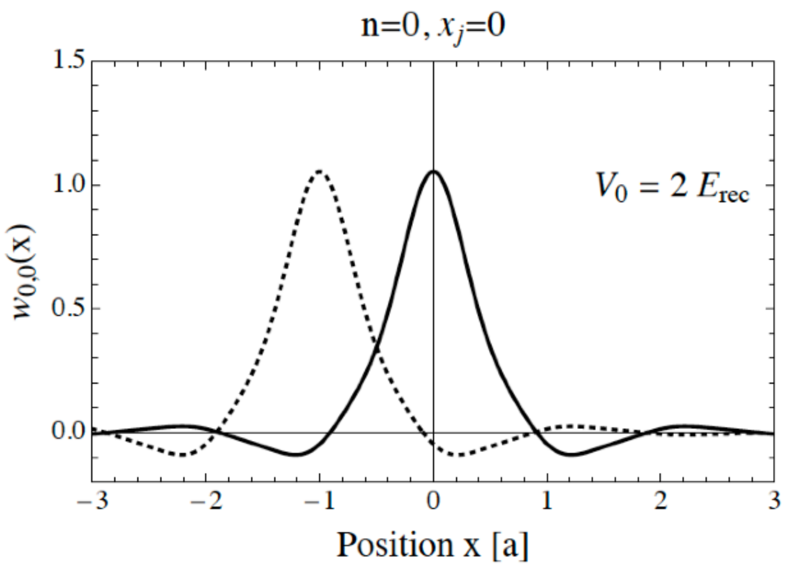


Wannier functions



Crédits H. Perrin

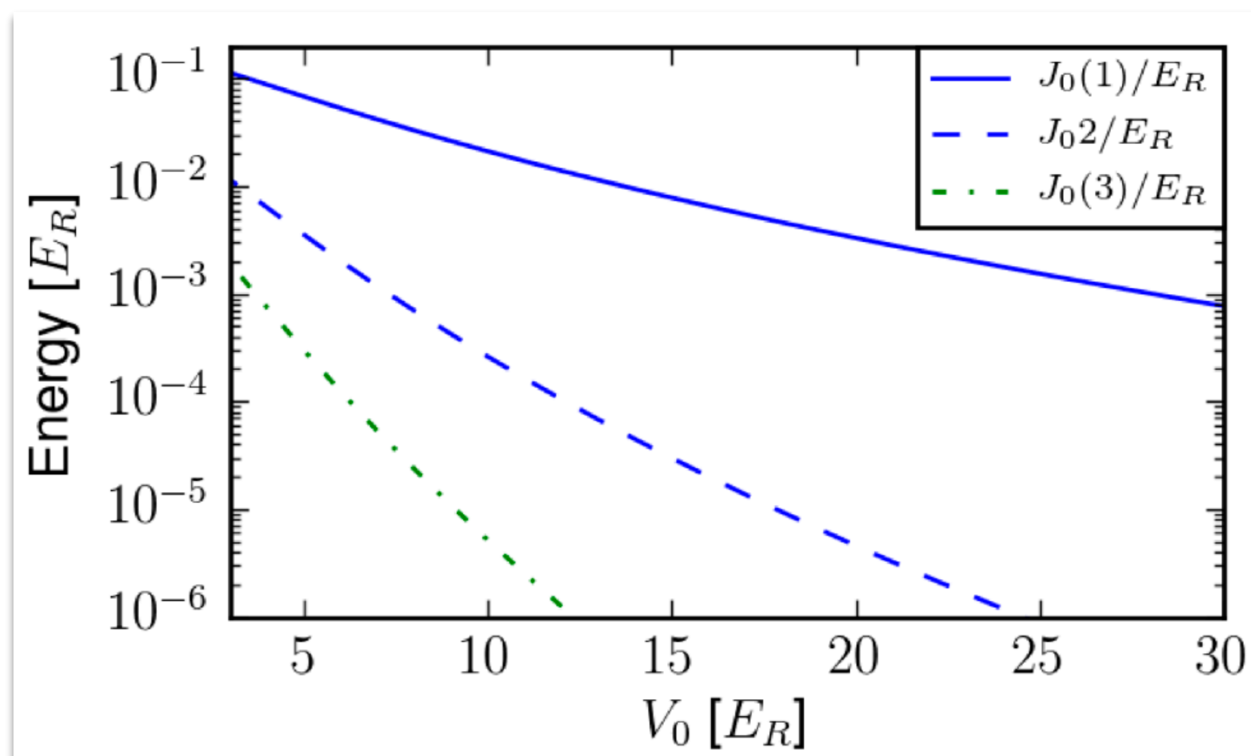
Tunnelling in the lattice



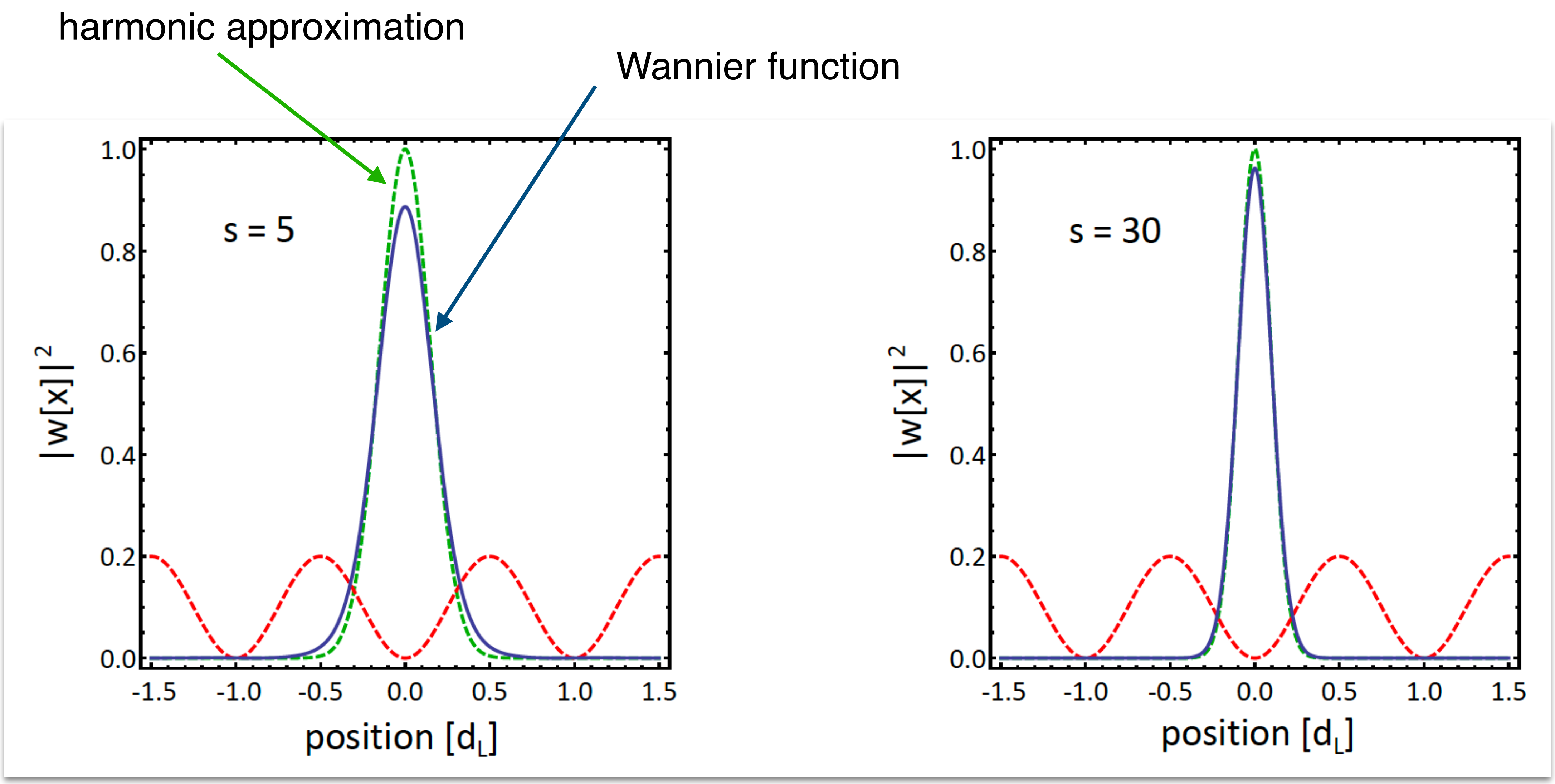
$$J = J_0 = \int dx\ \omega_0^*(x+a)\left(-\frac{\hbar^2}{2m}\Delta_x - V_{\mathrm{lat}}(x)\right)\omega_0(x)$$

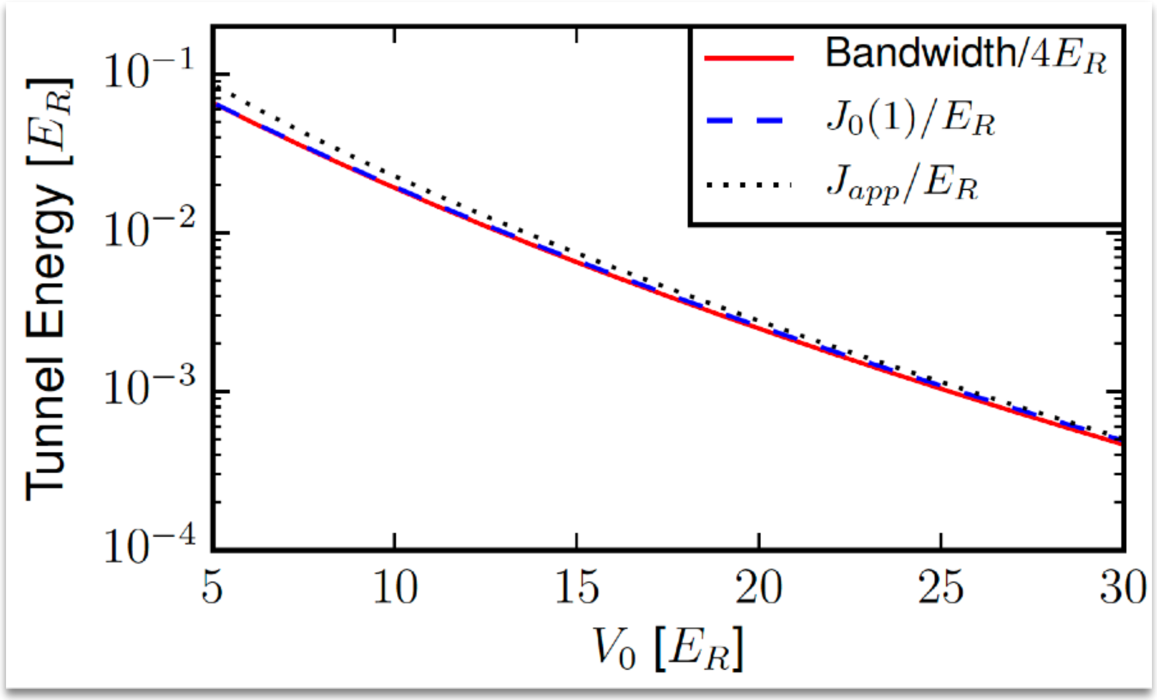
Tight-binding regime:

only nearest neighbour tunnelling



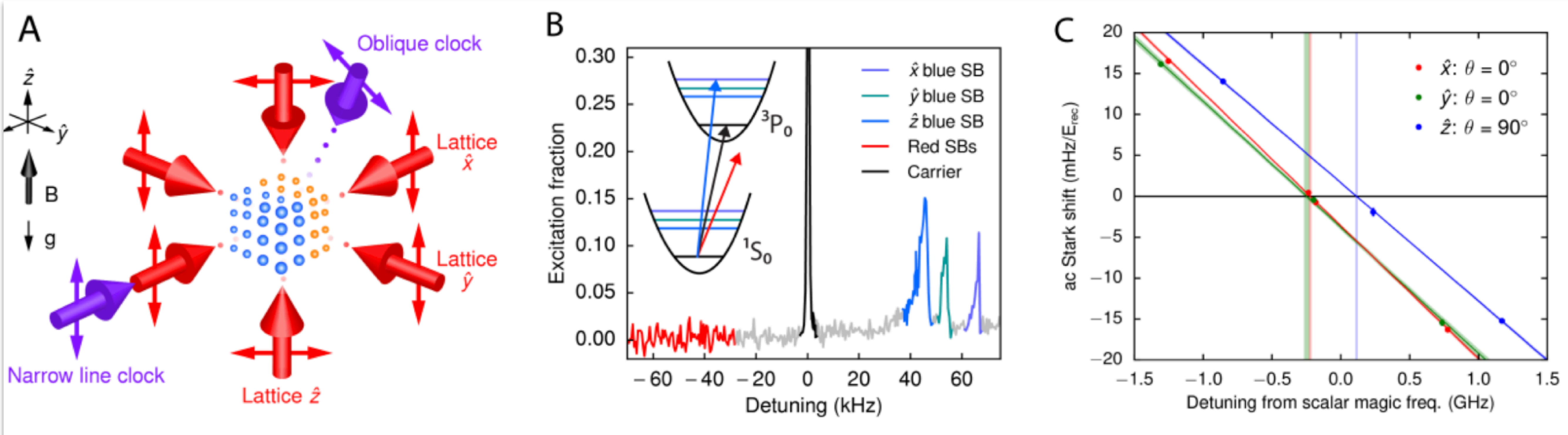
Deep lattices: harmonic approximation





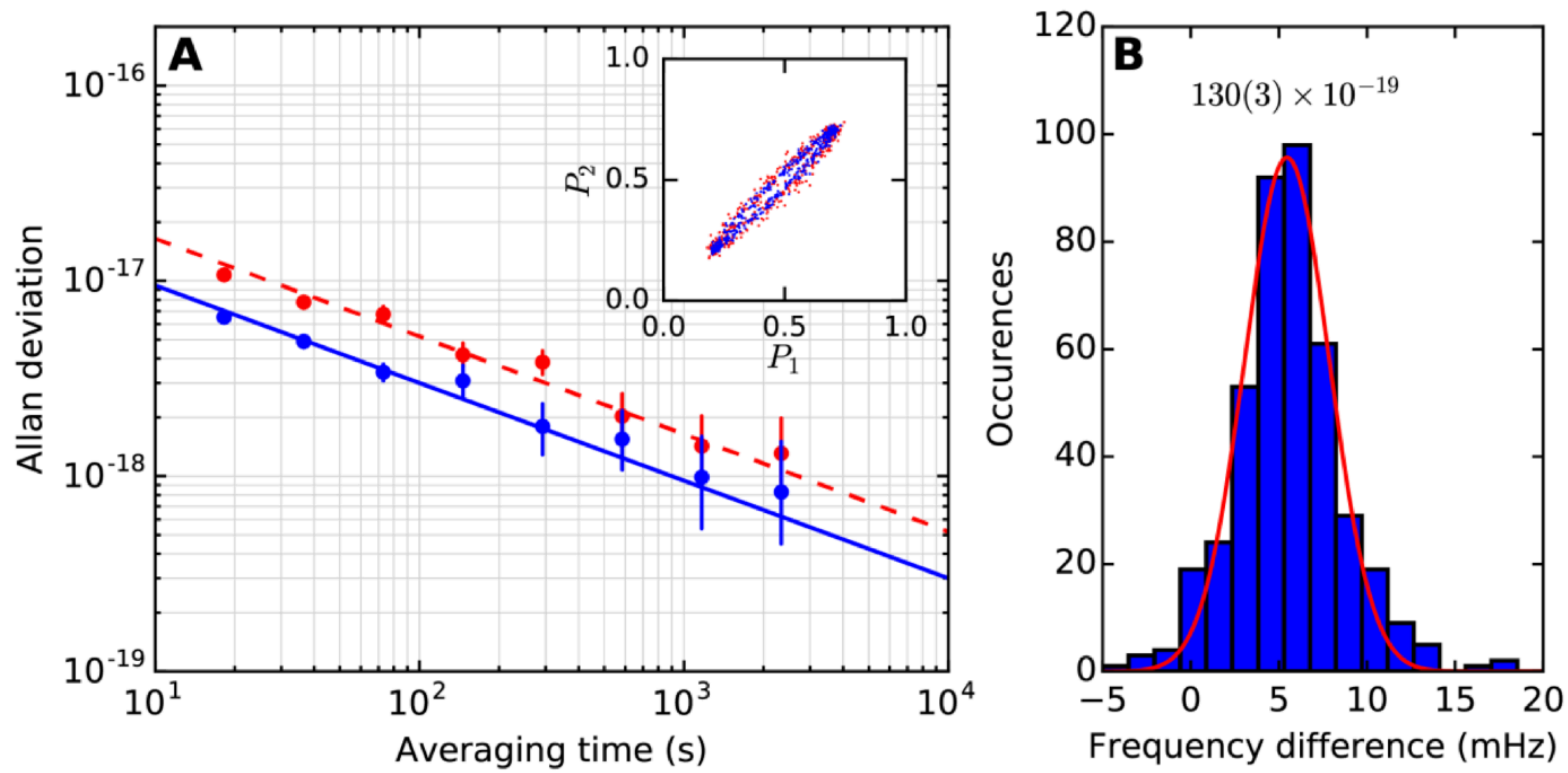
Tunnelling sensitive to the difference between Wannier and ground-state of harmonic trap, even in the tight-binding regime

Deep lattices: Dicke regime and optical clocks



Precision <10^{-18} over 1 hour

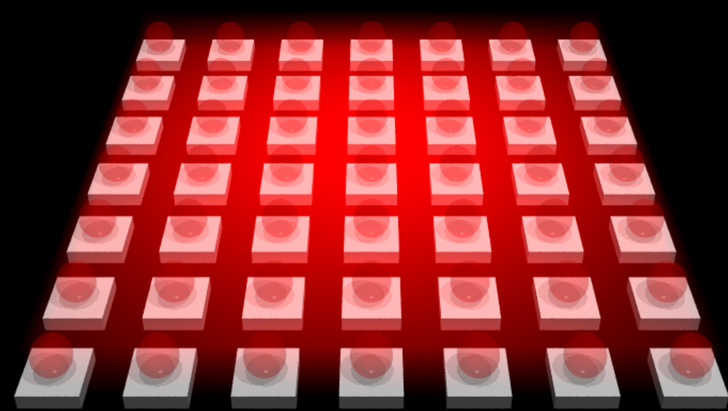
Campbell et al. Science ***358****, 90 (2017)*



Superfluid-to-Mott transition

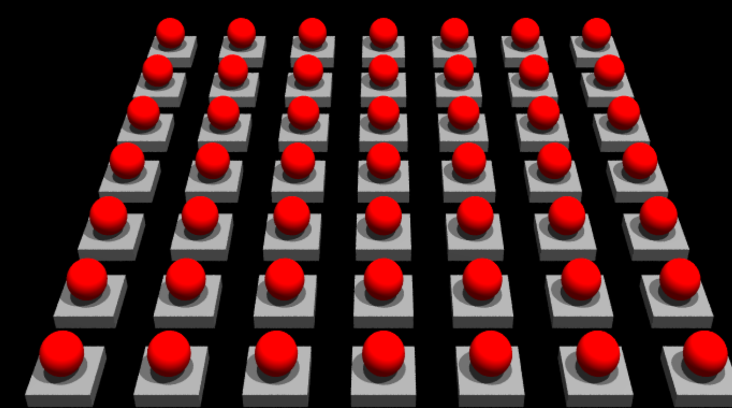
SUPERFLUID $J \gg U$

- ✓ Long-range phase coherence
- ✓ Poissonian number fluctuations
- ✓ Gapless excitation spectrum
- ✓ Compressible



MOTT INSULATOR $U \gg J$

- ✓ No phase coherence
- ✓ No number fluctuations (Fock states)
- ✓ Gap in the excitation spectrum
- ✓ Not compressible

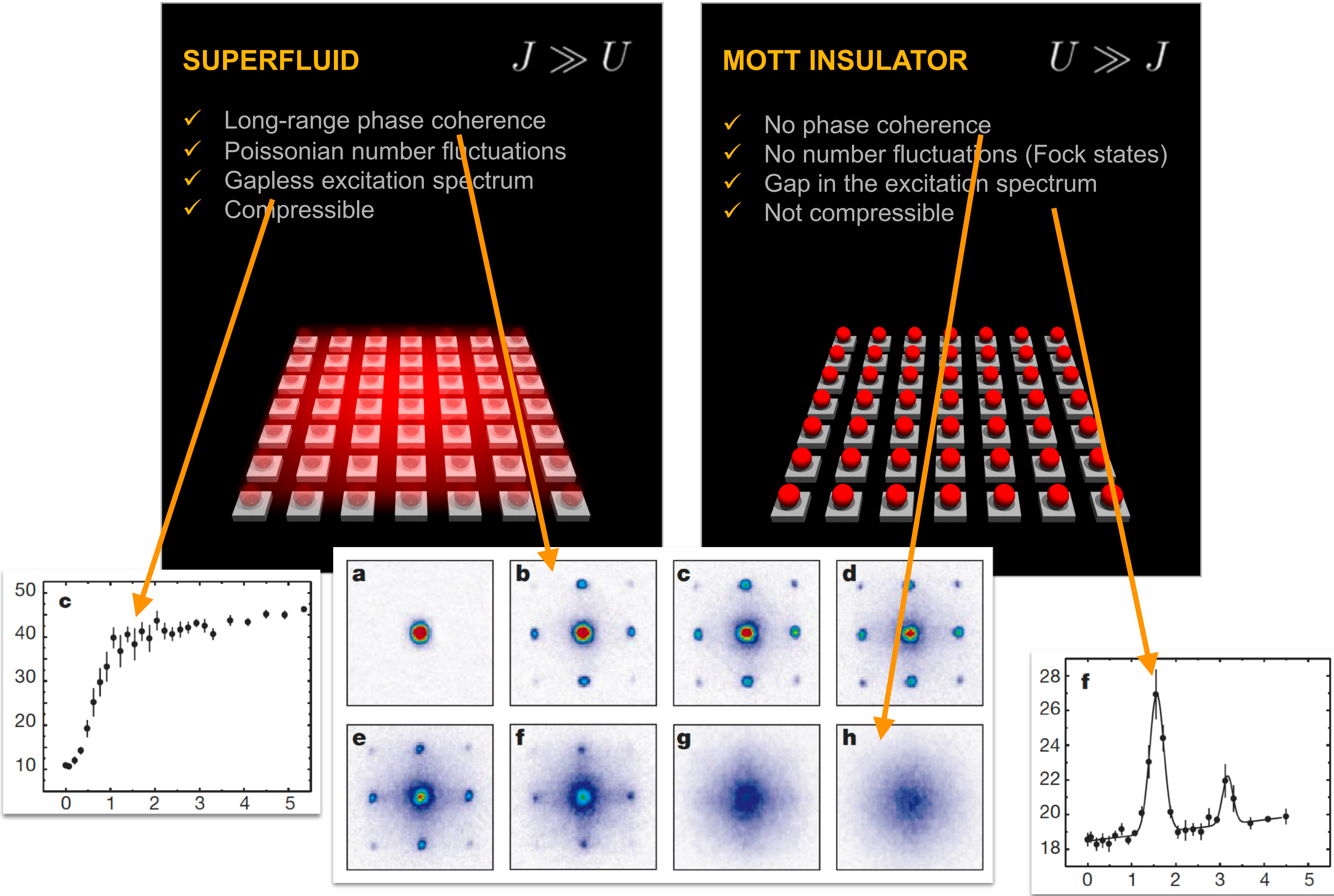


<u>Quantum phase transition</u> (T=0) induced by interactions: **Mott transition**

Sir N. Mott

Proc. Phys. Soc. **62** 416 (1949)

Superfluid-to-Mott transition



reversible loss of coherence (not shown)

Locating the Mott transition

(theoretical approaches)

$\bar{n} = 1$	$\left(\frac{U}{t}\right)_c$ Mean-Field	$\left(\frac{U}{t}\right)_c$ Exact	
1D	11.66	3.37	DMRG
2D	23.33	16.74	QMC
3D	34.98	29.36	QMC

Numerical techniques

- DMRG: density matrix renormalization group
- QMC: Quantum Monte Carlo

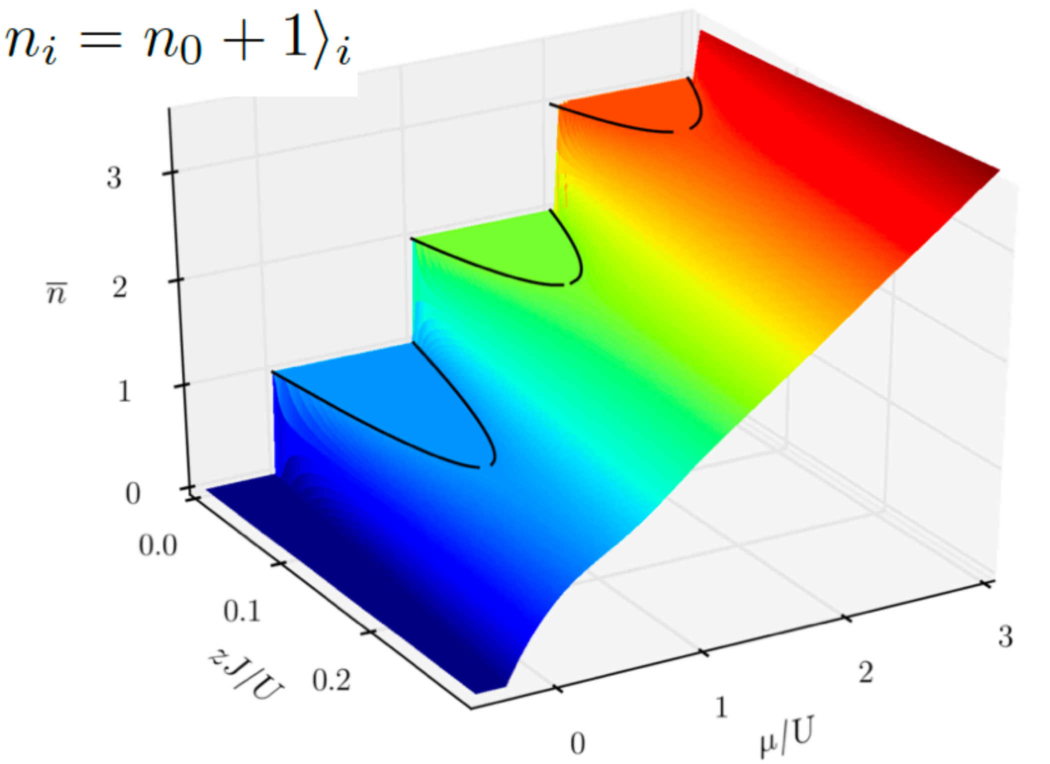
mean-field approaches over-estimates the extension of the order phase

in low dimensional systems, quantum fluctuations play a larger role

Alternative mean-field: truncate Hilbert space to three states/site:

$$|\phi_i\rangle = c(n_0 - 1)|n_i = n_0 - 1\rangle_i + c(n_0)|n_i = n_0\rangle_i + c(n_0 + 1)|n_i = n_0 + 1\rangle_i$$

$$\mu_{n_0}^{(\pm)} = U\left(n_0 - \frac{1}{2}\right) \pm \frac{zJ}{2} \pm \sqrt{U^2 - 2UzJ(2n_0 + 1) + (zJ)^2}$$



http://www.lkb.upmc.fr/boseeinsteincondensates/gerbier/

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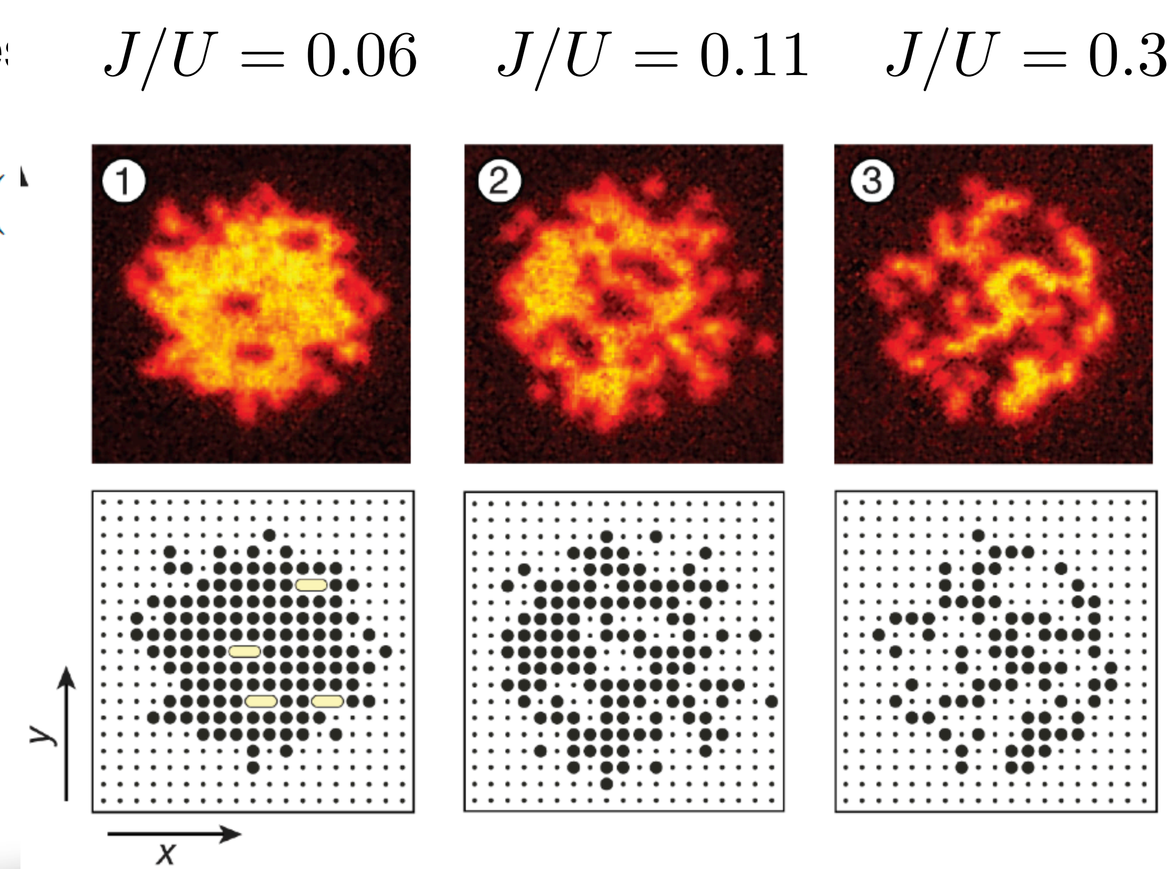
Approach also "naturally" suited to investigate properties of the Mott state! *(quasi-particles: particle-hole excitations)*

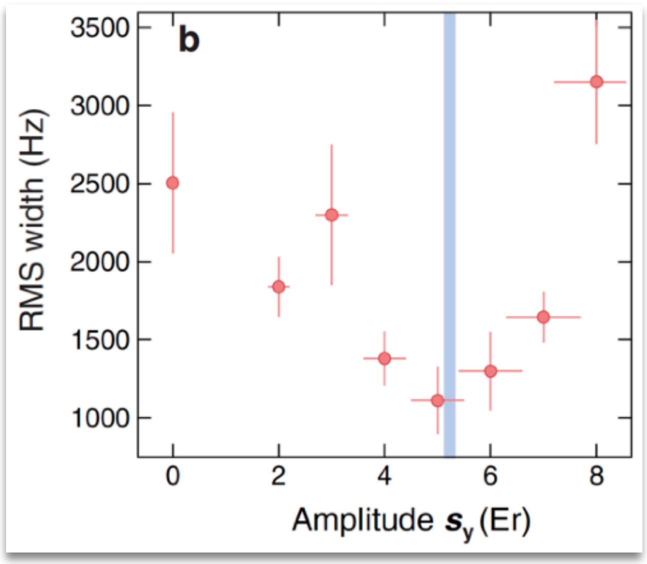


Endres et al. Science ***334****, 200 (2011)*

Locating the Mott transition

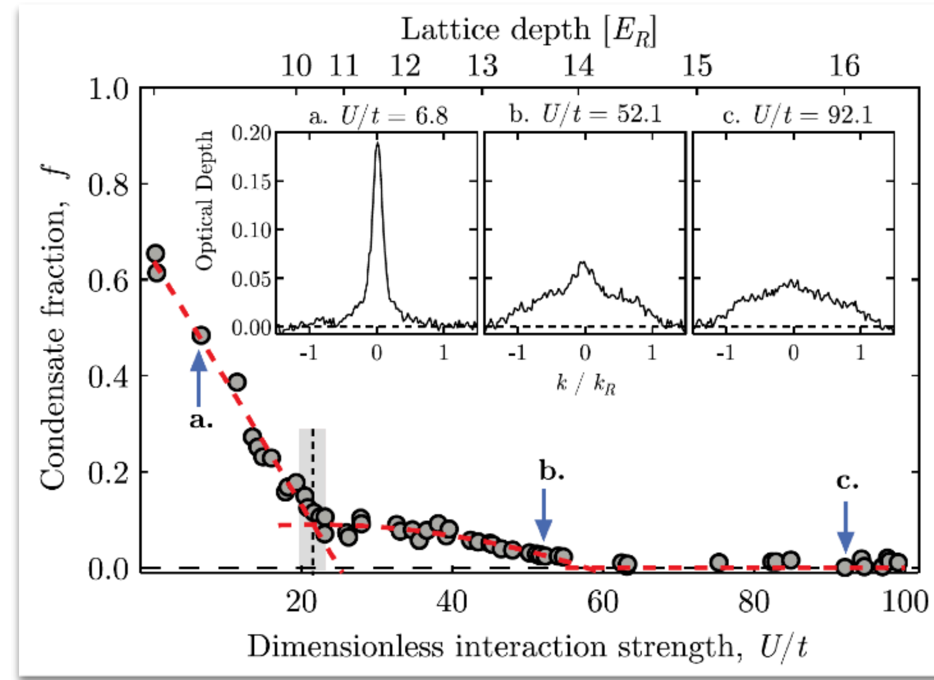
(experimental approaches)

1D - Bragg spectroscopy compatible with QMC prediction (not accurate though)

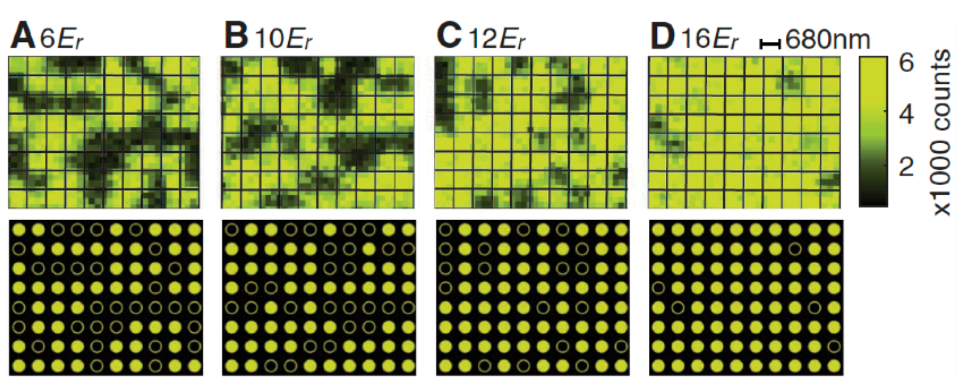


Clément et al. Phys. Rev. Lett. ***102****, 155301 (2009)*

2D - BEC fraction compatible with QMC prediction (not accurate though)

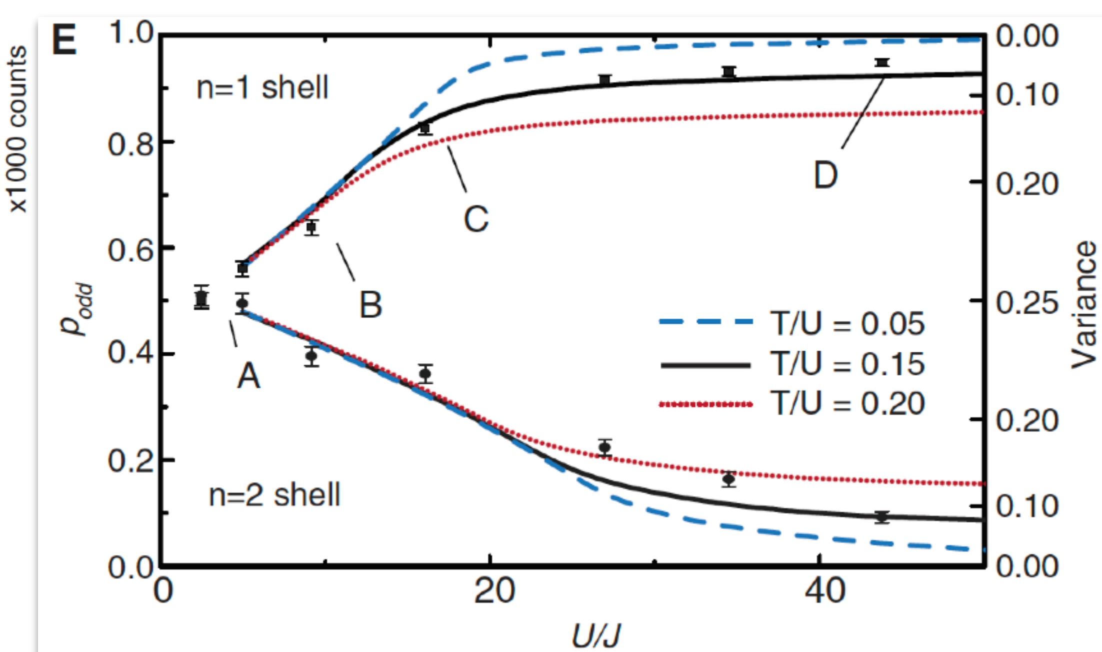


Jimenez-Garcia et al. Phys. Rev. Lett. ***105****, 110401 (2010)*



2D - Reduced atom number fluctuations in the Mott regime (not accurate though)

(parity measured)



Bakr et al. Science ***329****, 547 (2010)*

Locating the Mott transition

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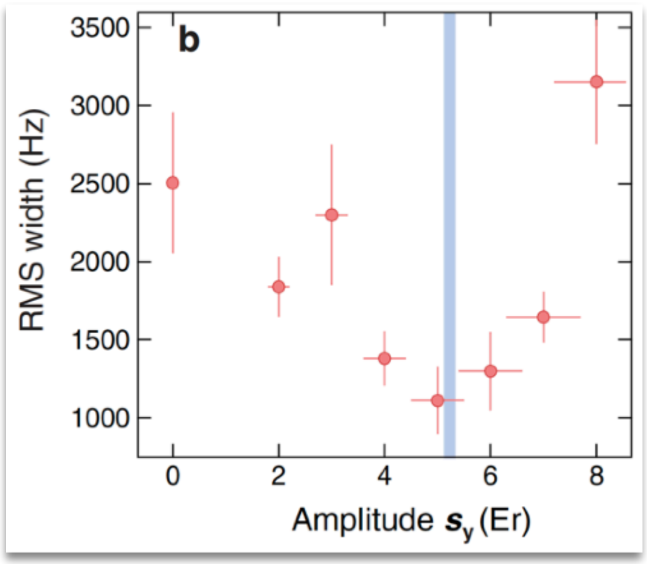

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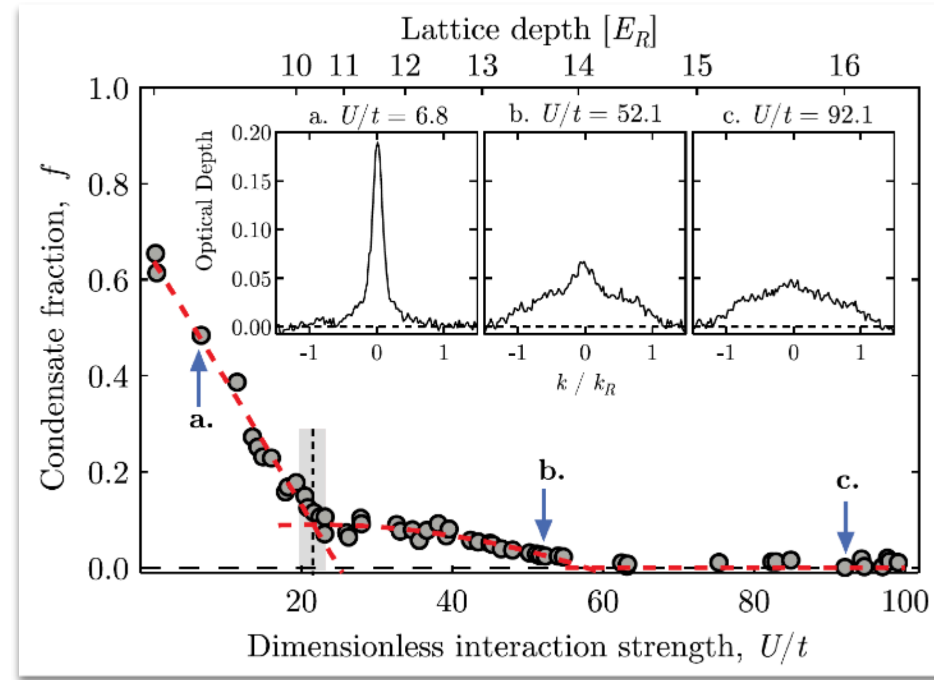

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3D - different measures compatible with mean-field predictions...

Mun et al. Phys. Rev. Lett. ***99****, 150604 (2007)*

Becker et al. New J. Phys. ***12****, 065025 (2010)*

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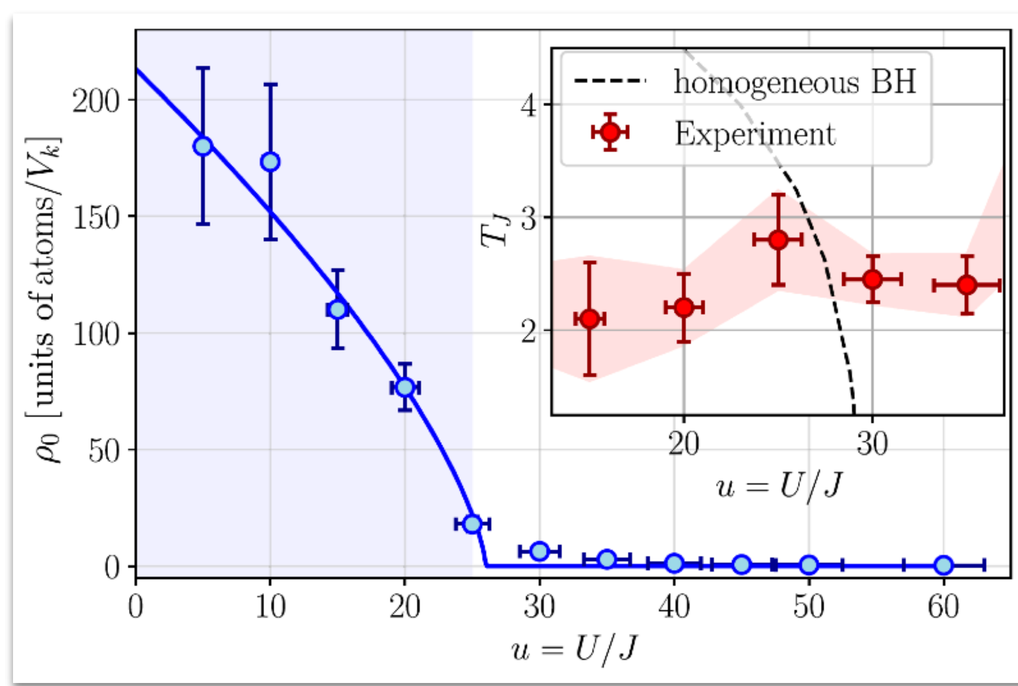
Thomas et al. Phys. Rev. Lett. ***119****, 100402 (2017)*



Hercé et al. Phys. Rev. A ***104****, L011301 (2021)*

compatible with QMC (exclude mean-field)!

...but....

Locating the Mott transition

(experimental approaches)

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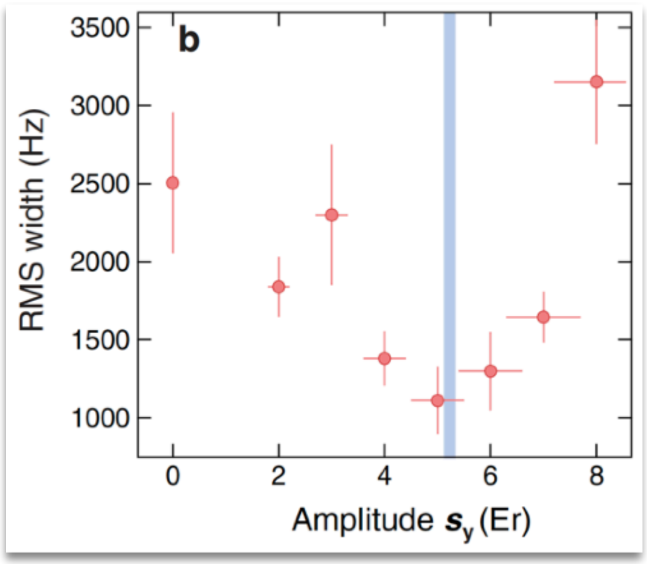

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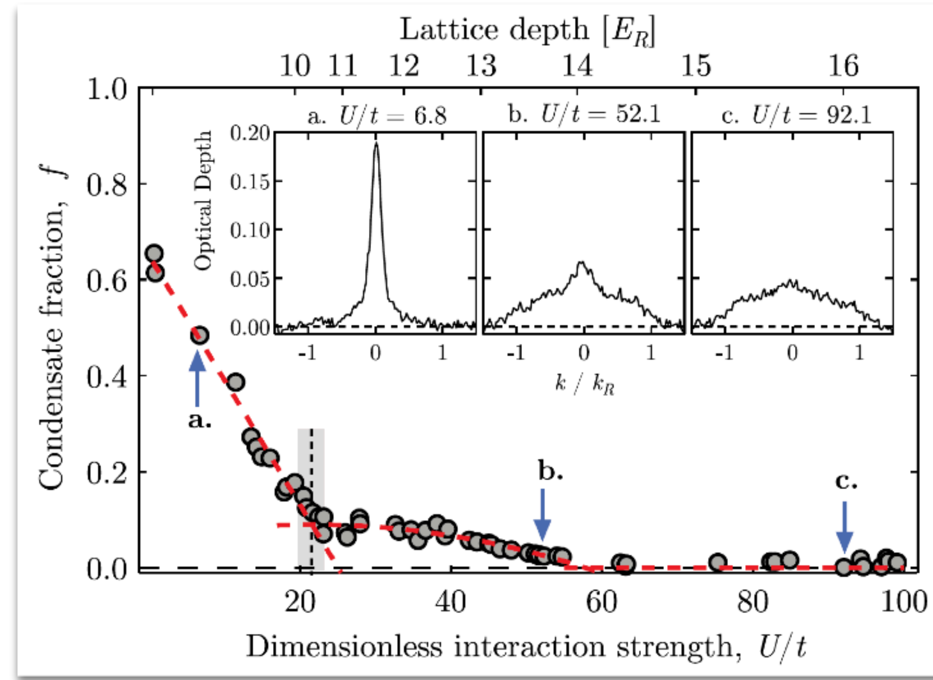

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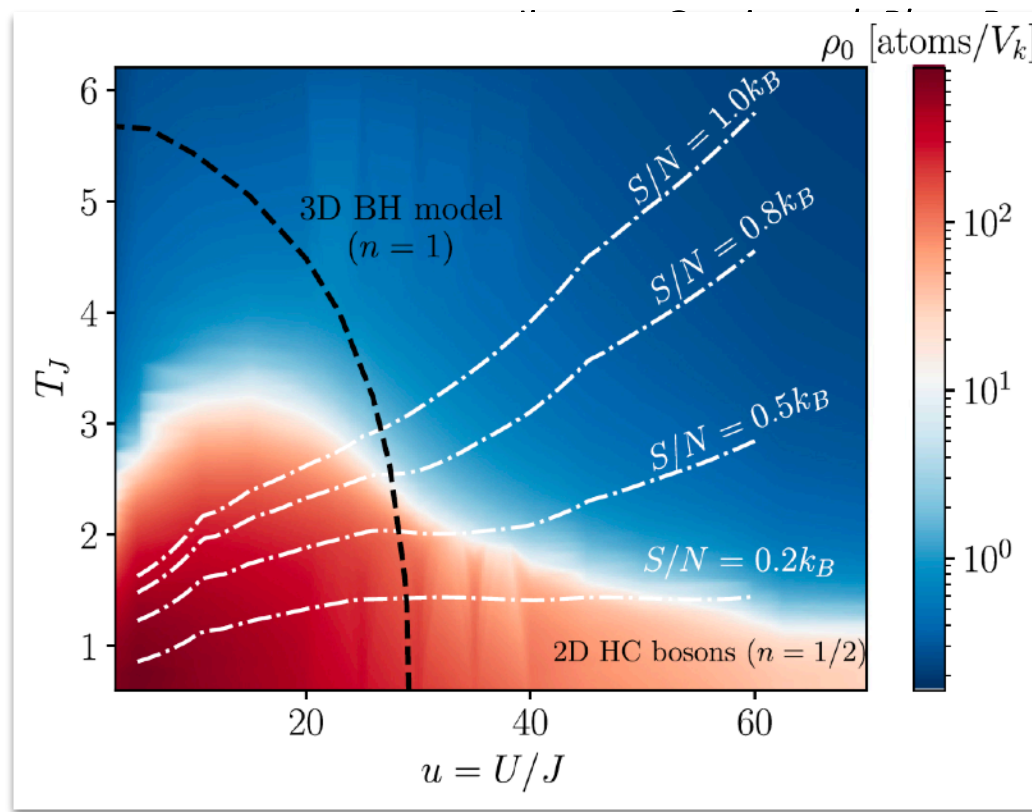


Hercé et al. Phys. Rev. A ***104****, L011301 (2021)*

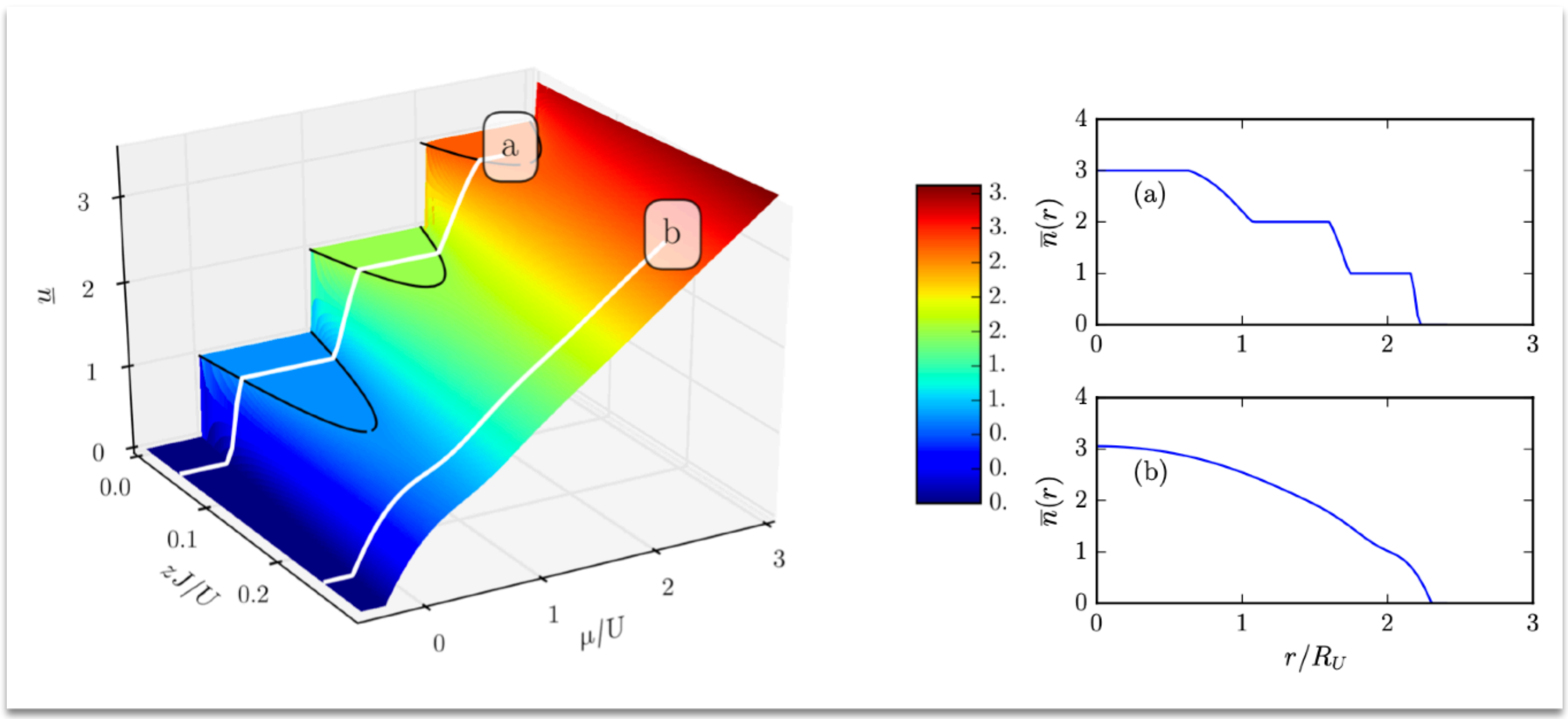
compatible with QMC (exclude mean-field)!

...but....

strongly depends on exp. parameters

Inhomogeneous lattice Bose gases

$$H = -J\sum_{j,j'} b_j^\dagger b_{j'} + \frac{U}{2}\sum_j n_j(n_j-1) + \sum_j \frac{m\omega a^2}{2} j^2$$



co-existence of several phases at once!

Inhomogeneous lattice Bose gases

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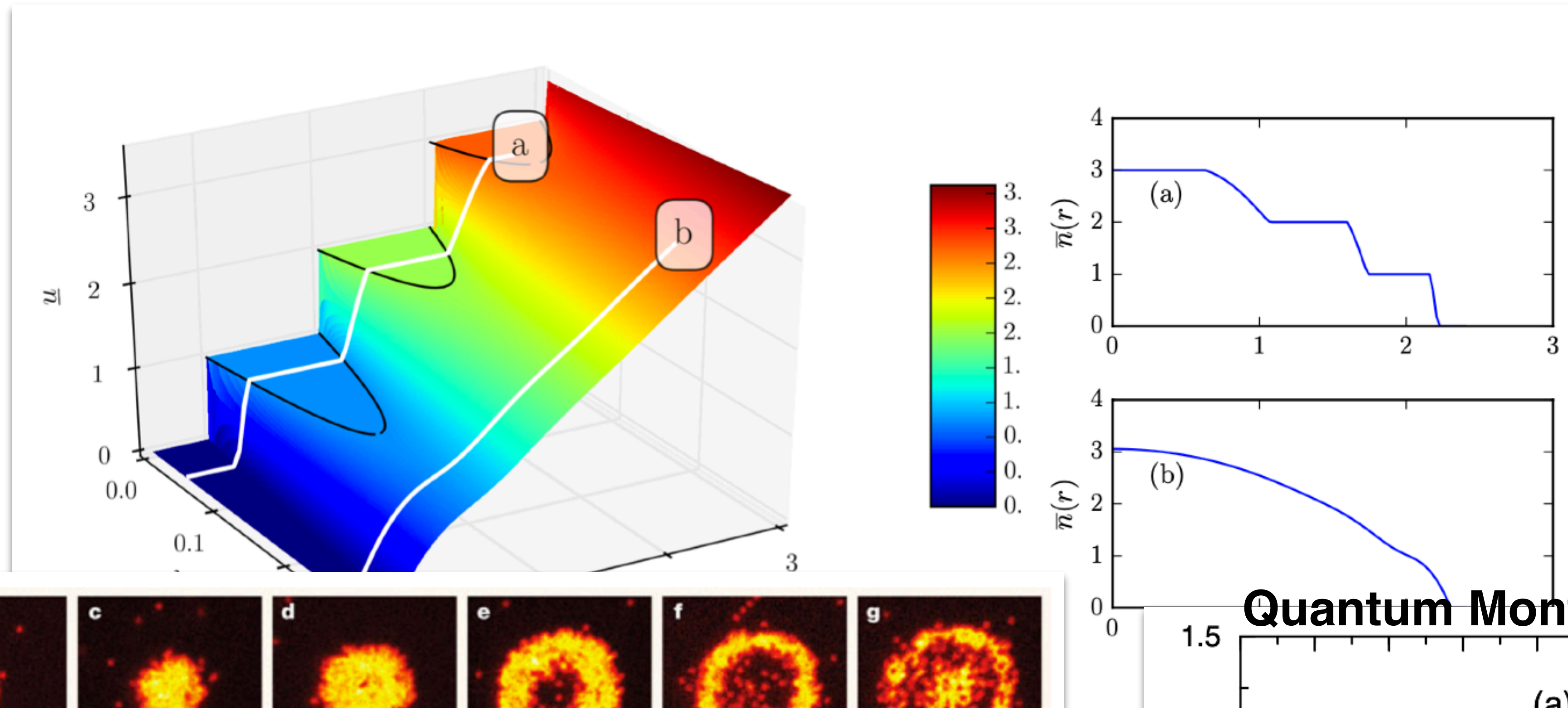
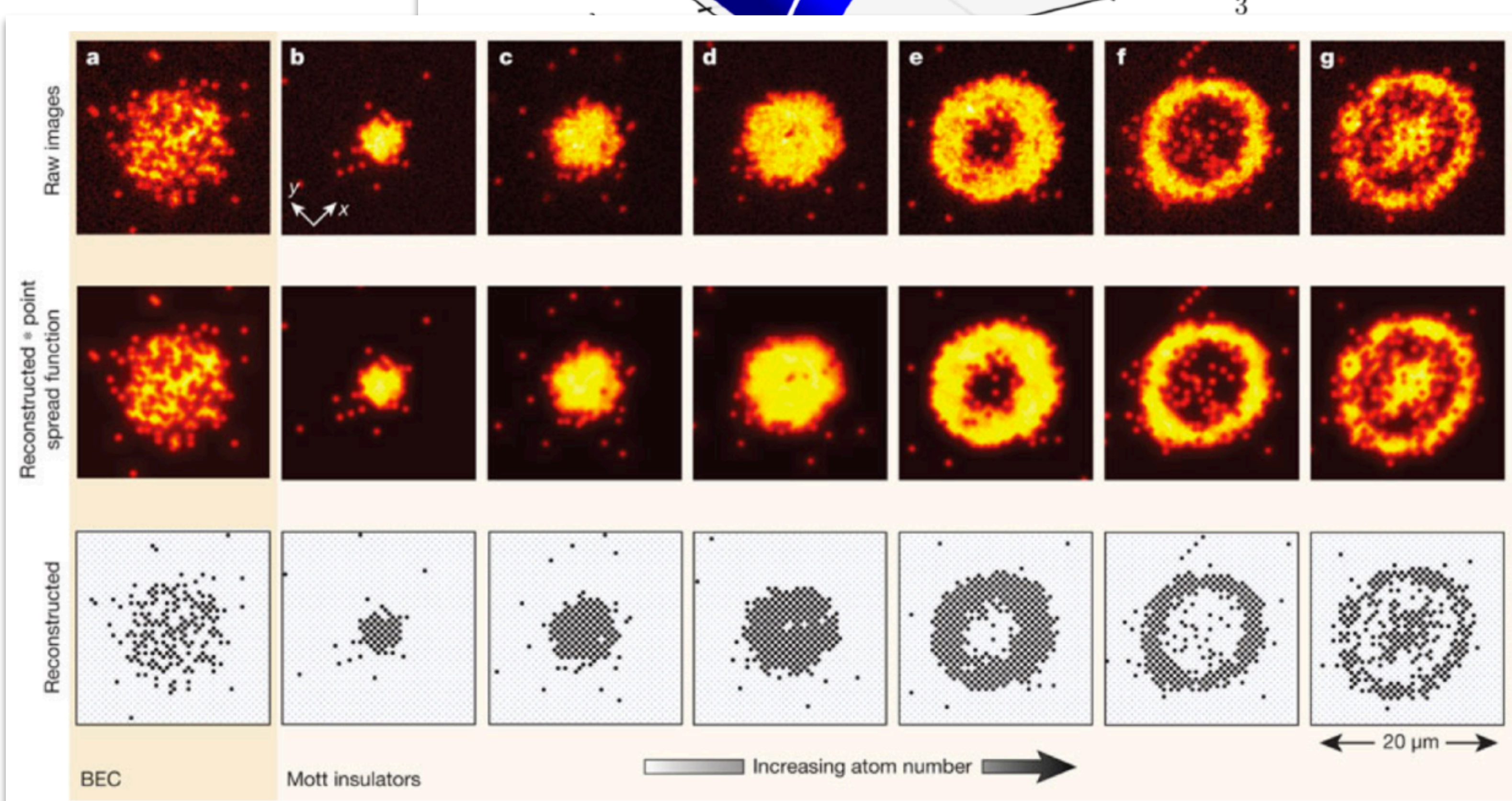
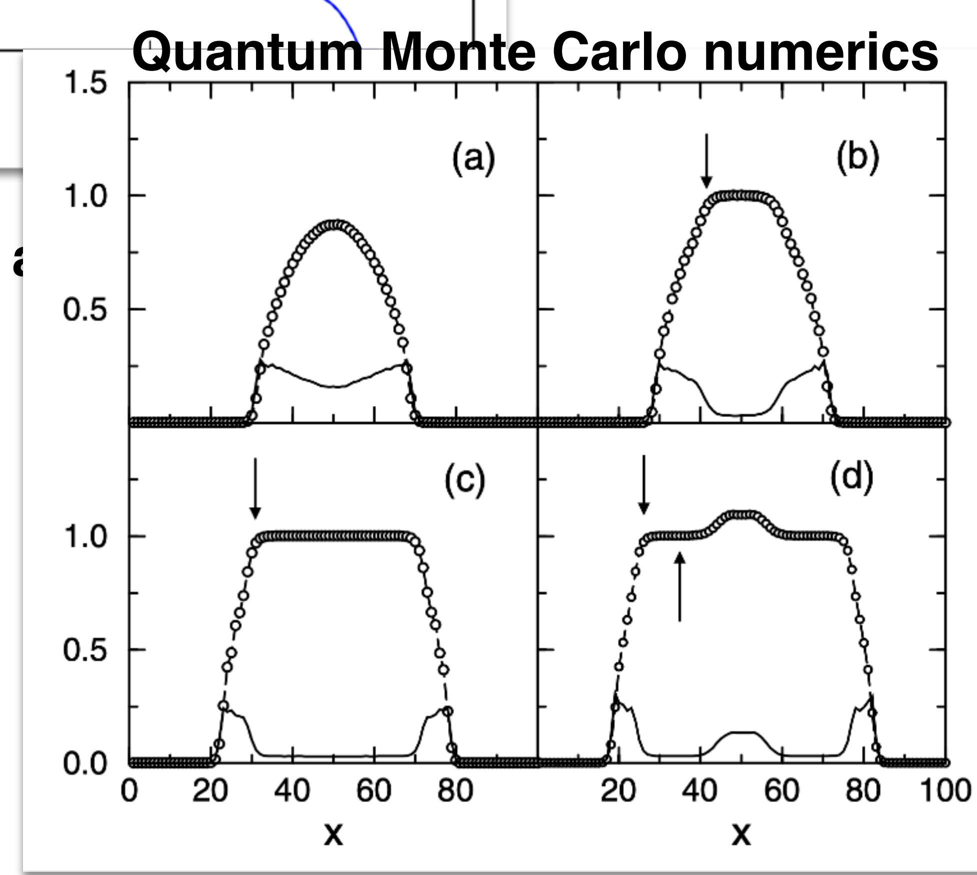


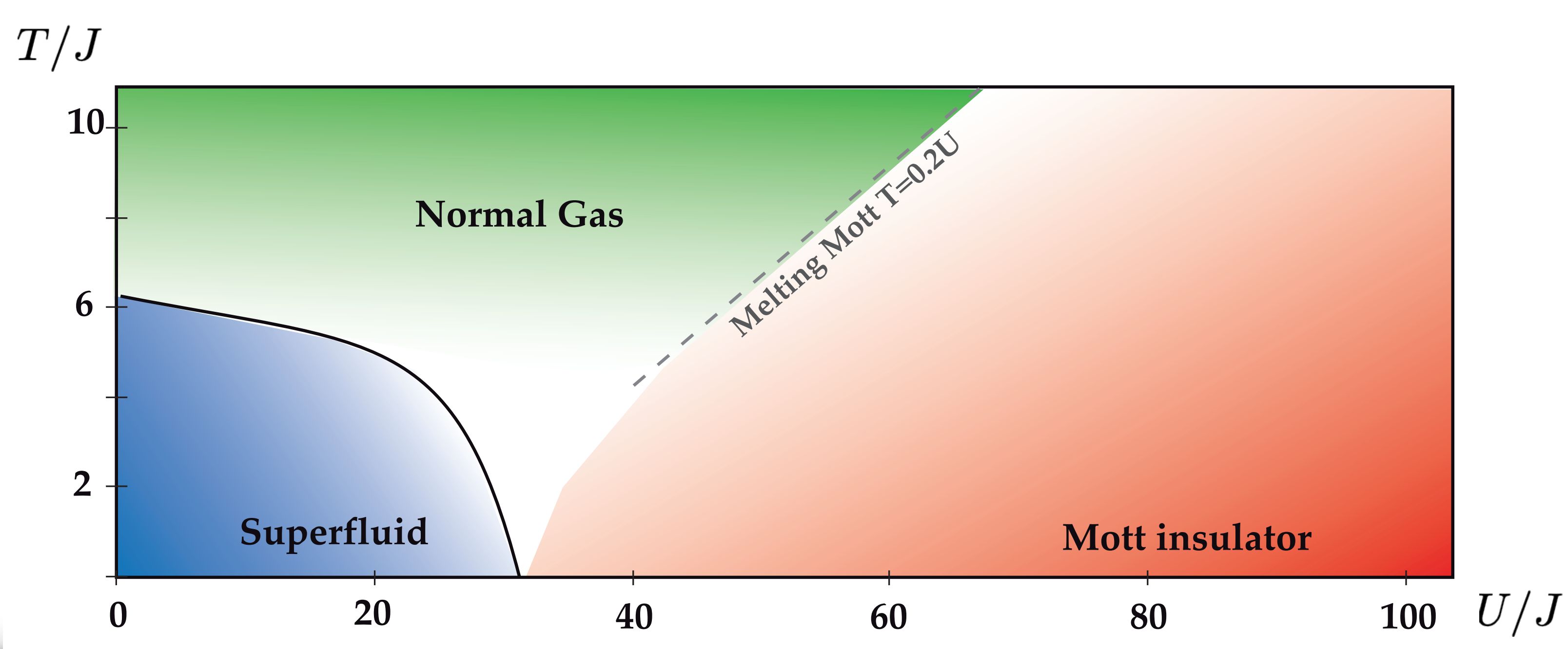


Sherson, *et al.* Nature **467**, 68-72 (2010)



G. Batrouni, *et al.* Phys. Rev. Lett. **89**, 117203 (2002)

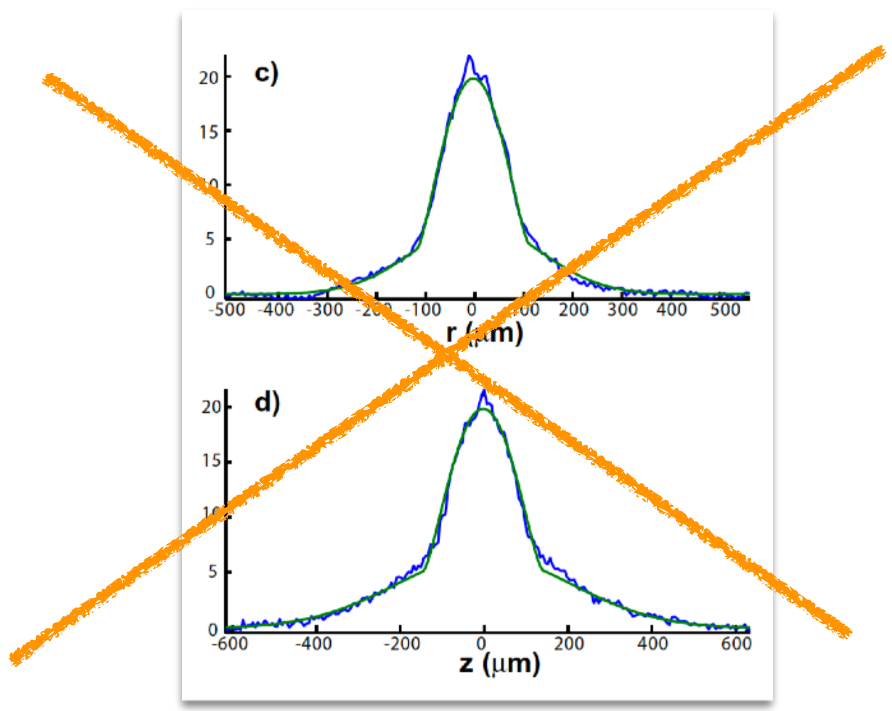
Thermometry of lattice Bose gases



Experiments are conducted at finite temperature!

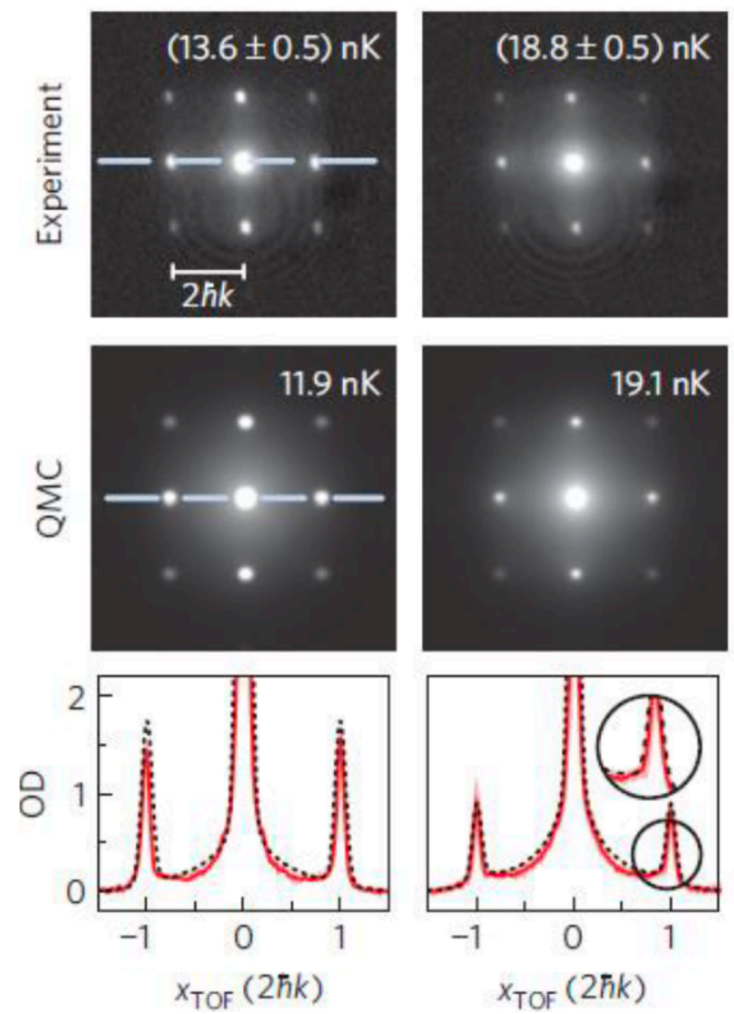
→ **observation of a finite temperature crossover rather than the quantum phase transition (T=0)**

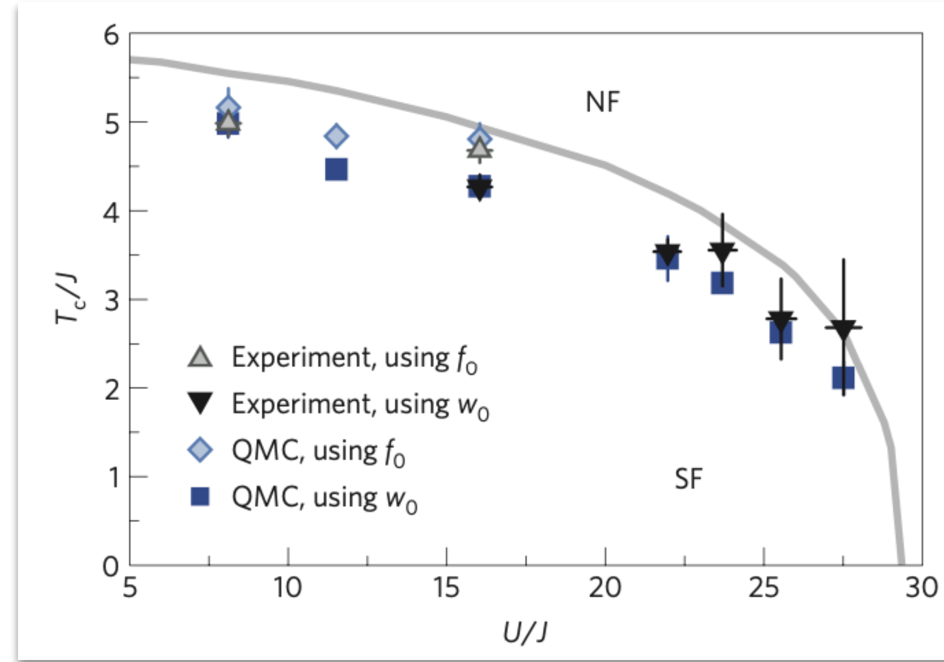
How to measure the temperature?



Thermometry of lattice Bose gases

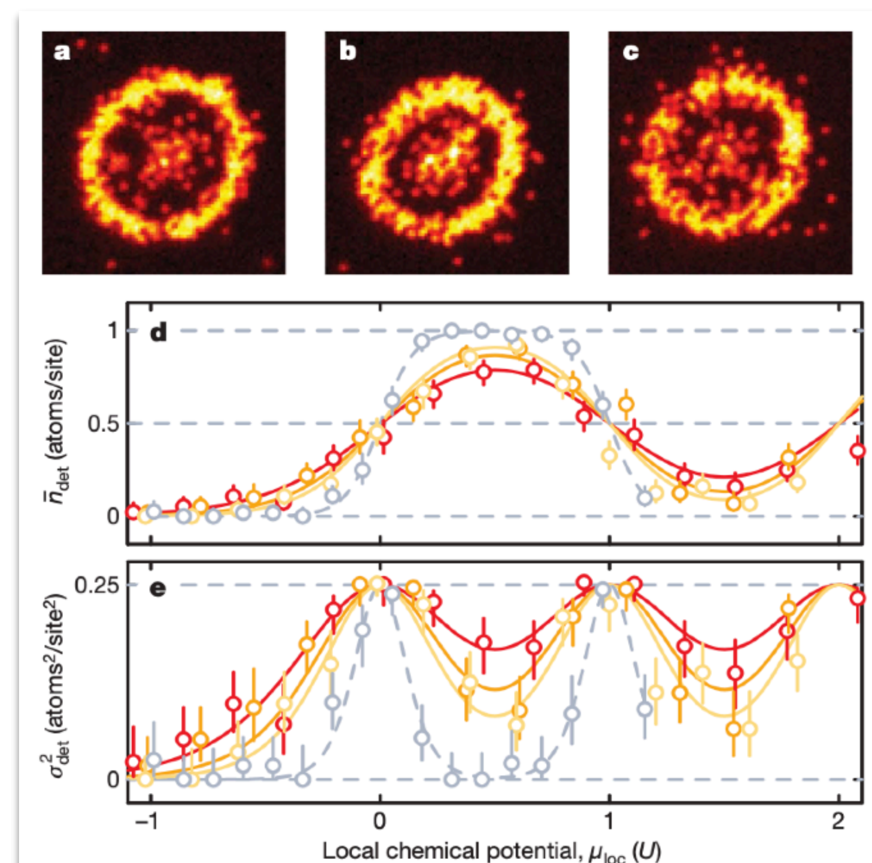
3D - comparison with ab-initio QMC calculations after TOF





Trotzky et al. Nat. Phys. ***6****, 998 (2010)*

2D - site-resolved atom number fluctuations in the Mott regime



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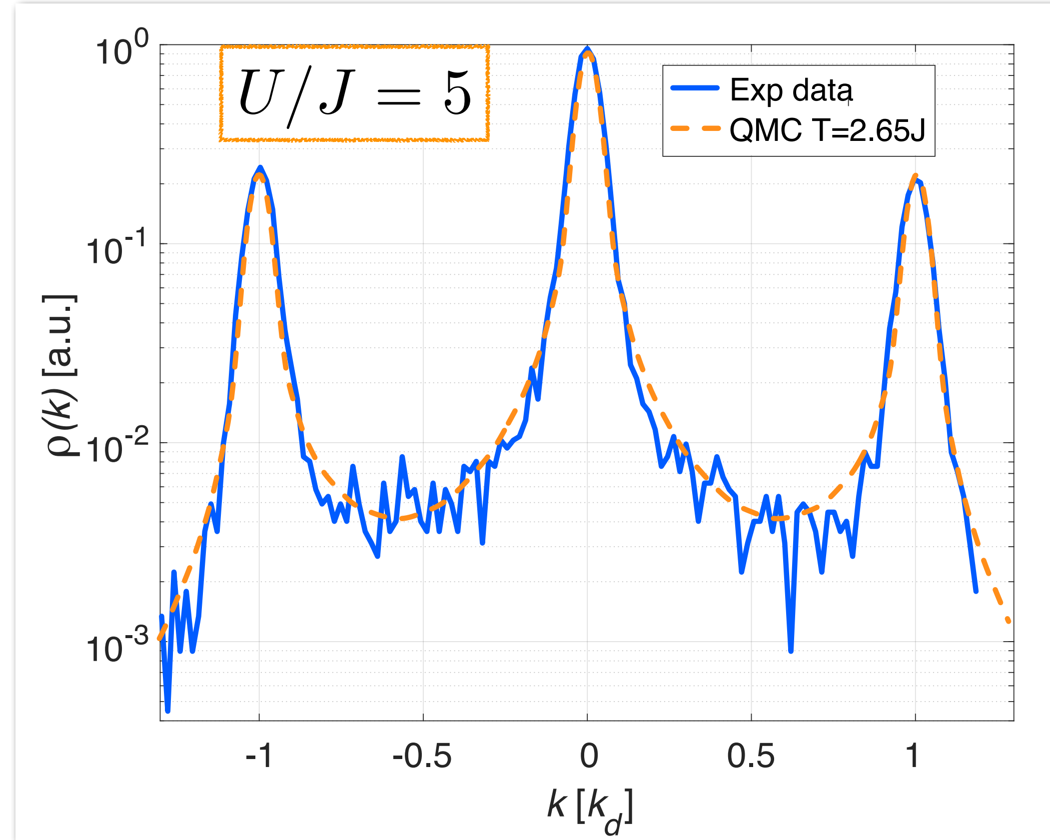
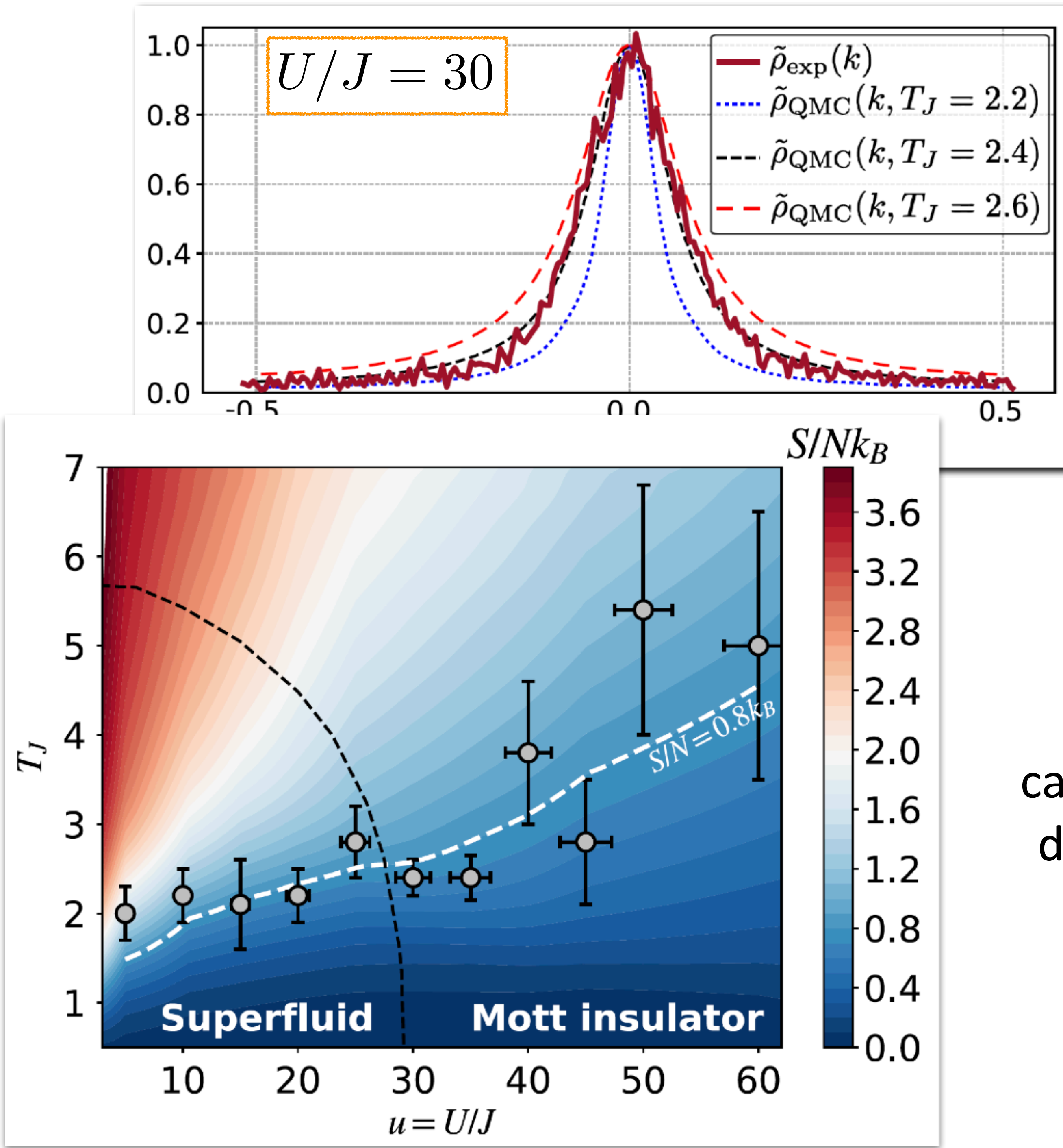


Cayla et al. Phys. Rev. A ***97****, 061609 (2018)*



Carcy et al. Phys. Rev. Lett. ***126****, 045301 (2021)*

3D - comparison with ab-initio QMC calculations after long TOF in the entire phase diagram: **compatible with adiabatic loading**

saturates Cramer-Rao bound everywhere!

$$\delta T = \frac{1}{\sqrt{I(T)M}}$$