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SYRTE
Observatoire de Paris, PSL, CNRS,
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Cold Atom Predoc School
Quantum simulations with ultracold atomic gases
Les Houches, 13-24 September 2021
SYRTE is a Department of the Paris Observatory

SYRTE research activities:
- Time and Frequency Metrology (LNE-SYRTE)
- Fundamental Astronomy
- History of sciences (astronomy)

LNE-SYRTE research activities:
- Atomic time scales (French Atomic Time, TAI)
- Atomic clocks (µwave + optical)
- Links (µwave, optical links)
- Atom Interferometry and Inertial Sensors
Atomic Interferometry and Inertial Sensors Team

8 permanent staff (5 CNRS, 1 LNE, 1 MDC, 1 Emeritus) + 10 students and post-docs

Our activities:
Development of high performance atom interferometers (for inertial and gravity sensing)

https://syrte.obspm.fr/spip/science/iaci/
Laser cooling and trapping: a bit of history

1933: demonstration by Otto Frisch of radiation pressure with resonance lamp (deflection of a thermal beam of Na, limited by the low brightness of the lamp)

60's: intensity gradients shown (in theory) to induce forces, dipole force proposed to trap atoms (Letokhov) and used to trap small glass spheres (Ashkin)

70's: first proposals for the use of tunable lasers for selective radiative forces. Hansch and Schawlow (for atoms), and Wineland and Dehmelt (for ions): Doppler cooling

80's: (experimental) advent of laser cooling and trapping of neutral atoms

The Nobel Prize in Physics 1997 was awarded jointly to Steven Chu, Claude Cohen-Tannoudji and William D. Phillips "for development of methods to cool and trap atoms with laser light."
Laser cooling and trapping

Laser cooling methods were instrumental in several breakthroughs

1995: Bose Einstein condensation demonstrated in dilute gases

The Nobel Prize in Physics 2001 was awarded jointly to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman "for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates."
Laser cooling and trapping

The Nobel Prize in Physics 2012 was awarded jointly to Serge Haroche and David J. Wineland "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems."

Wineland: First laser cooling of ions (Mg II)

Laser cooling is instrumental for realizing accurate ion clocks (Be+, Hg+, Al+)

Second first laser cooling of ions (Ba+)
W. Neuhauser, M. Hohenstatt, P. Toschek, and H. Dehmelt

The Nobel Prize in Physics 1989 was divided, one half awarded to Norman F. Ramsey "for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks", the other half jointly to Hans G. Dehmelt and Wolfgang Paul "for the development of the ion trap technique."
Organization of the lecture

1 : Radiative forces

2 : Doppler cooling

3 : The magneto optical trap

4 : Technological aspects
Organization of the lecture

1 : Radiative forces

2 : Doppler cooling

3 : The magneto optical trap

4 : Technological aspects
Basic principles

A two level atom, with ground state \( g \) and excited state \( e \)

A photon with energy \( h\nu \) and momentum \( \hbar k \)

The atom, initially in its ground state, absorbs the photon.

Conservation of energy → it changes electronic state

Conservation of momentum → it changes momentum

\[ p \rightarrow p + \hbar k \]

Corresponding change of velocity : \( \Delta v = v_r = \frac{\hbar k}{M} \) (recoil velocity)

For Rb, \( \Delta v \sim 6 \text{ mm/s} \) to be compared with thermal velocity \( v_T \sim \sqrt{k_B T / M} \sim 100 \text{ m/s} - 1 \text{ km/s} \)
Basic principles

![Energy level diagram]

Excited state with a finite lifetime $\tau = 1/\Gamma$

The atom will then deexcite via spontaneous decay by scattering a photon in a random direction

Absorption/emission cycles occur at a maximum rate $\sim \Gamma$

This leads to a (mean) force $F \sim \hbar k \Gamma$

And to a (maximal) acceleration $a = F/M \sim \hbar k \Gamma / M$

For Rb atoms, $a \sim 105 \, g$

With such an acceleration, we could slow down an atom at 100 m/s in ~1 ms and over a distance of ~ 5 cm
Basic principles

Let’s have a closer look at **energy conservation**

Before

An atom in state $g$, with momentum $p$

A photon with frequency $\nu$

\[
E_g + \frac{p^2}{2M} + h\nu
\]

After

An atom in state $e$, with momentum $p + \hbar k$

\[
E_e + \frac{(p + \hbar k)^2}{2M}
\]
Conservation of energy:

\[ E_g + \frac{p^2}{2M} + \hbar \nu = E_e + \frac{(p+\hbar k)^2}{2M} \]

\[ \hbar \omega_g + \frac{p^2}{2M} + \hbar \omega_L = \hbar \omega_e + \frac{p^2}{2M} + \hbar k \frac{p}{M} + \frac{(\hbar k)^2}{2M} \]

\[ \omega_L = \omega_e - \omega_g + \vec{k} \cdot \vec{v} + \frac{\hbar k^2}{2M} \]

Recoil shift: \( \frac{\delta \omega_r}{2\pi} \sim \text{few kHz} \)

Recoil energy: \( E_r = \frac{(\hbar k)^2}{2M} \)

For \( v = v_T \), Doppler (frequency) shift

\[ \delta v = \frac{\delta \omega}{2\pi} = \frac{\nu_T}{\lambda} \sim 100 \text{ MHz} - 1 \text{ GHz} \]
Atoms interacting with the electromagnetic field (laser+vacuum) is a quantum problem.

Laser field \( \hbar \omega_L \) \( \xrightarrow{\hat{V}_{AL}} \) \( \xleftarrow{\hat{V}_{AR}} \) Atoms \( \hbar \omega_0 \) \( \xrightarrow{\hat{V}_{AL}} \) Vacuum field

\( \omega_L \) close to \( \omega_0 \): Two level system

Coupling to the vacuum field: spontaneous decay of excited states

Hamiltonian of the system:

\[
\hat{H} = \hat{H}_A + \hat{H}_R + \hat{V}_{AL} + \hat{V}_{AR}
\]

with \( \hat{H}_A = \hbar \omega_0 |e\rangle\langle e| + \frac{\hat{p}^2}{2m} \) and \( \hat{H}_R = \sum \hbar \omega_l \hat{a}_l^+ \hat{a}_l \)
Interaction between the atoms and the field is the electric dipole interaction.

The electric field: \( E(\vec{r}, t) = \frac{\varepsilon(\vec{r})}{2} (\varepsilon(\vec{r}) e^{-i\omega t - i\varphi(\vec{r})} + c. c.) \) is coupled to the electric dipole moment \( D = d(|e\rangle \langle g| + |g\rangle \langle e|) \)

The coupling term \( \hat{V}_{AL} = -D.E(\vec{R}, t) \) is given (in RWA and semiclassical approximation) by:

\[
\hat{V}_{AL} \approx \frac{\hbar \Omega(\vec{r})}{2} (|e\rangle \langle g| e^{-i\omega t - i\varphi(\vec{r})} + h. c.)
\]

with \( \Omega(\vec{r}) \) the Rabi frequency, which characterizes the strength of the coupling.

\[
\Omega(\vec{r}) = \frac{(d. \varepsilon(\vec{r})) \varepsilon(\vec{r})}{\hbar}
\]

**NB**: Link with the laser intensity: \( I \propto \varepsilon^2 \propto \Omega^2 \)

\[
\frac{2\Omega^2}{\Gamma^2} = \frac{I}{I_{sat}}, \quad I_{sat} \text{ the saturation intensity}
\]
Atom light interaction

Important parameters that govern this evolution:

- For the coherent part, that corresponds to the interaction of the atoms with the laser:

  - $\Omega(\vec{r})$ the Rabi frequency
  - $\Delta = \omega_L - \omega_0$ the (laser) detuning to (the atomic) resonance

- For the incoherent part, that corresponds to the interaction of the atoms with the vacuum field:

  - $\Gamma$ the spontaneous decay rate
Calculation of the force:

Force operator in the Heisenberg operator picture

\[
\frac{d\hat{R}}{dt} = \frac{1}{i\hbar} [\hat{R}, \hat{H}] = \frac{1}{i\hbar} [\hat{R}, \hat{H}_A] = \hat{P}
\]

\[
\hat{F} = \frac{d\hat{P}}{dt} = \frac{1}{i\hbar} [\hat{P}, \hat{H}] = -\nabla\hat{V}_{AL} - \nabla\hat{V}_{AR}
\]

The force \( F \) is the expectation value of the force operator \( F = \langle \hat{F} \rangle = -\langle \nabla\hat{V}_{AL} \rangle \)
\( \langle \nabla\hat{V}_{AR} \rangle = 0 \) since spontaneous photons are scattered in random directions

\[
F = \langle \hat{F} \rangle = -\langle \nabla\hat{V}_{AL} \rangle = \langle \nabla \mathbf{D} \cdot \mathbf{E}(\hat{R}, t) \rangle = \langle \mathbf{D} \cdot \nabla \mathbf{E}(\hat{R}, t) \rangle
\]

Semiclassical approximation:

\( F \) is calculated at position \( \hat{r} = \langle \hat{R} \rangle \) and for a momentum \( \hat{p} = \langle \hat{P} \rangle \)

\[
\rightarrow \langle \mathbf{D} \cdot \nabla \mathbf{E}(\hat{R}, t) \rangle = \langle \mathbf{D} \rangle \cdot \nabla \mathbf{E}(\hat{r}, t)
\]

\( \mathbf{D} \) evolves at a rate of order \( 1/\Gamma \), much faster than the atom position/velocity
\( \mathbf{D} \) replaced by its steady state \( \mathbf{D}_{st} \)

\[
\rightarrow F = \langle \mathbf{D}_{st} \rangle \cdot \nabla \mathbf{E}(\hat{r}, t)
\]
Atom light interaction

Evolution of the atomic system and thus of $D$ can be calculated by using the Optical Bloch Equations, which allow to treat both coherent and incoherent processes.

OBE are equations of the evolution of the internal state matrix density $\hat{\sigma}$ given by

$$i\hbar \frac{d\hat{\sigma}}{dt} = [\hat{H}_A + \hat{V}_{AL}, \hat{\sigma}] - i\hbar \hat{\Gamma} \hat{\sigma}$$

The average dipole is given by $\langle D \rangle = Tr\{D\hat{\sigma}\} = d(\sigma_{eg} + \sigma_{ge})$

After solving OBE, one finds

$$\langle D_{st} \rangle \cdot \epsilon(\vec{r}) = 2d \cdot \epsilon(\vec{r}) \frac{s(\vec{r})}{1+s(\vec{r})} \left( \frac{\Delta}{\Omega(\vec{r})} \cos(\omega t + \varphi(\vec{r})) - \frac{\Gamma}{2\Omega(\vec{r})} \sin(\omega t + \varphi(\vec{r})) \right)$$

With $s = \frac{\Omega(\vec{r})^2/2}{\Delta^2 + \Gamma^2/4}$

In phase with the electric field
Related to the real part of the polarizability

Out of phase with the electric field
Related to the imaginary part of the polarizability and thus to absorption

Conservative force
Zero at resonance

Dissipative force
Maximum at resonance
Radiative forces

One finds two contributions:

- A dissipative force, the **radiation pressure force**, linked to the transfer of momentum in resonant scattering processes. It is governed by the scattering rate $\Gamma'$.

  \[ F_{rp} = \hbar k \Gamma' = \hbar k \frac{\Gamma}{2} \frac{s(\vec{r})}{1 + s(\vec{r})} \]

  with \( s(\vec{r}) = \frac{\Omega(\vec{r})^2}{\Delta^2 + \Gamma'^2/4} \)

  the saturation parameter

- A dispersive force, the **dipole force**, which can be interpreted as a position dependent light shift due to spatially varying intensity. It is governed by the intensity gradient. It results from a redistribution of photons in the laser field in absorption-stimulated emission cycles

  \[ F_{dip} = -\frac{\hbar \Delta}{2} \frac{\nabla s(\vec{r})}{1 + s(\vec{r})} \]

  It derives from the dipole potential \( U_{dip} = \frac{\hbar \Delta}{2} \ln(1 + s(\vec{r})) \)
The radiation pressure

It is due to absorption-spontaneous emission cycles that happen at a rate \( \Gamma' = \frac{\Gamma s}{2(1+s)} \)

\[
F_{rp} = \hbar k \frac{\Gamma}{2} \frac{s}{1+s} \quad \text{with} \quad S = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4}
\]

Or

\[
F_{rp} = \hbar k \frac{\Gamma}{2} \frac{\Omega^2/2}{\Omega^2/2 + \Delta^2 + \Gamma^2/4}
\]

It saturates at \( F_{max} = \hbar k \frac{\Gamma}{2} \)

At low intensity \( (\Omega < \Gamma) \), the width versus \( \Delta \) is given by \( \Gamma \)

At high intensity \( (\Omega \gg \Gamma) \), the width is given by \( \Omega \) (power broadening)
The radiation pressure

Let us consider an atom with a velocity $v$ and a laser at resonance.

\[ \tilde{k} \quad \tilde{v} \]

The Doppler shift corresponds to a detuning $\Delta = \tilde{k} \cdot \tilde{v}$

\[
F_{rp} = \hbar k \frac{\Gamma}{2} \frac{\Omega^2/2}{\Omega^2/2 + (\tilde{k} \cdot \tilde{v})^2 + \Gamma^2/4}
\]

At low intensity, the force reduces drastically when $\tilde{k} \cdot \tilde{v} \gg \Gamma$

This corresponds to $v \gg v_L = \lambda \Gamma/2\pi$

For Rb, $\lambda = 780$ nm, $\Gamma/2\pi = 6$ MHz, this gives $v_L = 4.7$ m/s

This gets somehow larger when increasing the intensity.

**But then how could I stop an atom at thermal velocities (1 km/s) ?**
Zeeman slower

How to keep the atoms in resonance?

Idea (B. Phillips) : tune the energy of the atomic states with a magnetic field

The B field amplitude is inhomogeneous.

It varies so as to compensate via the Zeeman shift the change in Doppler shift

The atoms remain on resonance

Need of a B field amplitude of order of 1000 G for a change of frequency of order of 1 GHz \((\gamma \sim 1 \text{ MHz/G})\)
Atoms slowed down by about 40% of their initial thermal velocity

15000 photons absorbed
The dipole force

Expression: \( F_{\text{dip}} = -\frac{\hbar \Delta}{2} \frac{\nabla s(\vec{r})}{1+s(\vec{r})} \) with \( s = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} \)

It is null at resonance and changes sign with the detuning (dispersive)

It derives from the dipole potential \( U_{\text{dip}} = \frac{\hbar \Delta}{2} \ln(1 + s(\vec{r})) \)

In the low saturation regime \( s \ll 1 \), \( U_{\text{dip}} = \frac{\hbar \Delta}{2} s(\vec{r}) \) and \( F_{\text{dip}} = -\frac{\hbar \Delta}{2} \nabla s(\vec{r}) \)

• For large detuning, \( s = \frac{\Omega^2/2}{\Delta^2} \) and \( U_{\text{dip}} = \frac{\hbar \Omega^2}{4\Delta} \)

• \( \Delta < 0 \): Attractive potential:
The atoms are attracted towards the intensity maxima

• \( \Delta > 0 \): Repulsive potential

• Spontaneous emission rate: \( \Gamma' = \frac{\Gamma}{2} s = \frac{\Gamma}{\Delta} U_{\text{dip}} \)

When \( \Delta \gg \Gamma \), spontaneous emission may become negligible

The (total) force is dominated by the dipole force, which is conservative
The dipole force

- $\Delta < 0$ (red detuning) : Attractive potential

Crossed dipole trap

$w_a = 30 \ \mu m$

$w_b = 200 \ \mu m$

$\lambda = 1070 \ \text{nm}$

Absorption imaging of atoms trapped at the crossing
X. Alauze, SYRTE

Optical lattices

I. Bloch Nature Physics 1, 23 (2005)

Courses to come by David Clément
The dipole force

- $\Delta > 0$ (blue detuning) : repulsive potential

Mirror for atom waves

Box-shape potential with one hollow “tube” beam + two “sheet” beams

Aminoff et al., PRL 71, 3083 (1993)

Gaunt et al, PRL 110, 200406 (2013)
Organization of the lecture

1: Radiative forces
2: Doppler cooling
3: The magneto optical trap
4: Technological aspects
Doppler cooling

Let us send two red-detuned counterpropagating laser beams

In the frame of the atom, the lasers are Doppler shifted in opposite directions

The force exerted by the « right » beam is larger than the one by the « left »

As a result, the net force is opposed to the motion of the atom
Doppler cooling

In the low saturation regime, the net force is the sum of the two forces

\[ F_{\text{sum}} = F_{+k} + F_{-k} = \hbar k (s_{+k} + s_{-k}) \]

with

\[ s_{\varepsilon k} = \frac{\Omega^2/2}{(\Delta - \varepsilon k v)^2 + \Gamma^2/4} \]

At low velocity, \( v \ll \Gamma/k \), the (linearised) force is a friction force: \( F_{\text{sum}} = -\alpha v \)

with \( \alpha = \frac{-2\Delta \Gamma}{\Delta^2 + \Gamma^2/4} \hbar k^2 s_0 \) and \( s_0 = \frac{2\Omega^2}{\Gamma^2} = \frac{I}{l_{\text{sat}}} \)

The friction \( \alpha \) is maximum for \( \Delta = -\frac{\Gamma}{2} \) and \( \alpha_{\text{max}} = 2\hbar k^2 s_0 \)

Equation of motion : \( m \ddot{v} = -\alpha v \). The solution is \( v(t) = v(0)e^{-\frac{\alpha}{m}t} \).

Damping time : \( \tau = \frac{m}{\alpha} = \frac{\hbar}{4E_r s_0} \)

Order of magnitude (Rb with \( s_0 = 0.1 \)) : \( \sim 100 \mu s \)
Optical molasses

Generalization to 3D:
3 pairs of counterpropagating beams

In the low saturation regime, the net force is the sum of the 6 forces

\[ F_{\text{sum}} = \sum_{i=1}^{6} F_j(v) \]

If all beams have the same intensities, and for small velocities, we find

\[ F_{\text{sum}} = -\alpha v \]

Optical molasses for sodium atoms
Credit: K. Helmerson (NIST).
Optical molasses

The friction is related to the average force experienced by the atoms,

but there are fluctuations of the force as well related to the scattering events.

\[ m \dot{v} = -\alpha v + F(t) \]

The dynamic in the momentum space is a Brownian motion with momentum kicks \( \hbar k \) in random directions when photons are absorbed and emitted

\[ m \mathbf{v}(t) = m \mathbf{v}(0)e^{-\frac{\alpha}{m}t} + \int_0^t e^{-\frac{\alpha}{m}(t-t')} F(t')dt' \]

The evolution of the kinetic energy is given by

\[ \frac{d\langle p_j^2(t) \rangle}{dt} = 2 \left( p_j(t) \frac{dp_j}{dt} \right) = -2 \frac{\alpha}{m} \langle p_j^2(t) \rangle + 2 \langle p_j(t)F_j(t) \rangle \]

The correlation between the force fluctuations and the momentum is

\[ \langle p_j(t)F_j(t) \rangle = \langle p_j(0)F_j(t) \rangle e^{-\frac{\alpha}{m}t} + \int_0^t e^{-\frac{\alpha}{m}(t-t')} \langle F_j(t')F_j(t) \rangle dt' = 0 + D \]

With \( D \) the momentum diffusion coefficient
Optical molasses

The evolution of the kinetic energy is then given by

\[ \frac{d\langle p_j^2(t) \rangle}{dt} = -2 \frac{\alpha}{m} \langle p_j^2(t) \rangle + 2D \]

Whose steady state corresponds to

\[ \langle p_j^2 \rangle = \frac{D}{\alpha} \]

This corresponds to an effective temperature given by

\[ \frac{1}{2} k_B T = \frac{1}{2} \langle p_j^2 \rangle = \frac{1}{2} \frac{D}{\alpha} \]

\[ k_B T = \frac{D}{\alpha} \]
Optical molasses

How to calculate the momentum diffusion coefficient?

\[
\frac{d\langle p_j^2(t) \rangle}{dt} = -2 \frac{\alpha}{m} \langle p_j^2(t) \rangle + 2D \quad D \text{ is related to the rate at which the kinetic energy increases in absorption/emission cycles}
\]

Over a time \( \Delta t \) there are \( \Delta N \) momentum kicks \( \hbar k \) in a random direction

This gives an increase of \( \Delta \langle p_j^2 \rangle = \frac{1}{3} (\hbar k)^2 \Delta N \)

And a rate of increase of \( \frac{\Delta \langle p_j^2 \rangle}{\Delta t} = \frac{1}{3} (\hbar k)^2 \frac{\Delta N}{\Delta t} = \frac{1}{3} (\hbar k)^2 R \), where \( R \) the rate of scattering events

This leads to \( D = \frac{1}{6} (\hbar k)^2 R \)

In the molasses, the rate of event is \( R = 6 \left( \frac{\Gamma}{2} s_0 + \frac{\Gamma}{2} s_0 \right) \)

Rate of spontaneous emission

6 beams \quad Rate of absorption

Finally we get \( D = (\hbar k)^2 \Gamma s_0 \)
Optical molasses

Temperature

\[ k_B T = \frac{D}{\alpha} \]

Friction

\[ \alpha = \frac{-2\Delta\Gamma}{\Delta^2 + \Gamma^2/4} \hbar k^2 s_0 < \alpha_{\text{max}} = 2\hbar k^2 s_0 \]

Momentum diffusion coefficient

\[ D = (\hbar k)^2 \Gamma s_0 \]

\[
T = \frac{\hbar \Delta^2 + \Gamma^2/4}{2k_B |\Delta|} > T_{\text{min}} = \frac{\hbar \Gamma}{2k_B}
\]

For Rb, \( T_{\text{min}} = 145 \, \mu K \)

The momentum distribution is Gaussian:

\[ f(p) = \frac{1}{(2\pi mk_B T)^{3/2}} e^{\frac{-p^2}{2mk_B T}} \]

For Rb, the rms velocity width:

\[ \sigma_v = \sqrt{\frac{k_B T}{m}} = 12 \, \text{cm/s} \]
What is the capture velocity of the molasses? This depends on the interaction length (i.e., the size of the laser beam)

\[
m \dot{v} = F(v) \Rightarrow m v \frac{dv}{dx} = F(v) \Rightarrow dx = \frac{mv}{F(v)} dv
\]

\[
L = \int dx = \int_{v_c}^{0} \frac{mv}{F(v)} dv
\]

This gives \( v_c = \frac{\Gamma}{k} \left( 3 \frac{L}{l} \frac{I}{I_{sat}} \right)^{\frac{1}{4}} \) with \( l = \frac{M\Gamma}{\hbar k^3} \)

For Rb, \( l = 10 \mu m \)
For \( L = 1 \) inch and \( s_0 = \frac{l}{I_{sat}} = 0.1 \), we find \( v_c = 13 \) m/s

In practice, larger \( s_0 \) are used, which increases \( v_c \)

For typical parameters (\( s_0 \sim 1 \)), the optimal detuning is around \( \Delta = -3\Gamma \)
Optical molasses

Does this theory work?

Observation of Atoms Laser Cooled below the Doppler Limit

Paul D. Lett, Richard N. Watts, Christoph I. Westbrook, and William D. Phillips

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(Received 18 April 1988)

We have measured the temperature of a gas of sodium atoms released from “optical molasses” to be as low as $43 \pm 20 \mu K$. Surprisingly, this strongly violates the generally accepted theory of Doppler cooling which predicts a limit of $240 \mu K$. To determine the temperature we used several complementary measurements of the ballistic motion of atoms released from the molasses.
Optical molasses

Does this theory work?

With metastable He

With Mercury (Hg)

It works for isotopes with zero (nuclear) spin!

R. Chang et al, PRA 90, 063407 (2014)

Organization of the lecture

1: Radiative forces

2: Doppler cooling

3: The magneto optical trap

4: Technological aspects
Magneto optical trap

Let us consider a $J = 0 \rightarrow J = 1$ transition

Let us add a magnetic field gradient $B = bx$
and use for the counterpropagating laser beams opposite circular polarisation

\[ \sigma_+ \] \hspace{1cm} \sigma_- \]

The magnetic field and polarisation induce position dependent detunings

\[ \Delta = \Delta_0 \pm g_J \mu_B bx \]

And thus unbalanced radiation pressures for atoms away from the zero magnetic field

This leads to a **trapping force** that pushes the atoms back to the center
Magneto optical trap

For small displacement $x$, the force is linear:

$$F = -\kappa x$$

with $$\kappa = g_j \mu_B b \frac{-2\Delta \Gamma}{\Delta^2 + \Gamma^2 / 4}$$

The overall equation of motion for small displacements and velocities is:

$$m \frac{dv}{dt} = F = -\alpha v - \kappa x$$

This is the equation of a **damped harmonic oscillator**!

Doppler cooling is still active and determines the temperature of the atoms, but also the size of the atomic cloud.

The energy being equally distributed between kinetic and potential energy, we have

$$\frac{1}{2} k_B T = \frac{1}{2} m \sigma_v^2 = \frac{1}{2} \kappa \sigma_x^2$$

For $\Delta = -\frac{\Gamma}{2}$, we have

$$\sigma_x = \left( \frac{\hbar \Gamma}{4 k g_j \mu_B b s_0} \right)^{1/2}$$

Rb, $b = 10 \text{G/cm}$, $s_0 = 0.1$

$$\sigma_x = 36 \mu\text{m}$$
Magneto optical trap

Generalization to 3D:

- 3 pairs of counterpropagating beams with opposite circular polarisations

- A pair of « quadrupole » coils, in anti-Helmholtz configuration which realize gradients along the three directions
Beauty of magneto optical traps

And many more …
Magneto optical trap

Capture velocity: \[ v_c = \frac{\Gamma}{k} \left( \frac{L}{l} \frac{1}{l_{sat}} \right)^{\frac{1}{2}} \] with \( l = \frac{M \Gamma}{\hbar k^3} \)

For \( s_0 = 0.1 \), and \( L = 1 \) inch, this gives for Rb \( v_c = 23 \) m/s

In practice, one operates

- at larger saturation parameters \( s_0 \geq 1 \)
- at detunings \( \Delta \) in the range \([-5\Gamma; -3\Gamma]\)

The parameters are set to optimize the capture efficiency (and not the temperature)

Also, the temperature of the MOT is influenced by the magnetic field gradient.

\( T \) is typically in the order of tens of \( \mu \)K
Magneto optical trap dynamics

Equation that governs the evolution of the number of atoms trapped in the MOT:

\[
\frac{dN}{dt} = R - \gamma N - \beta \int n^2(r, t) d^3r
\]

- \( R \) the loading rate is the flux of incoming trapped atoms. 
  \( R \) is in the range \([10^8; 10^{10}] \) at/s

- \( \gamma \) is the one-body loss rate, linked to collisions with the background gas. 
  \( \gamma \) depends on the background pressure. 
  It is in the range \([0.01; 1] / s \) for pressures in the range \([10^{-9}; 10^{-11}] \) mbar

- \( \beta \) is a two body loss rate related to binary collisions between trapped atoms. 
  It depends on the atoms, but is usually in the range \([10^{-9}; 10^{-11}] \) cm³/s
Magneto optical trap dynamics

For small R and/or large $\gamma$, one can neglect two body losses.

$$\frac{dN}{dt} = R - \gamma N \Rightarrow N(t) = \frac{R}{\gamma} (1 - e^{-\gamma t}) \Rightarrow N_{stat} = \frac{R}{\gamma}$$

For large R and small $\gamma$, one can neglect one body losses

$$\frac{dN}{dt} \approx R - \beta \frac{N^2}{V} \Rightarrow N_{stat} = \sqrt{\frac{RV}{\beta}}$$

For large MOTs, an other process is at play : photon reabsorption

The cloud become so dense that photons scattered by the atoms can be reabsorbed before leaving the cloud. This leads to a repulsive force that increases the size of the cloud (up ~ a few mm size).

This occurs at densities of order of $10^{10}$at/cm$^3$ and for atom numbers of order of $10^8$

Very large MOTs of order of $10^{10}$ atoms exhibit (dynamical) instabilities.
2D Magneto optical traps

The 2D MOT


« Pure 2D » MOT

J. Schoser et al., PRA, 66, 023410 (2002)

2D MOTs produce intense flux of $>10^{10}$ atoms/s at mean velocities of around 10-20 m/s

2D MOTs can be used as the 3D MOT source
Organization of the lecture

1 : Radiative forces

2 : Doppler cooling

3 : The magneto optical trap

4 : Technological aspects
b) « Surface MOT » or « mirror MOT » with 4 beams.
The 2 missing beams are generated by reflections on the surface.

c) « Grating MOT »
A single beam normal to the grating
4 other beams are generated by diffraction on the grating
the 6th being the reflected beam (0th order)
Magneto optical traps

Pyramidal (and even conical) MOTs

Lee et al., Optics Letters 21, 1177 (1996)

Replacing coils by (straight) wires

Magneto optical trap sources

• Atomic vapour in a cell

The reservoir can be an ampoule or a dispenser

• Effusive beam

Heated ovens produce intense beams of thermal (fast) atoms, slowed down using Zeeman slower or chirped lasers

• 2D MOTs

Intense source of slow atoms
### Magneto optical trap sources

Comparison between the different sources

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux</th>
<th>Size</th>
<th>Vacuum</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapour</td>
<td>$10^8$ atoms/s</td>
<td>negligible</td>
<td>Limited to $&gt;10^{-9}$ mbar</td>
<td>Low</td>
</tr>
<tr>
<td>Atomic beam</td>
<td>$10^{11}$-$10^{12}$ atoms/s</td>
<td>meter long setup</td>
<td>Differential pumping</td>
<td>High</td>
</tr>
<tr>
<td>2D-MOT</td>
<td>$10^9$-$10^{10}$ atoms/s</td>
<td>1 liter</td>
<td>Differential pumping</td>
<td>Medium</td>
</tr>
</tbody>
</table>
Articles:

On radiation forces:

A. Ashkin, Science, 210, 1081 (1980)
V.S. Letokhov and V.G. Minogin, Phys. Reports, 73, 1 (1981)
S. Stenholm, Rev. Mod. Phys. 58, 699 (1986)

On Doppler cooling:


On laser cooling:

Lectures of the 1997 Nobel Prize:
S. Chu, Rev. Mod. Phys. 70, 685 (1998),
C. Cohen-Tannoudji, Rev. Mod. Phys. 70, 707 (1998),
References

Books:


Harold J. Metcalf, Peter van der Straten, Laser Cooling and Trapping (Springer)

Lectures (in French):