Laser cooling and trapping II

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Cold Atom Predoc School
Quantum simulations with ultracold atomic gases
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Organization of the lecture

1 : What can be (laser) cooled?

2 : Narrow line cooling

3 : Sub-Doppler cooling

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1 : What can be (laser) cooled?

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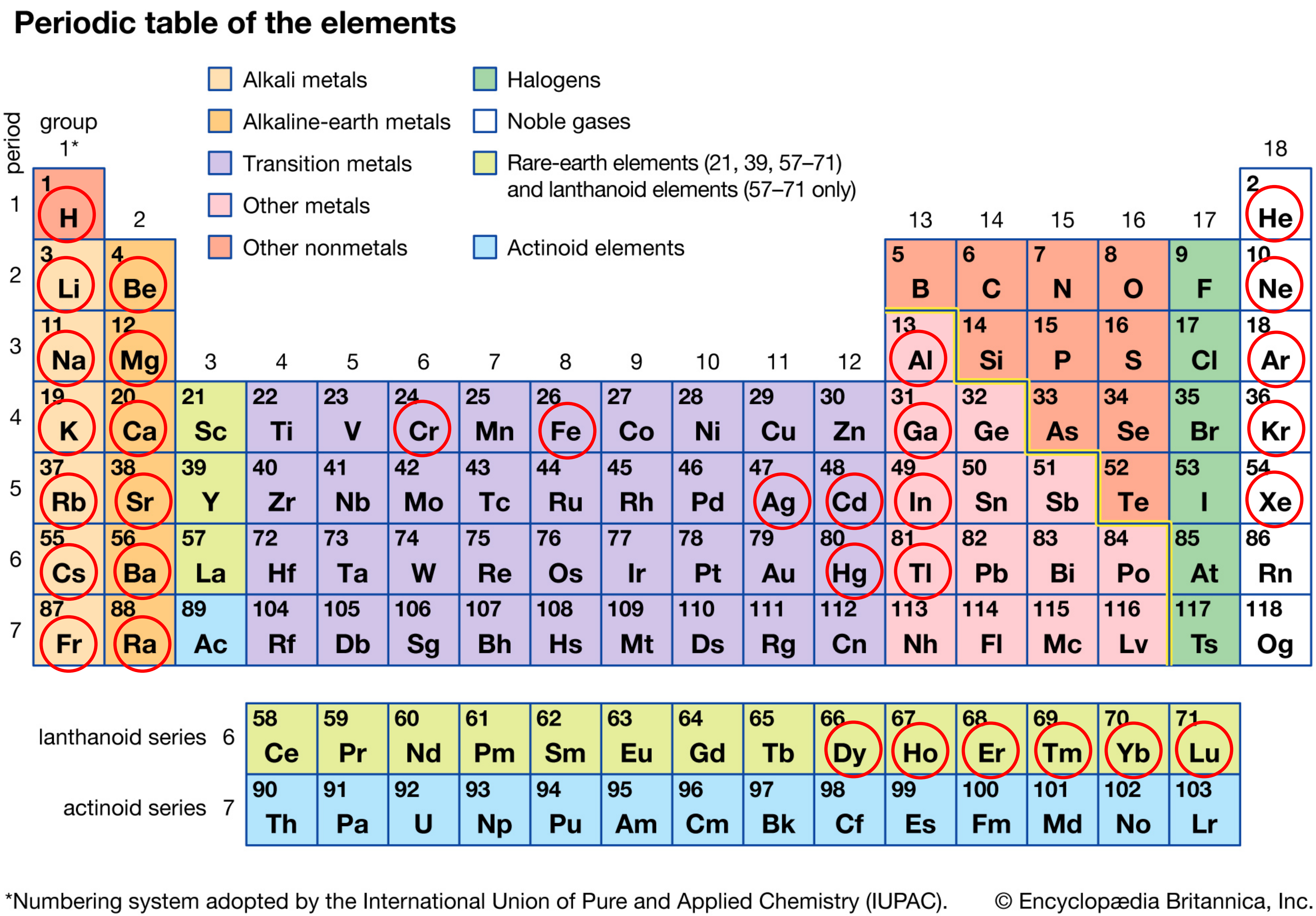
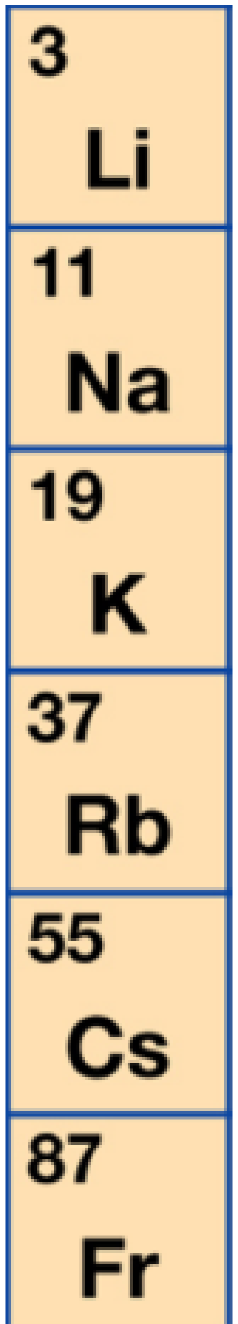
Laser cooled atoms

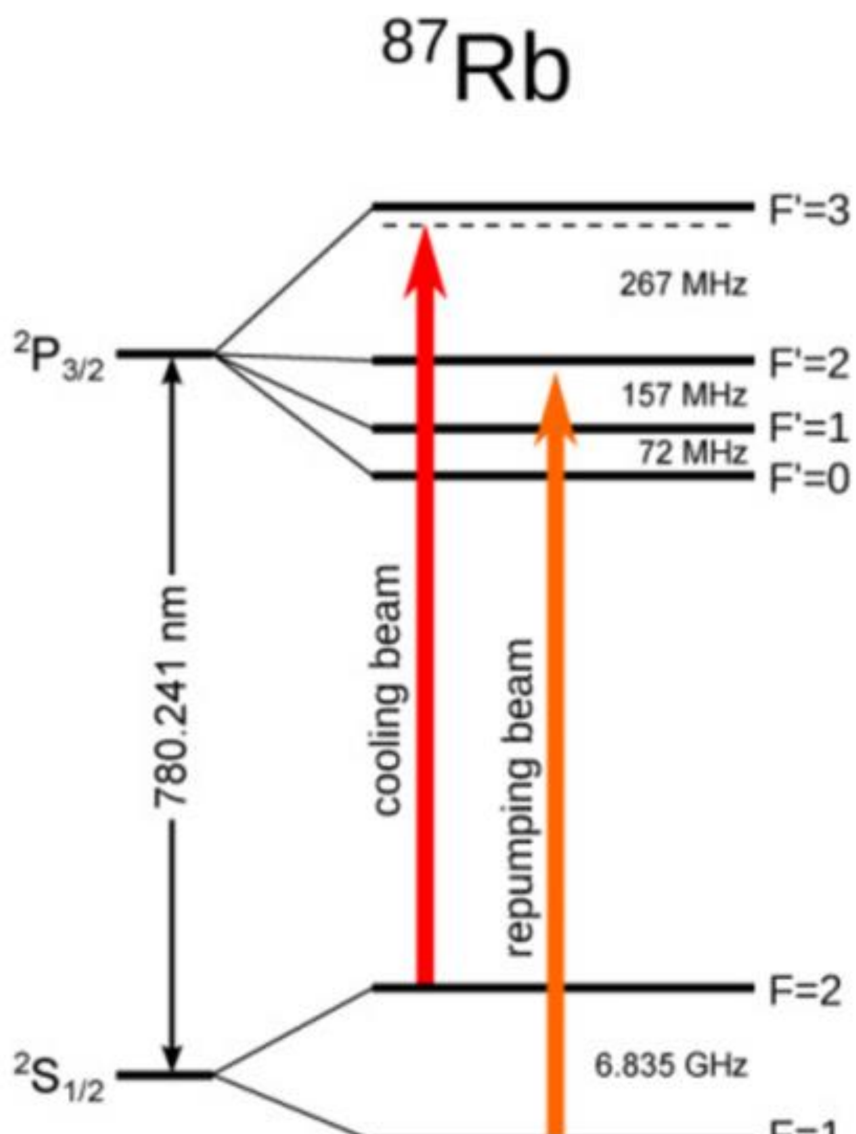
A large number of atomic species have been laser cooled (neutral + ions)



+ Anti-hydrogen *(C. J. Baker et al., Nature 592, 35–42 (2021))* + a few molecules (diatomic + polyatomic)

Alkali metals





Simplified energy diagram for rubidium

Strong ($\Gamma/2\pi$~10 MHz) transitions in the visible-near infrared

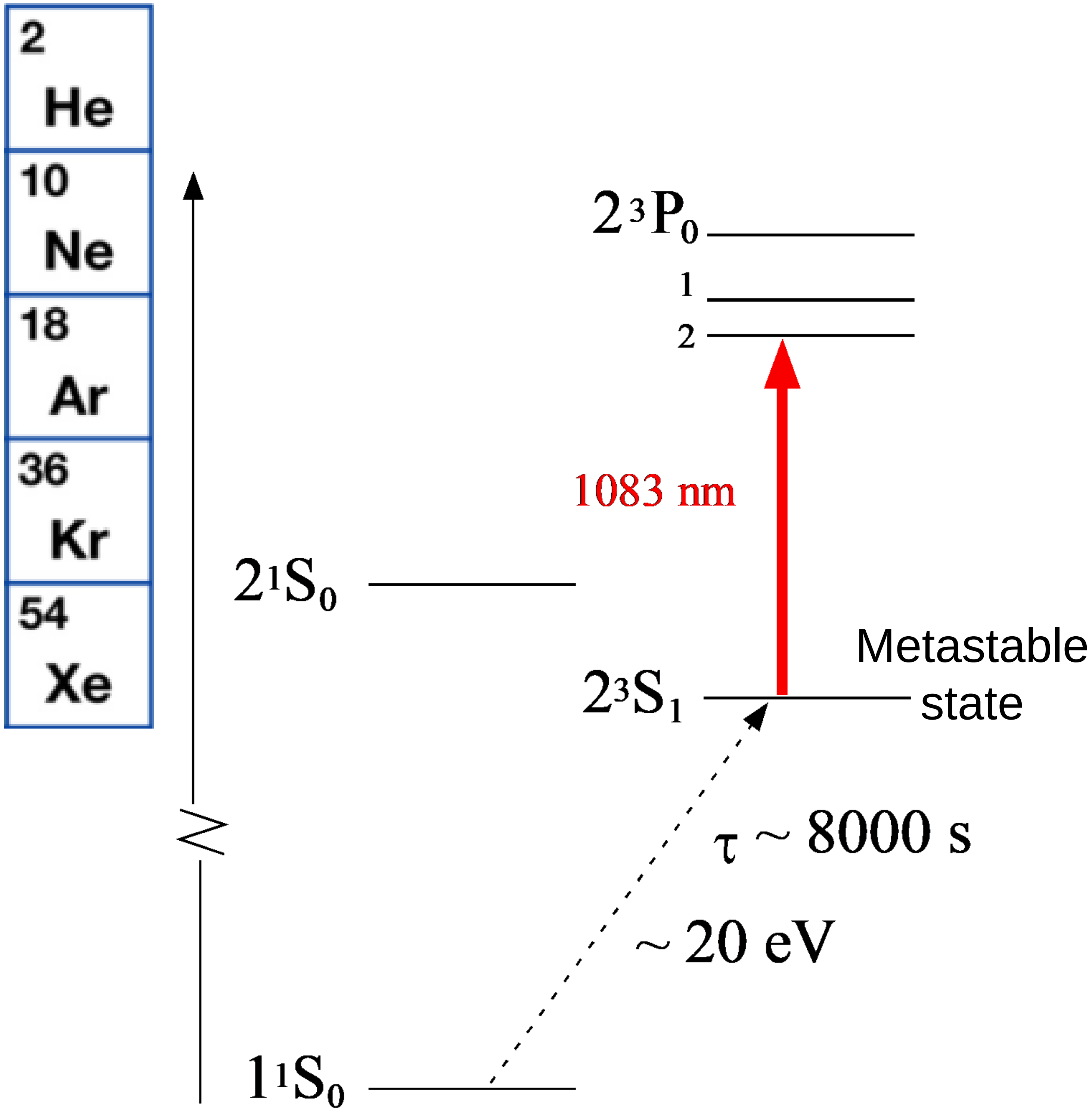
Hyperfine splitting
Repumping beam required

Relatively high vapour pressure (up to ~ 10^{-7} mbar) at room temperatures

Vapour in a cell or beam with ovens ~ 100-200°C

A few (one or two) stable isotopes

Noble gases



^{4}He (partial) level structure

Strong ($\Gamma/2\pi\sim$ 1 to few MHz) transitions in the visible-near infrared **starting from metastable states**

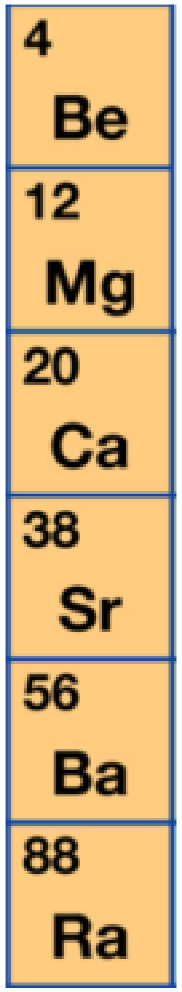
Several stable isotopes

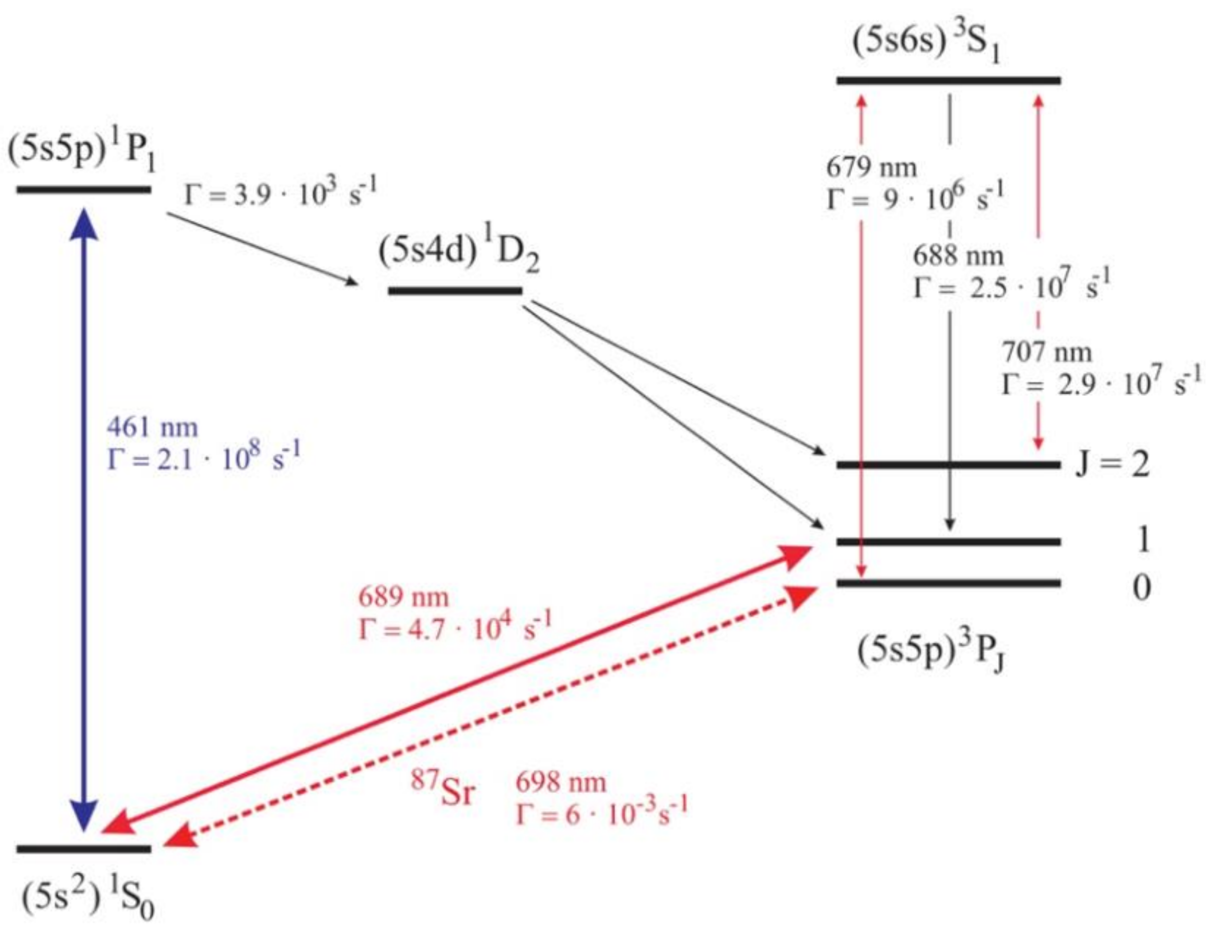
Production of metastable atoms requires discharge (in a beam)

Relatively large inelastic processes (Penning collisions)

Atoms can be easily detected via ionization

Alkaline-earth metals





Simplified energy diagram for strontium

Strong ($\Gamma/2\pi$~ tens of MHz) transitions in the UV-blue

(Ultra-)narrow lines

Need for (several) repumpers

Relatively low vapour pressure at room temperatures

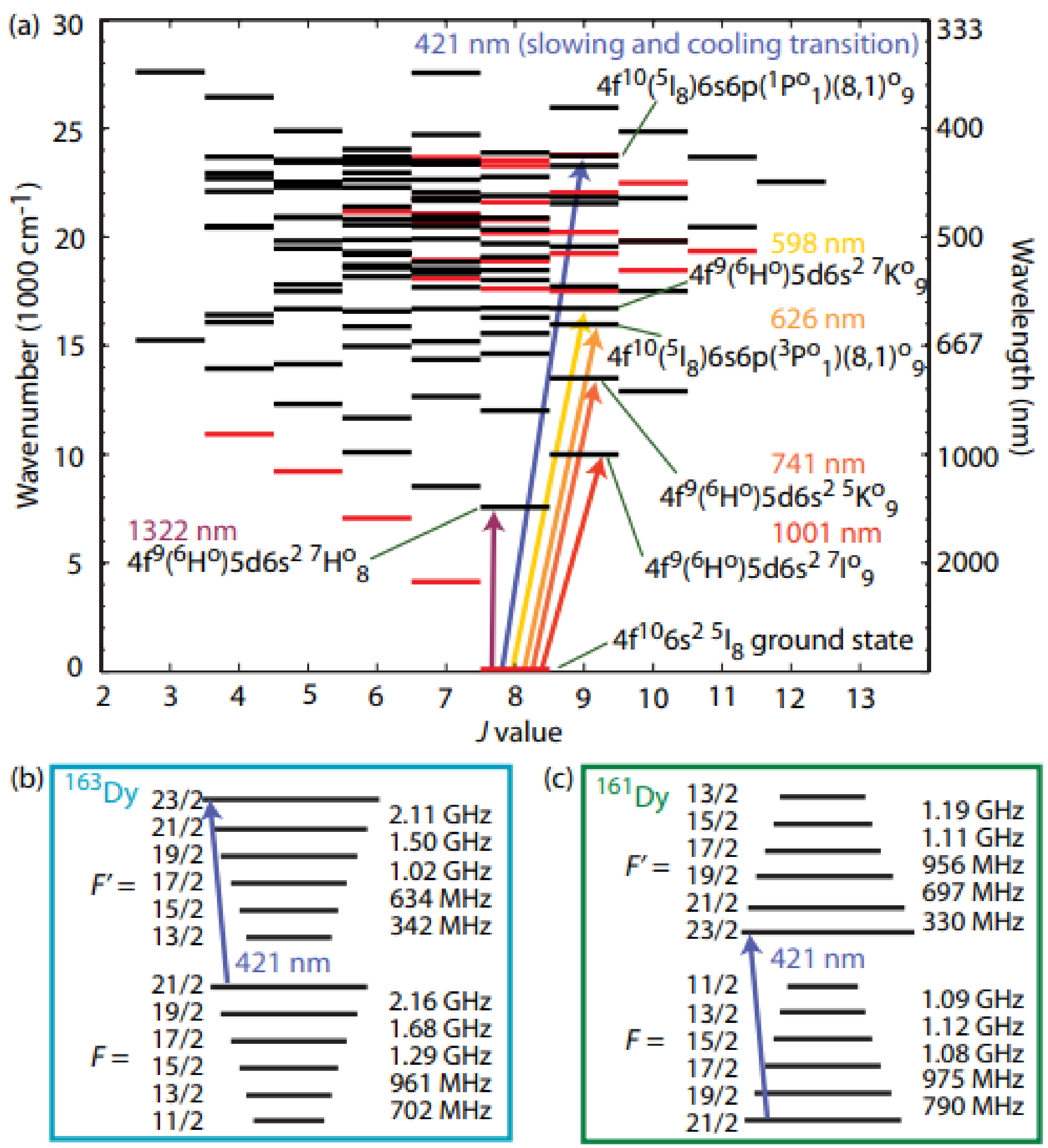
Most often, atomic beam machines, with oven ~ 500 °C and Zeeman slower

Several (many) stable isotopes

Similar structure for « alkaline-earth like » atoms (complete internal shells and a complete outer s-shell with two electrons) : Zn, Cd, Hg and Yb, No

Rare-earth atoms

66	67	68	69	70	71
Dy	Ho	Er	Tm	Yb	Lu



Dy energy level structure

From Youn et al, PRA 82, 043425 (2010)

More complex electronic structure

Strong transition in the blue

Several laser cooling transitions, (with narrow lines)

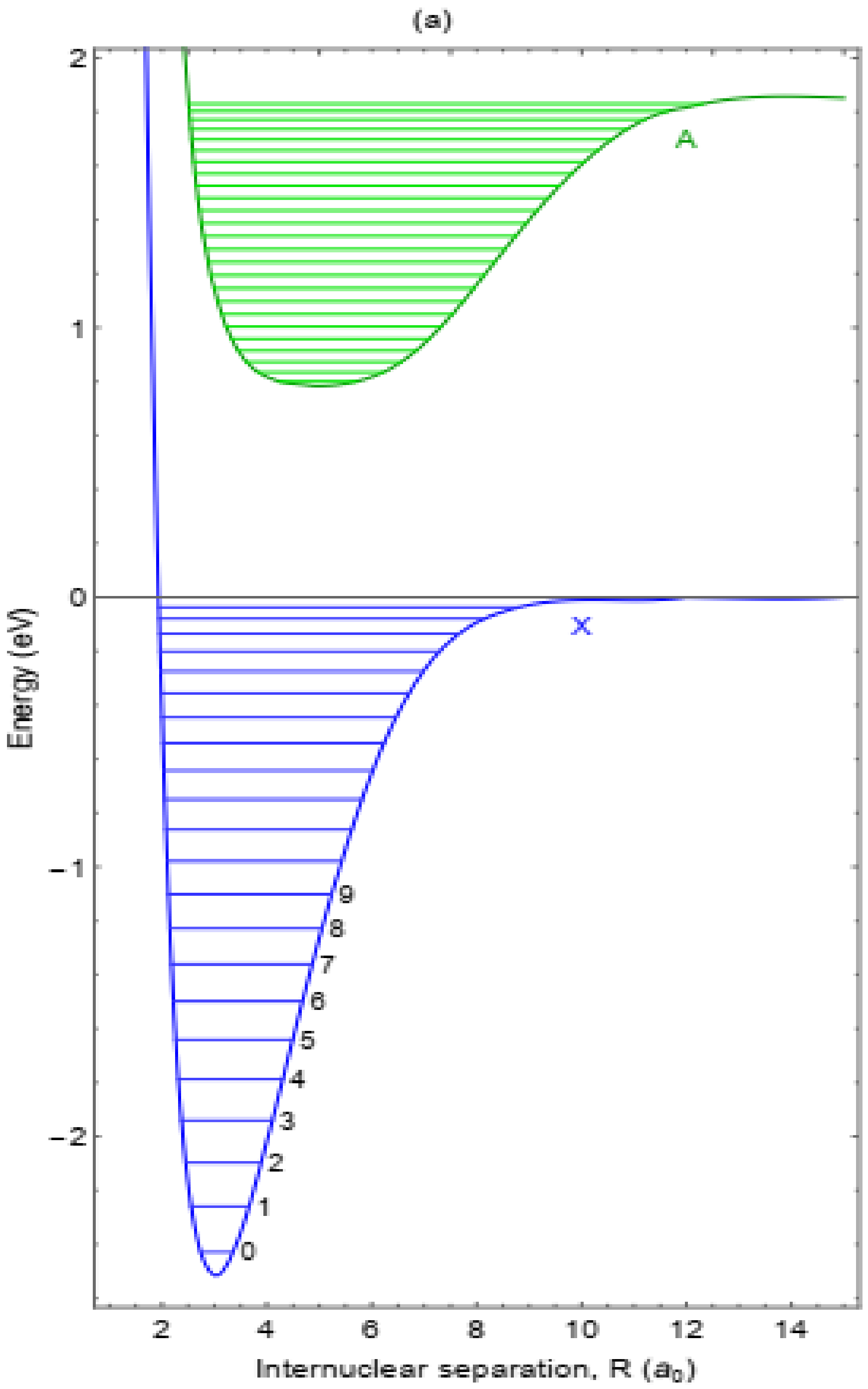
Low vapour pressure at room temperatures

Most often, atomic beam machines, with oven ~ 1000 °C and Zeeman slower

Several (many) stable isotopes

Laser cooling of molecules

Molecules are far from being « two level » molecules : absence of « closed » transitions



Potential energy curves of LiH

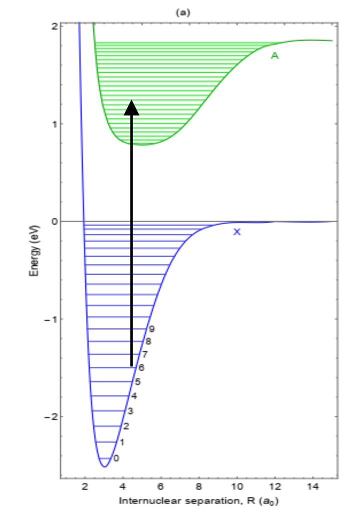
X : ground electronic state
A : first electronically-excited state

Many vibrational energy levels

Source: Laser cooling of molecules, M. R. Tarbutt, Contemporary Physics 59, 356-376 (2018)

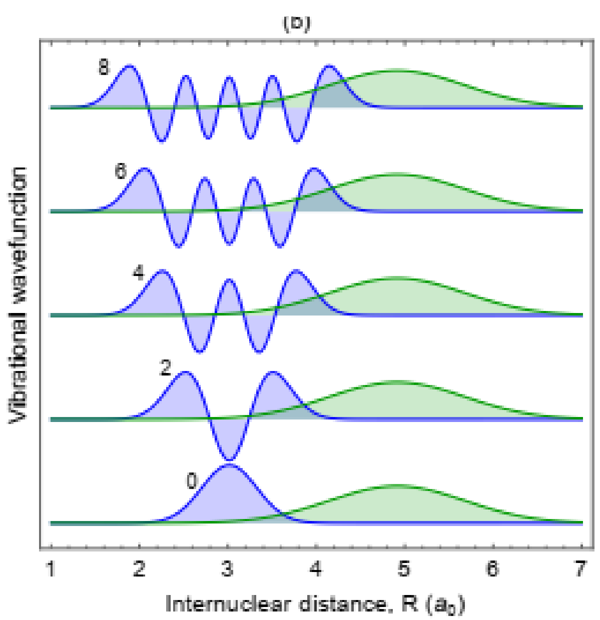
Laser cooling of molecules

Let us drive a transition between X, v and A, v'



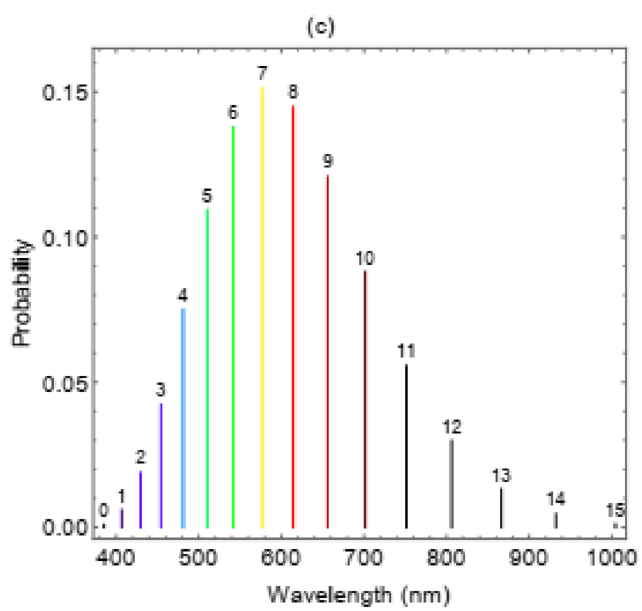
The molecule can decay to many states X, v''
with probabilities given by the Franck Condon integral overlap

$$P_{v',v''} = \left| \int \Phi_{X,v''}(R)\Phi_{A,v'}(R)dR \right|^2 .$$



(b) Lowest vibrational wavefunction (v'= 0) for the *A* state (green), and a selection (v''= 0*;* 2*;* 4*;* 6*;* 8) of vibrational wavefunctions for the X state (blue).

The square of the overlap integral between a vibrational wavefunction from the *X* state and one from the A state gives the corresponding Franck-Condon factor.



(c) Emission spectrum for molecules excited to the v'= 0 vibrational level of the A state.

One would need (too) many different frequencies

Laser cooling of molecules

How to get around the problem ?

If ground and excited potential curves have identical shapes, the probability $P_{v',v''} = \delta_{v'v''}$

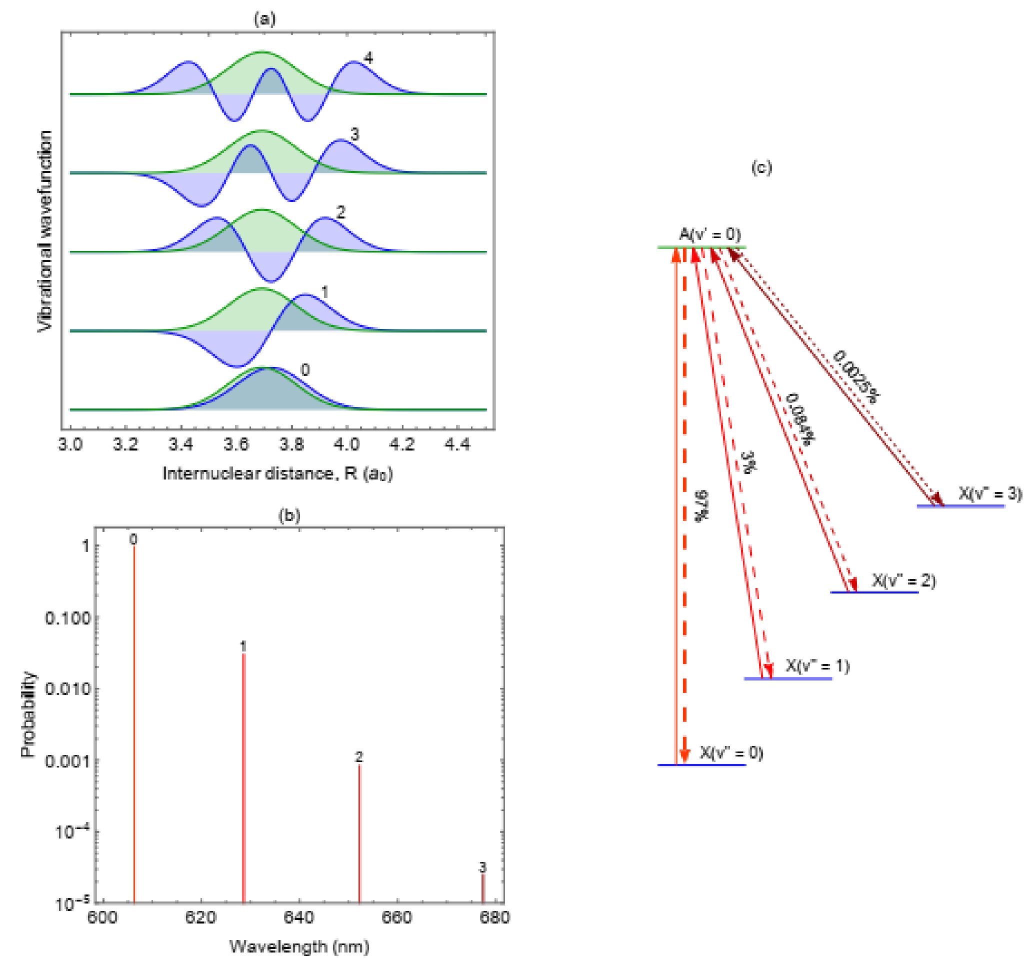
Some molecules, such as CaF, are quite like this.

Vibrational branching ratios are then suitable for laser cooling.

a) Lowest vibrational wavefunction ($v' = 0$) for the *A* state of CaF (green), and a selection ($v'' = 0; 1; 2; 3; 4$) of vibrational wavefunctions for the *X* state (blue).

b) Emission spectrum for molecules excited to the $v' = 0$ vibrational level of the *A* state.

c) Laser cooling scheme for CaF involving four lasers.



Laser cooling of molecules

Additional complexities :

- The spectrum of a molecule has also a rotational structure: each vibrational state has a ladder of rotational states.

Hopefully, there exist « rotationnally closed » transitions

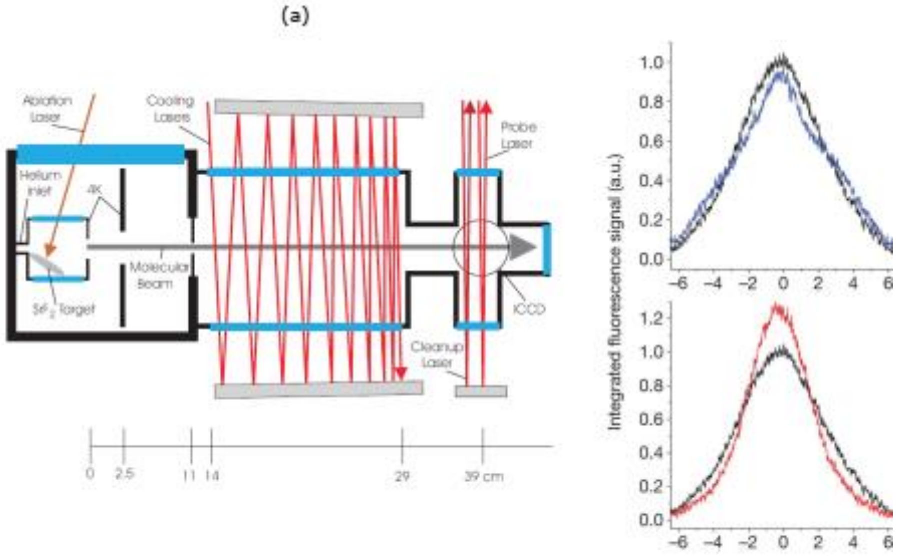
- These transitions have « dark » (Zeeman) states into which molecules get pumped

Dark state destabilization via polarisation modulation or using a magnetic field

Pioneering work by a team at Yale in 2010 :
1D (transverse) laser cooling of a beam of SrF
with two repumpers

Soon after YO, SrOH, YbF

Also slowing beams with frequency chirping
(CaF and YO)

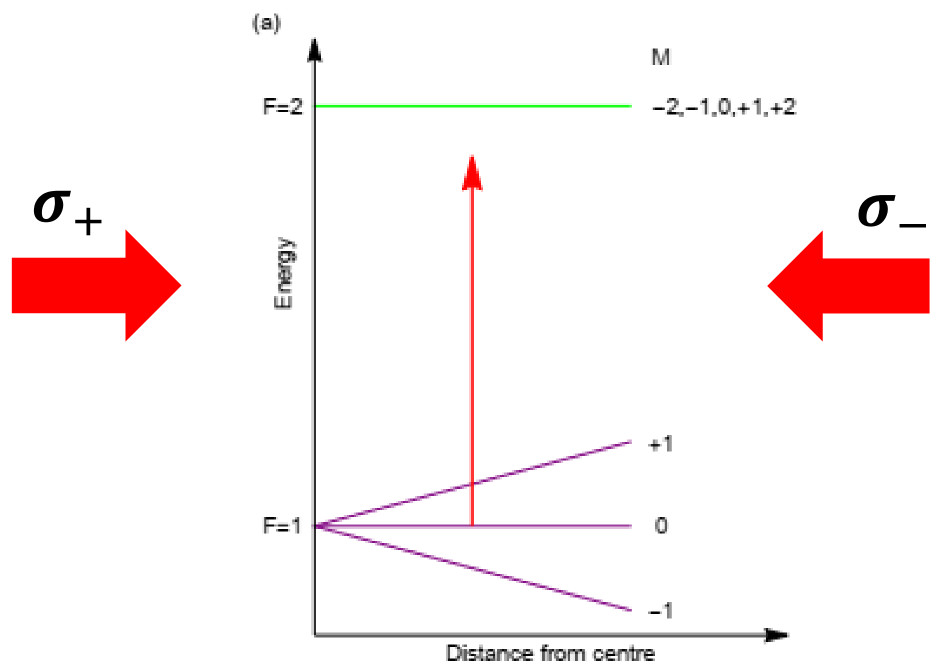


Shuman et al. Nature. 467, 820 (2010)

MOT of molecules

For making a MOT, Zeeman splitting of the excited state is required

But for the molecules that have been laser cooled so far, two difficulties:



1) Zeeman splitting of the excited state is very weak

Right and left laser beams have the same detuning, whatever the position or M state of the atoms

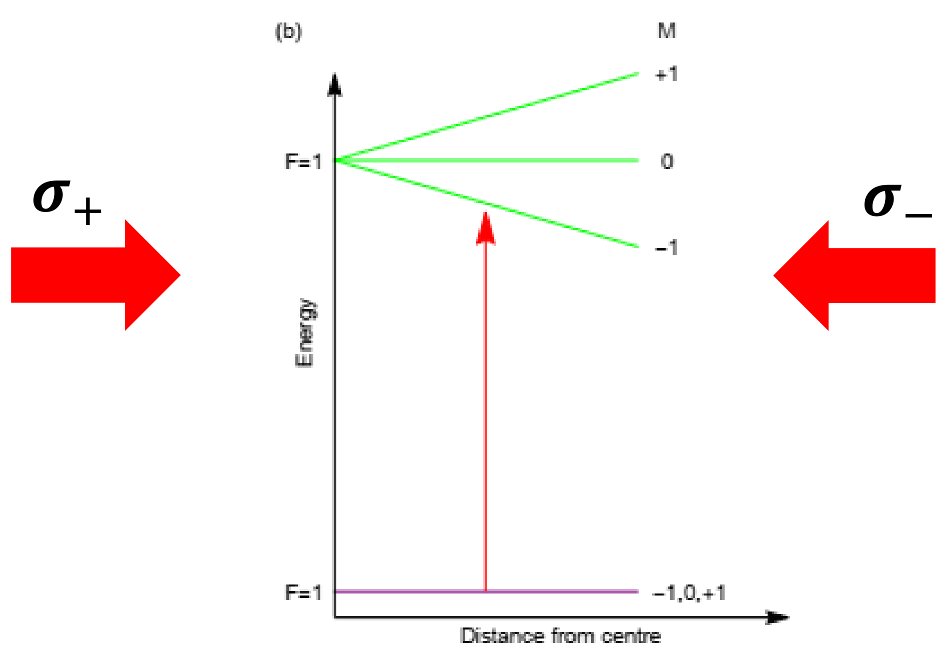
No restoring force

2) The cooling transitions are J = 1 → J = 1 transitions, which have dark states

An atom in M=+1 (-1) can only absorb from the σ_- (σ_+) beam.

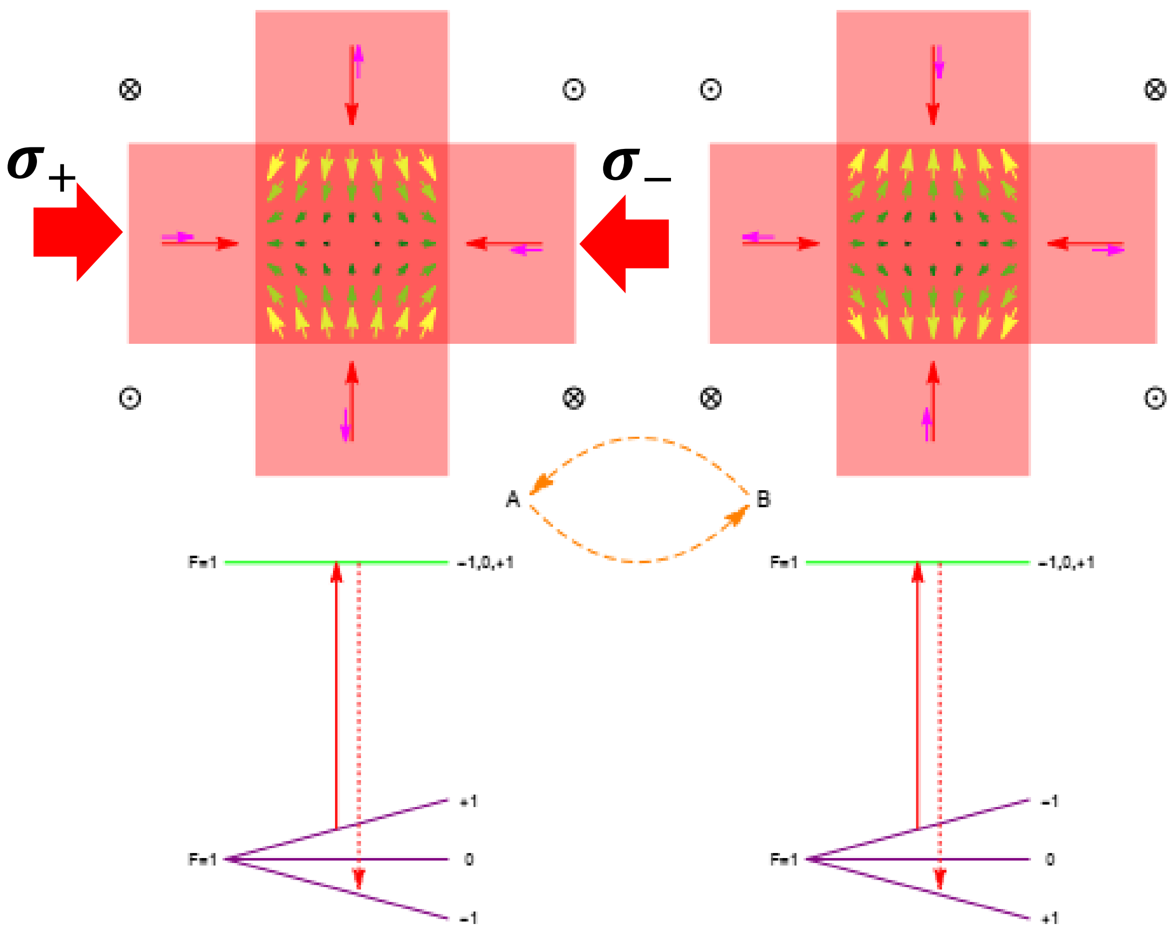
It gets excited to M'=0, falls back with equal probability in M=-1 and M=+1. It is then excited to M'=0 either with σ_- or σ_+(with a detuning independent on the position)

Net force is zero on average



MOT of molecules

The cooling transitions are J = 1 → J = 1 transitions
and the Zeeman splitting in the excited state is negligible



Consider an atom in M=1 displaced to the right.
It will scatter from the σ_- beam and get pushed to the center.
After a few cycles, it will be pumped into M=-1 (and then pushed away). But at a lower rate as the detuning to resonance is larger.
Finally, the atom spends most of his time in M=-1.

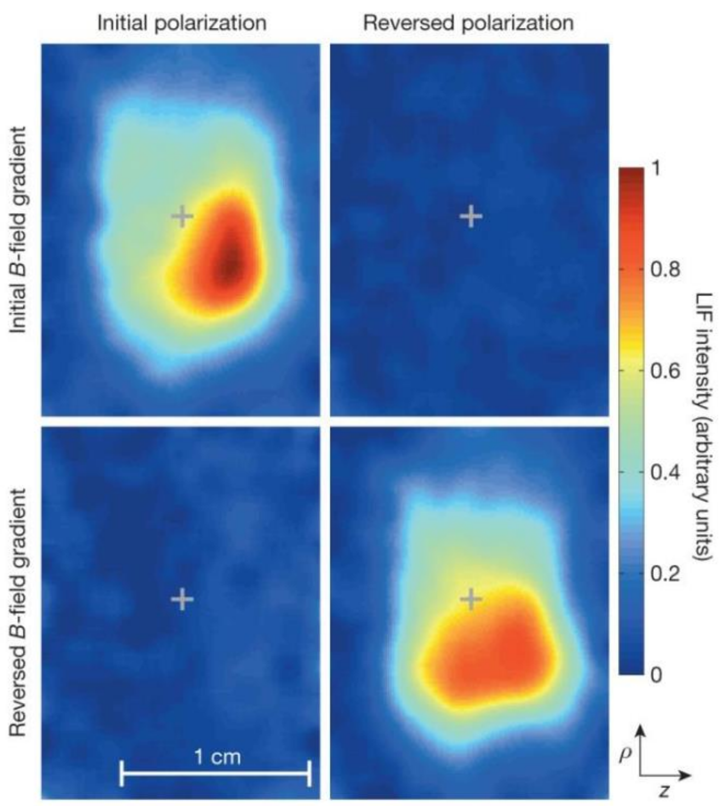
Trick to enhance the force : **RF MOT**
The magnetic field and laser polarizations are switched back and forth between two configurations, labelled A and B at a rate of a few MHz, of order of the optical pumping rate.

In A, molecules are optically pumped into *M* = -1 by the beam that pushes towards the centre.
In B, they are pumped back to *M* = +1, again by the beam that pushes towards the centre.

There is then a **strong preferential scattering** from the beam that pushes the molecule back towards the centre

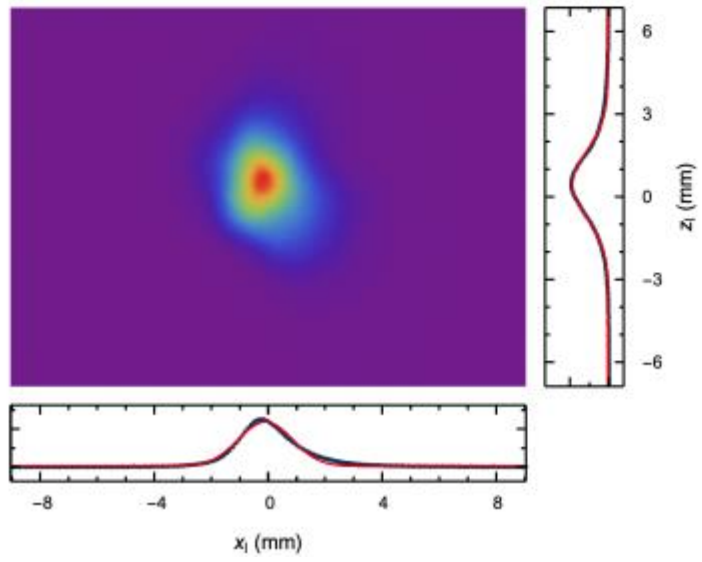
MOT of molecules

2014 : First 3D MOT of SrF molecules
Static B field and polarisation
300 molecules at a density of 600 molecules/cm^3
T = 2.3 mK
Barry et al., Nature. 512, 286 (2014)



2016 : RF MOT of SrF
10^4 molecules
Density of 2.5 10^5 /cm^3
T=250 µK

2017 : MOTs of CaF (with dual frequency and RF methods)
10^5 molecules
Density of 7 10^6 /cm^3
T=250 µK
Williams et al., New J Phys. 19, 113035 (2017)

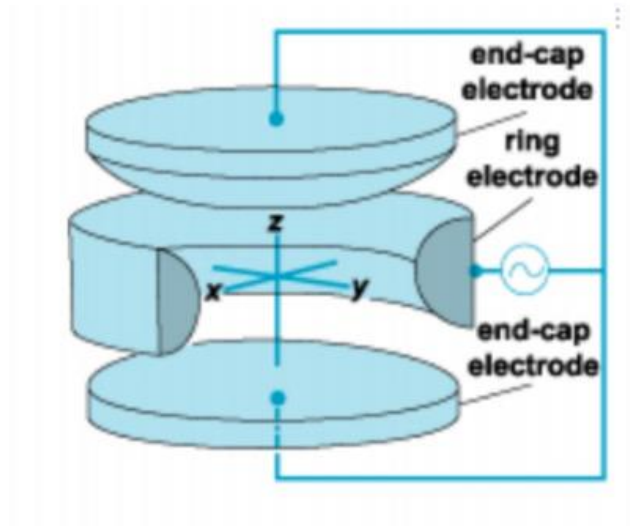


2018 : MOT of YO (Yttrium Monoxide), Collopy et al, Phys Rev Lett. 121, 213201 (2018)

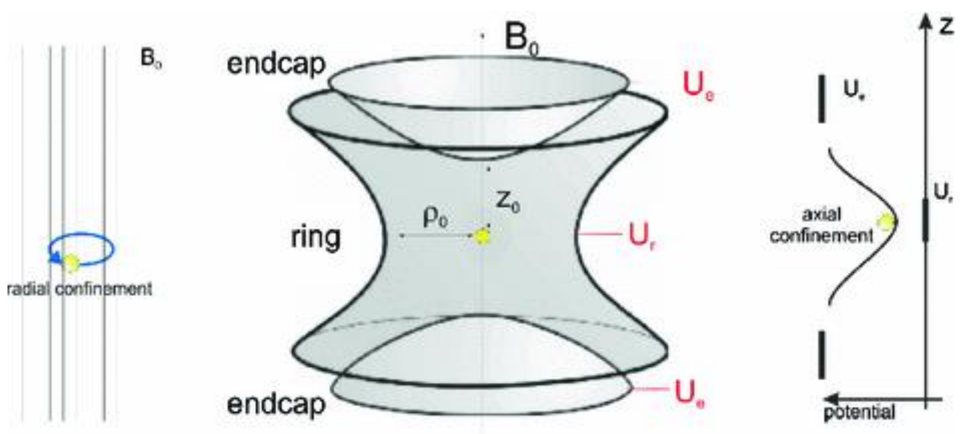
Laser cooling of trapped ions

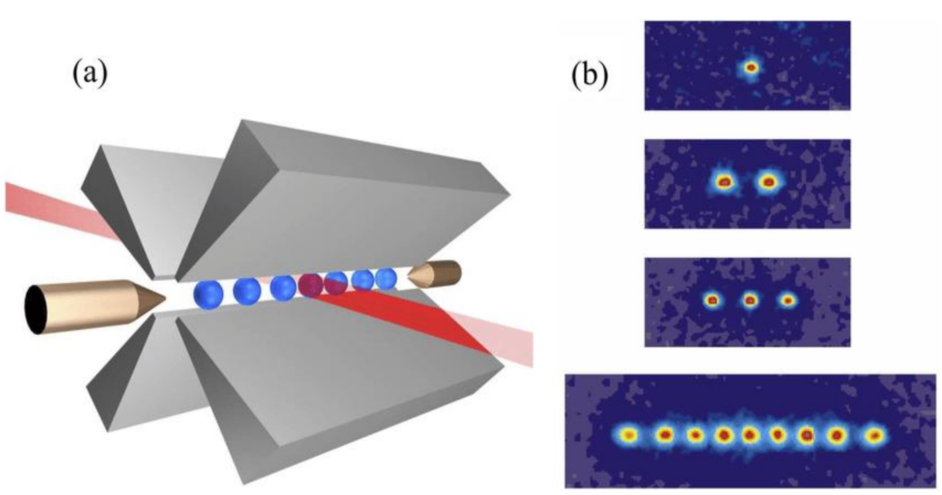
Ion traps

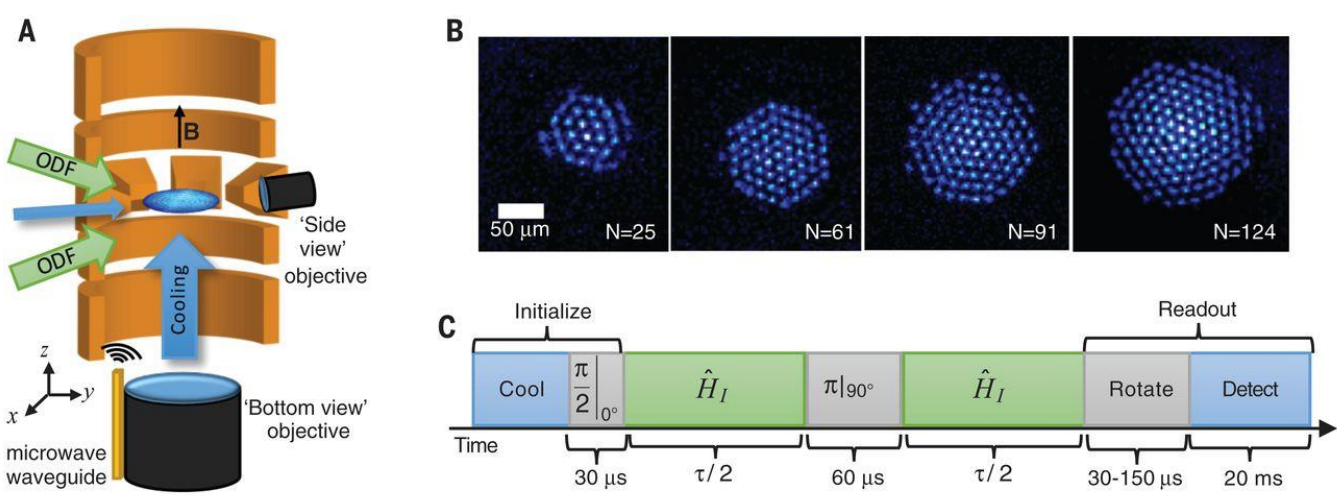
Paul trap:
Uses dynamic electric fields to trap charged particules



Penning trap:
Uses strong homogeneous axial magnetic field to confine particles radially
and a quadrupole electric field to confine the particles axially







Laser cooling of trapped ions

Implementation: a single laser beam is enough,
due to the oscillatory motion of the ion in the trap and to axis couplings

- Doppler cooling of 5 10^4 Mg+ in a Penning trap

Wineland et al., PRL 40, 1639 (1978)

Temperature reduced down to 40 K

- Doppler cooling of Ba+ ions in a Paul trap

W. Neuhauser et al. Phys. Rev. Lett. 41, 233–236 (1978)

- Later, first **single** laser cooled ion down to a temperature of a few tens of mK

W. Neuhauser; …, H. Dehmelt, Phys. Rev. A 22, 1137–1140 (1980)

Organization of the lecture

1 : What can be (laser) cooled?

2 : Narrow line cooling

3 : Sub-Doppler cooling

Narrow line cooling

Doppler temperature : $T_{min} = \frac{\hbar\Gamma}{2k_B}$

Does cooling on narrow lines allow for reaching arbitrary low temperatures ?

In fact, the Doppler cooling theory is a semiclassical theory
valid for broad lines : $\hbar\Gamma \gg E_r$

For $\hbar\Gamma \lesssim E_r$, one needs a quantum treatment, with quantized momenta

Y. Castin, H. Wallis, and J. Dalibard, JOSA B 6, 2046-2057 (1989)

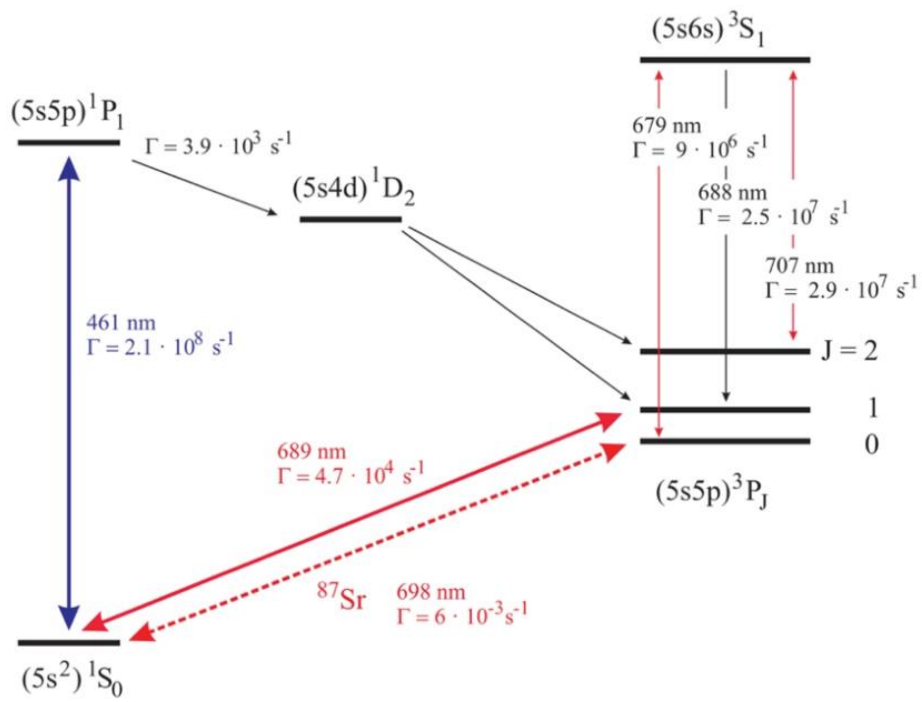
One finds $\boxed{T_{min} \simeq \frac{E_r}{k_B}}$, reached for $\Delta = 3.4\frac{E_r}{\hbar}$

Narrow line cooling

Optical molasses and MOT with a narrow line: *Katori et al., PRL 82, 1116 (1999)*

Start with a blue MOT on the $^1S_0$-$^1P_1$ transition @ 461 nm with $\frac{\Gamma}{2\pi} = 32\ \text{MHz}$

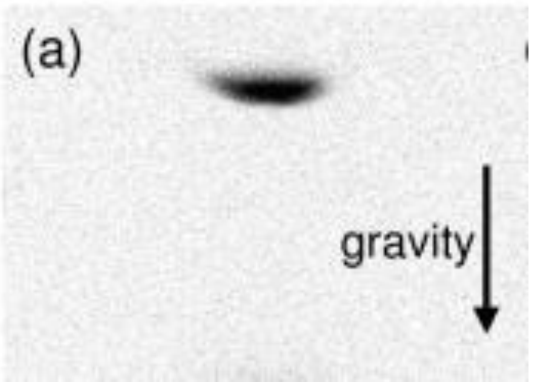
Switch to a red MOT or molasses on the $1S_0$-$^3P_1$ transition @689 nm with $\Gamma/2\pi = 7.6\ \text{kHz}$



$$\boxed{T_{min} \simeq 400\ \text{nK}}$$

when $\frac{E_r}{k_B} = 230\ \text{nK}$ (and $\frac{\hbar\Gamma}{2k_B} = 182\ \text{nK}$)

<u>Picture of the red MOT</u>



Atoms are located where $F + mg = 0$

F is weak

$\Rightarrow$ Gravitational sag

Narrow line cooling

Temperature depends on atomic density due to photon reabsorption.

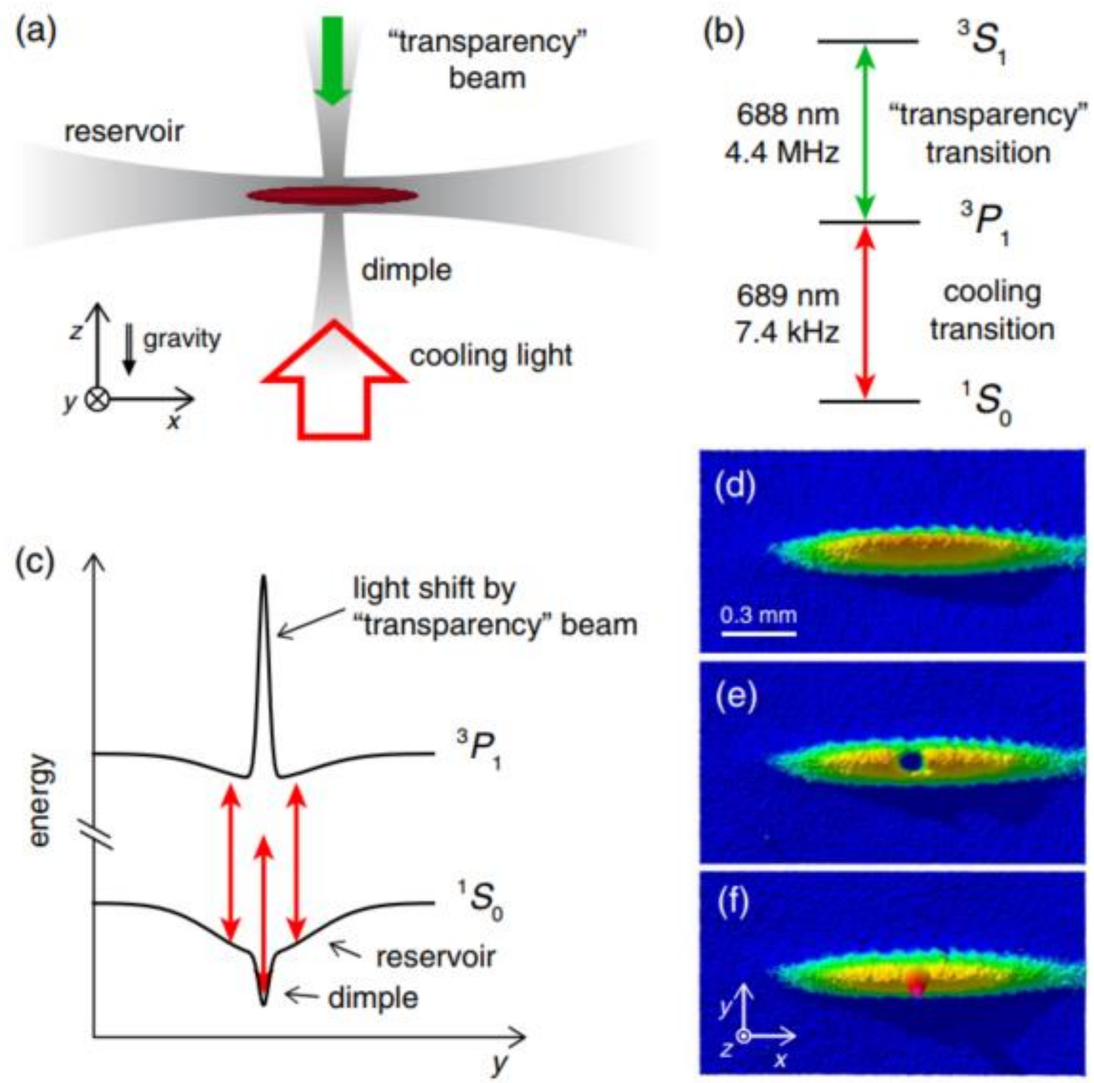
$$\boxed{T \simeq 400 \text{ nK}/(10^{12} \text{ at/cm}^3)}$$

Maximum observed phase space density:
$\rho = 0.01$ at a density of 6 10^{11} at/cm^3

How to do better?
And eventually reach BEC ($\rho \sim 1$) ?

Key: protect the atoms from reabsorption.

^{84}Sr in a (strongly elliptic) dipole trap
+ transparency laser
+ dimple laser



Stellmer at al, PRL 110, 263003 (2013)

Production of a BEC within a sample that is Doppler cooled to below 1 µK.
Thermal equilibrium between the gas in this central region and the surrounding laser cooled part of the cloud is established by elastic collisions.
Condensates of up to 10^5 atoms can be repeatedly formed on a time scale of 100 ms.

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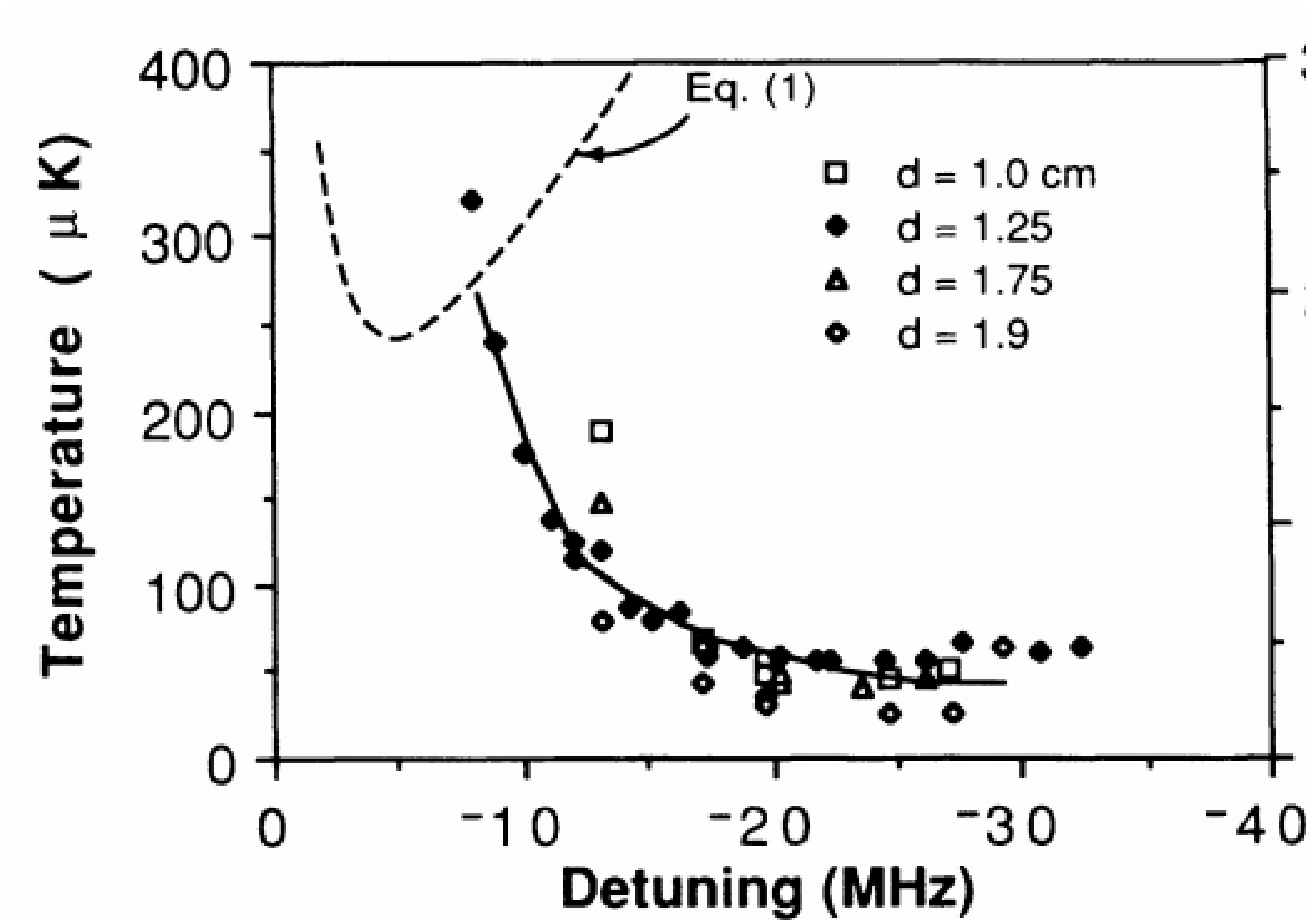
Sub-Doppler cooling

Temperatures well below the Doppler limit soon reported



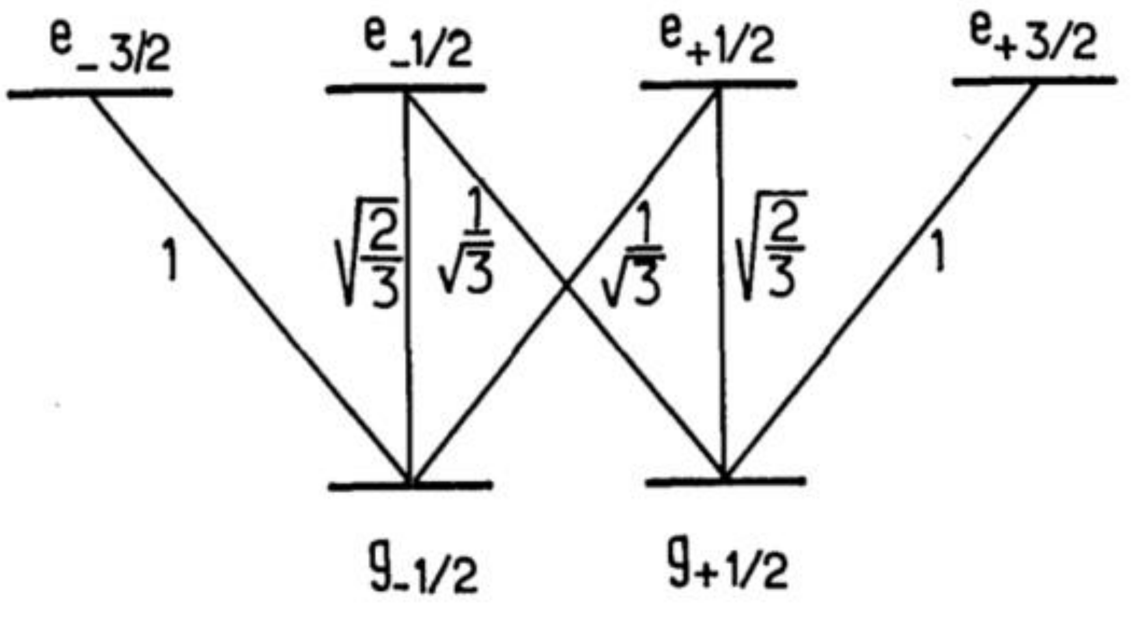
Doppler theory apply for some atoms (eg Hg bosonic isotopes, zero nuclear spin)

Key missing ingredient: the internal structure of the atomic states

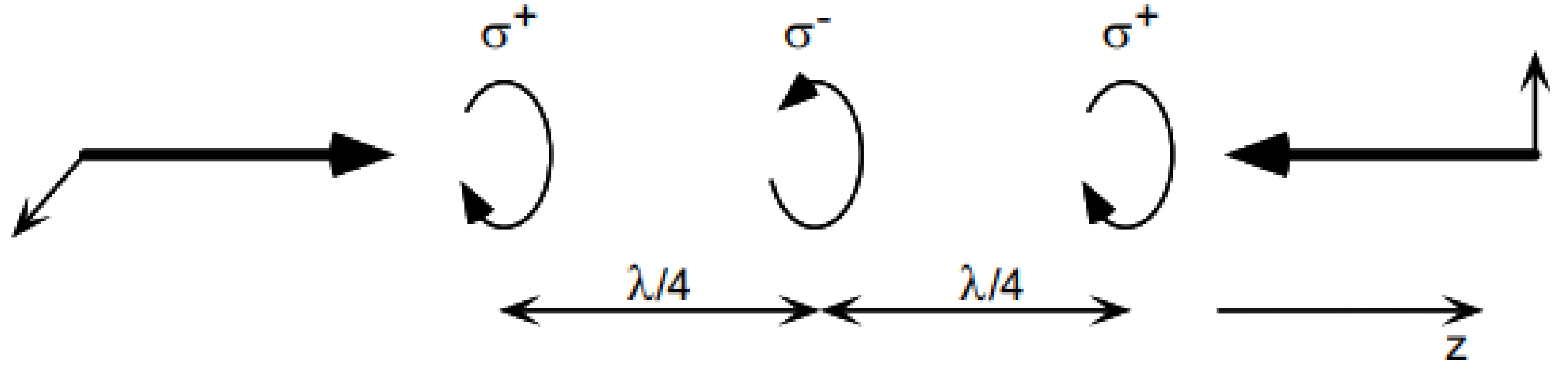
Sub-Doppler cooling

J. Dalibard and C.Cohen-Tannoudji, Laser cooling below the Doppler limit by polarization gradients: simple theoretical models, JOSAB 6, 2023 (1989)

Model transition J = 1/2 → J' = 3/2



Let's consider a 1D configuration with orthogonal linear polarisations



The resulting polarisation depends on the position
It is spatially modulated with a period λ/2

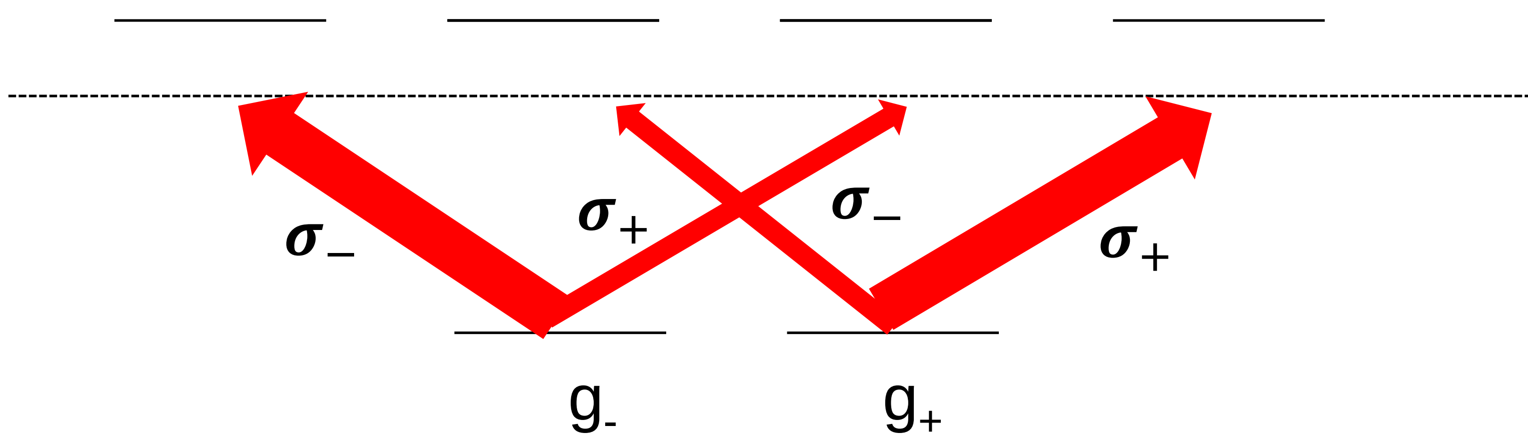
$$I(\sigma^+) = 2I_0 \sin^2(kz)$$
$$I(\sigma^-) = 2I_0 \cos^2(kz)$$

Sub-Doppler cooling

We choose for the light field a red detuning (as for Doppler cooling)

Light shifts of the ground states are given by:

$$\Delta E_i = \sum_k \frac{\hbar \Delta_{i,k}}{2} s_{i,k} \qquad \text{with} \qquad s_{i,k} = \frac{{\Omega_{i,k}}^2/2}{{\Delta_{i,k}}^2 + \Gamma^2/4}$$



$$\Delta E_+ = \frac{\hbar\Delta}{2} \frac{1}{\Delta^2 + \Gamma^2/4} \left(\frac{{\Omega_{+,3/2}}^2 + {\Omega_{+,-1/2}}^2}{2} \right)$$

$$\Delta E_+ = \frac{1}{4} \frac{\hbar\Delta\, {\Omega_0}^2}{\Delta^2 + \Gamma^2/4} \left(2\, {C_{+,3/2}}^2 \sin^2(kz) + 2\, {C_{+,-1/2}}^2 \cos^2(kz) \right)$$

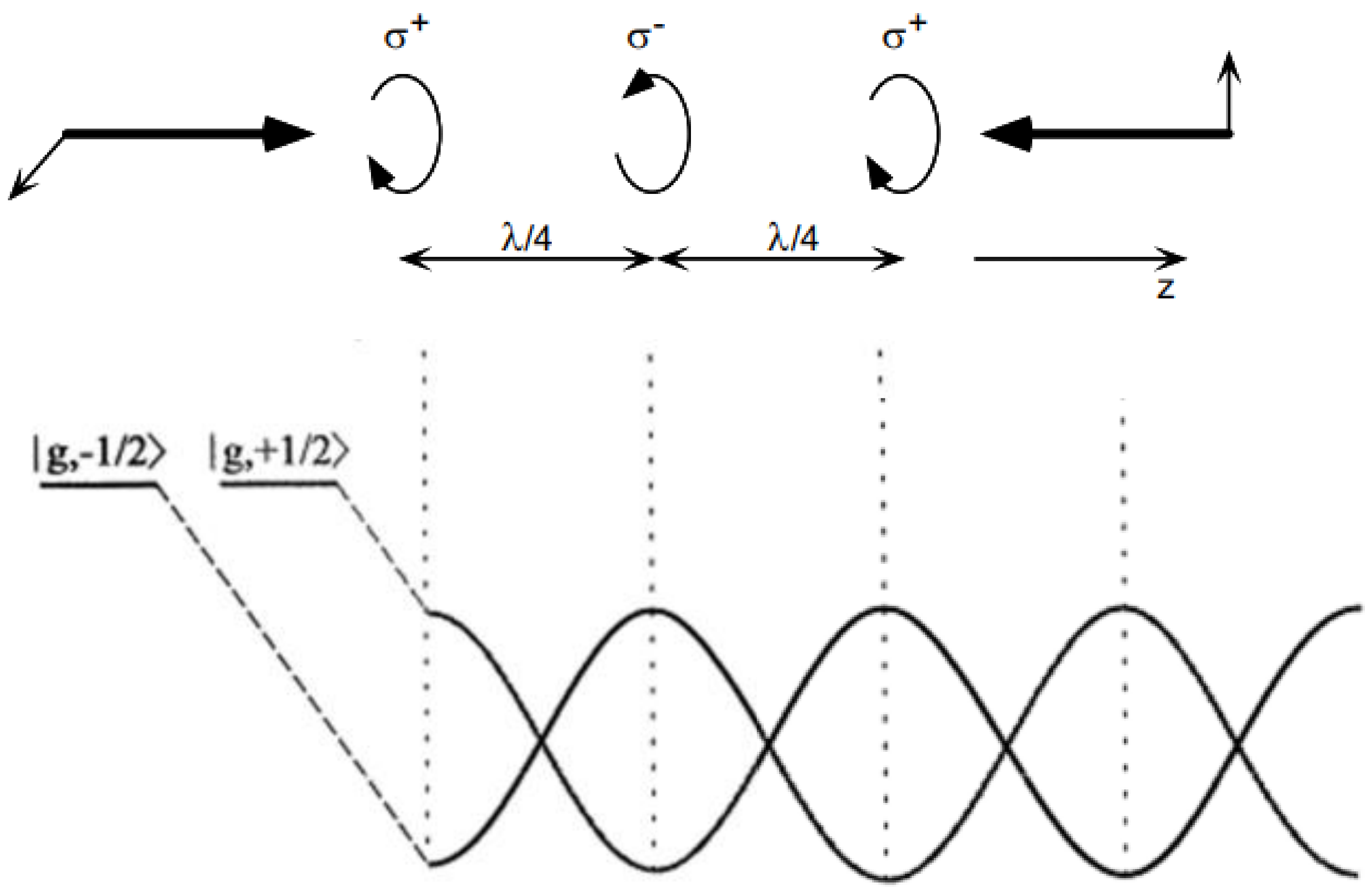
$$\Delta E_+ = \hbar\Delta s_0 \left(1 \sin^2(kz) + \frac{1}{3} \cos^2(kz) \right) = \frac{2}{3} \hbar\Delta s_0 - \frac{1}{3} \hbar\Delta s_0 \cos 2kz$$

Sub-Doppler cooling

Spatial modulation of the light shifts
$$\Delta E_+ = \frac{2}{3}\hbar\Delta s_0 - \frac{1}{3}\hbar\Delta s_0 \cos 2kz$$

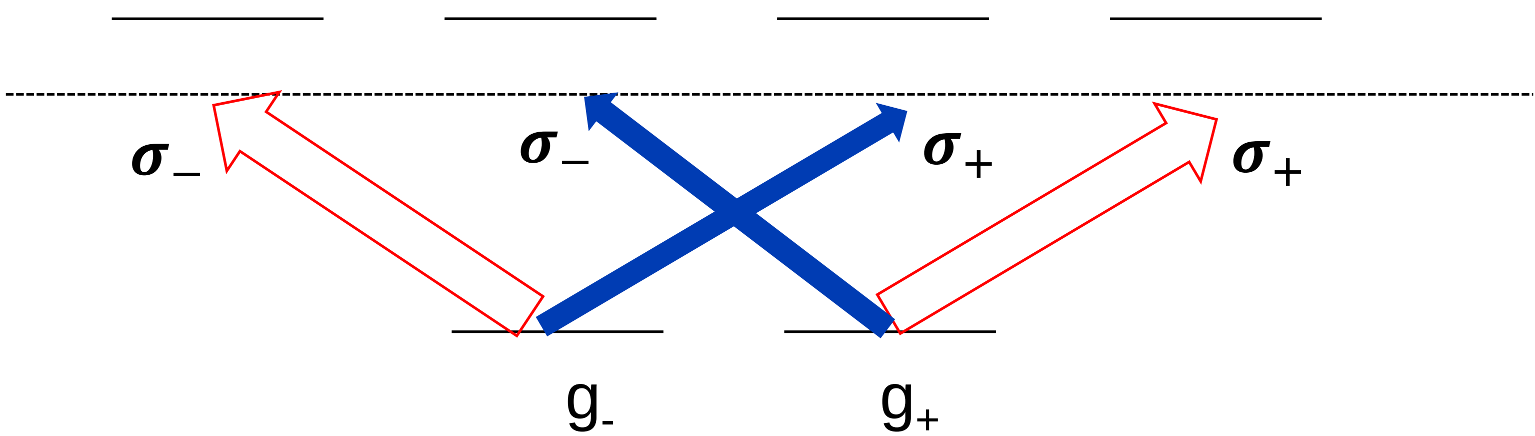
$$\Delta E_- = \frac{2}{3}\hbar\Delta s_0 + \frac{1}{3}\hbar\Delta s_0 \cos 2kz$$

Correlated with the spatial modulation of the light polarisation



Sub-Doppler cooling

Steady state populations for **an atom at rest**



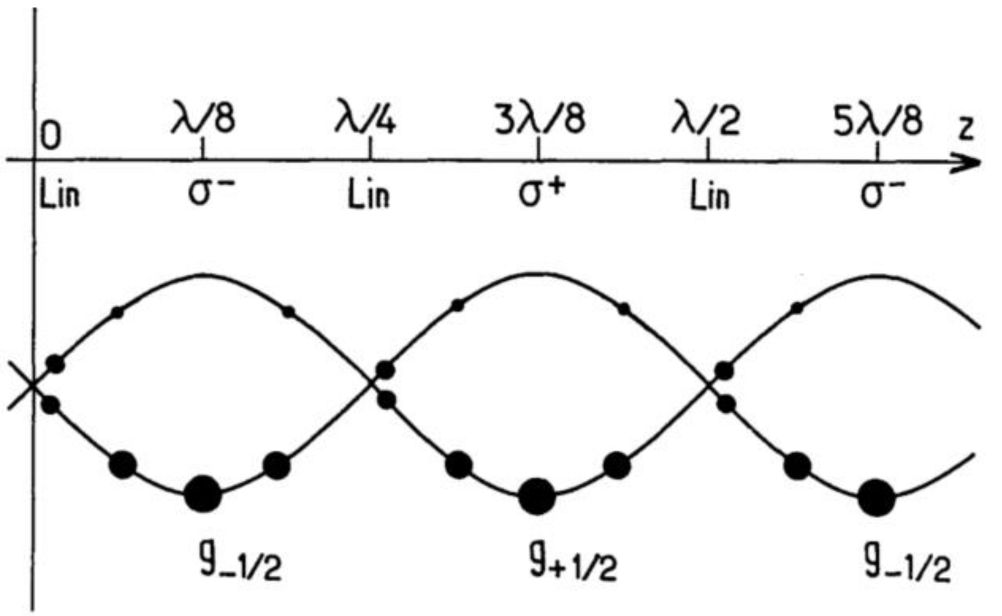
$$\Pi_- I(\sigma^+) = \Pi_+ I(\sigma^-)$$
$$\Pi_- \cos^2(kz) = \Pi_+ \sin^2(kz)$$

At low intensity, populations in the excited states are neglected, so that

$$\Pi_- + \Pi_+ = 1$$
$$\Pi_- \cos^2(kz) = \Pi_+ \sin^2(kz)$$

$$\Rightarrow$$

$$\Pi_- = \sin^2(kz)$$
$$\Pi_+ = \cos^2(kz)$$



Atoms are optically pumped
in the lowest energy state
where the polarisation are circular

Optical pumping rate of order of Γs_0

$$\gamma_0 = \frac{2}{9}\Gamma s_0$$

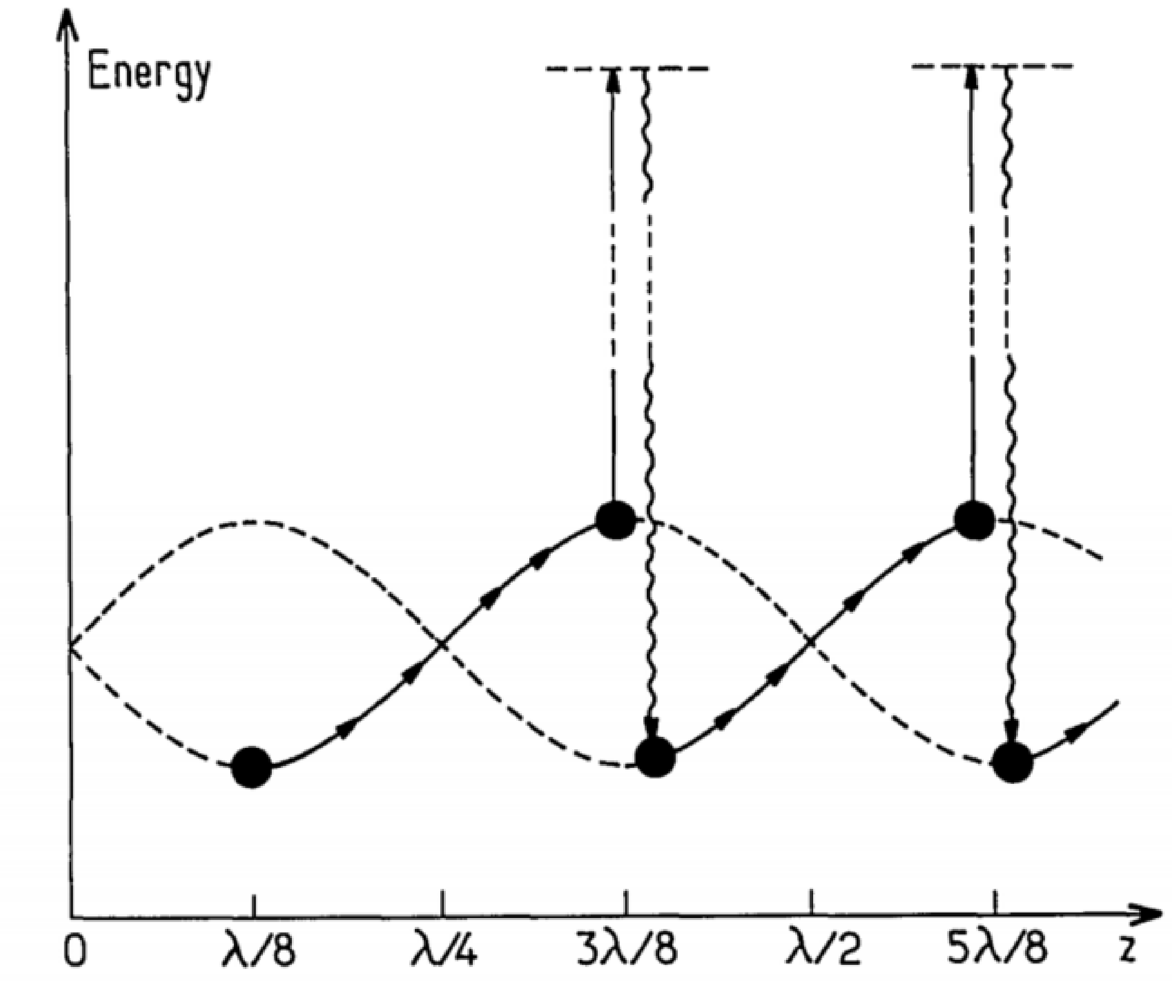
Sub-Doppler cooling

For a moving atom, the populations in the two ground states evolve to adjust to the steady state, but with a delay that corresponds to the optical pumping time

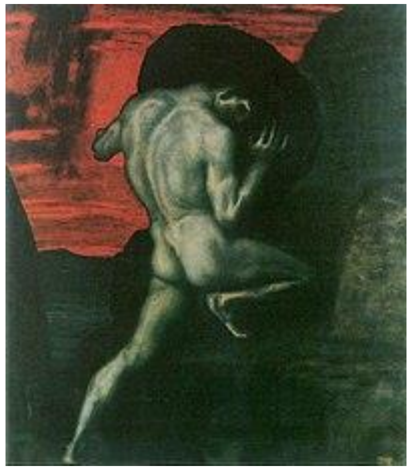
Let's consider an atom at a velocity v such that it travels over a distance $\frac{\lambda}{4}$ during a time $\tau = \frac{1}{\gamma_0}$

$$v = \gamma_0 \frac{\lambda}{4}$$

It climbs a potential well
and gets optically pumped back
into the lower energy state and so on



SISYPHUS COOLING



Sisyphus,
Franz von Stuck, 1920.

Sub-Doppler cooling

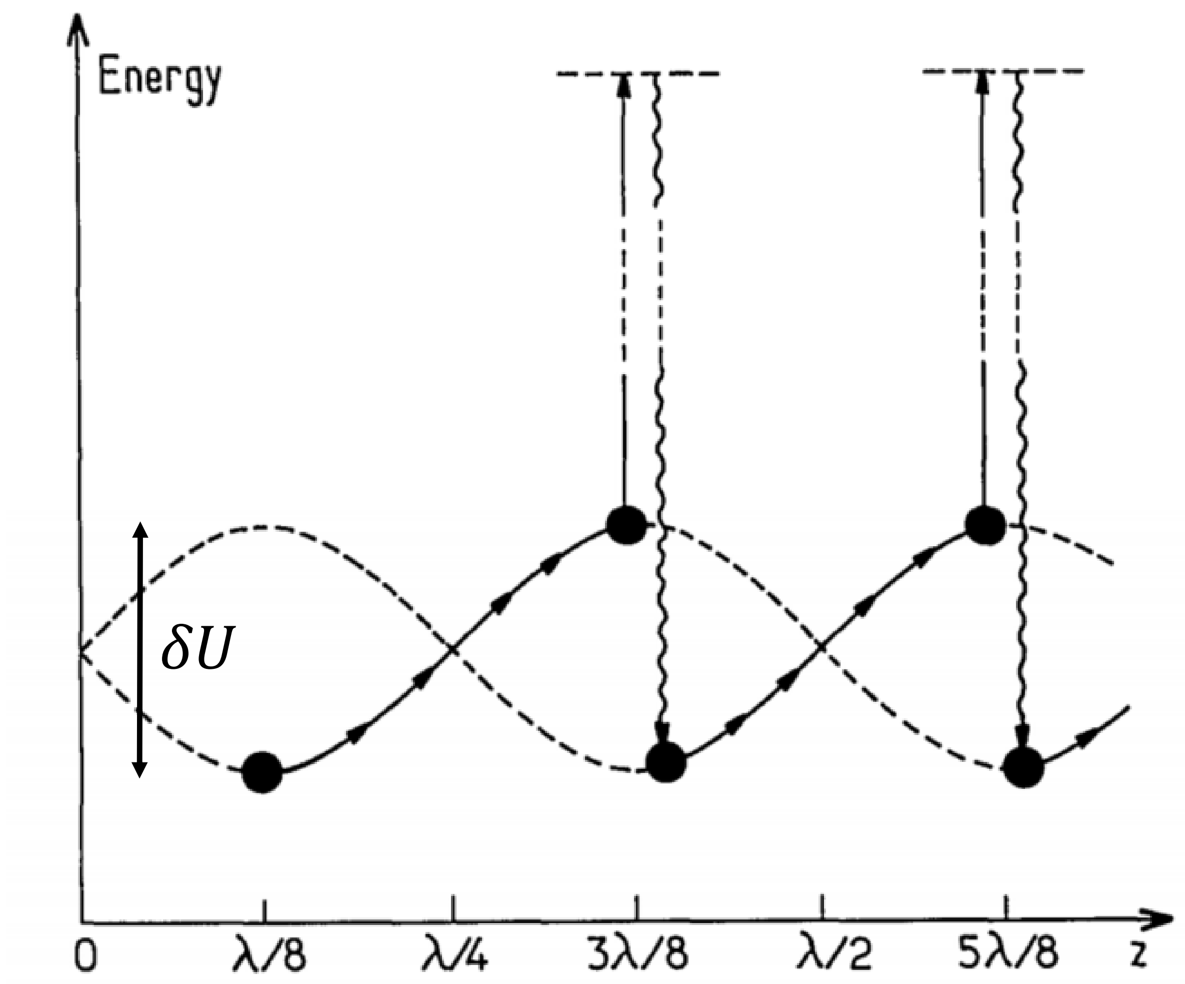
It looses an energy of $\delta U = \frac{4}{3}\hbar\Delta s_0$ over a time τ

This is equivalent to a force

$$F \sim \frac{1}{v}\frac{\delta U}{\tau}$$

and to a friction

$$\alpha = -\frac{F}{v} \sim -\frac{\gamma_0}{v^2}\delta U \sim -\left(\frac{4}{\lambda}\right)^2 \frac{\frac{4}{3}\hbar\Delta s_0}{\frac{2}{9}\Gamma s_0} \sim -\hbar k^2 \frac{\Delta}{\Gamma}$$



The friction is independent of the laser intensity

Reminder: max friction for Doppler cooling (in the low saturation regime): $2\hbar k^2 s_0$

At large detunings $\Delta \gg \Gamma$, it gets larger than $\hbar k^2$

Sub-Doppler cooling

More precisely …

$$\Delta E_+ = \frac{2}{3}\hbar\Delta s_0 - \frac{1}{3}\hbar\Delta s_0 \cos 2kz$$

$$\Delta E_- = \frac{2}{3}\hbar\Delta s_0 + \frac{1}{3}\hbar\Delta s_0 \cos 2kz$$

$$F = -\nabla E$$

$$F_+ = -\frac{2k}{3}\hbar\Delta s_0 \sin 2kz$$

$$F_- = \frac{2k}{3}\hbar\Delta s_0 \sin 2kz$$

Average force : $F = \Pi_- F_- + \Pi_+ F_+ = \frac{2k}{3}\hbar\Delta s_0 \sin 2kz\ (\Pi_- - \Pi_+)$

The populations evolve according to $\dot{\Pi}_\pm = -\gamma_0(\Pi_\pm - \Pi_{\pm,st})$

with $\gamma_0 = \frac{2}{9}\Gamma s_0$ and $\Pi_{-,st} = \sin^2(kz)$ and $\Pi_{+,st} = \cos^2(kz)$ and $z = vt$

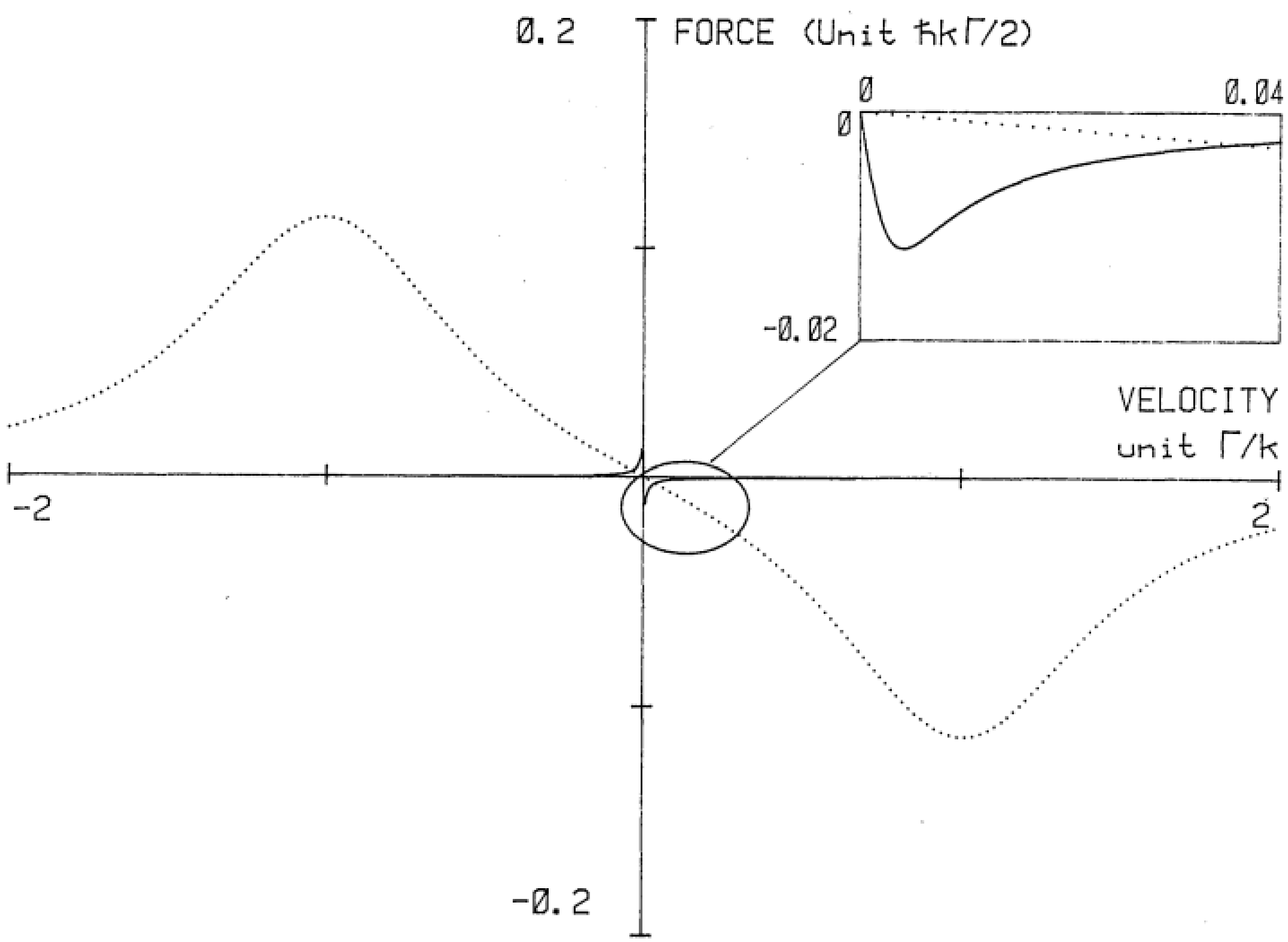
This leads to $\Pi_\pm = \frac{1}{2}\left(1 \mp \frac{\cos 2kz + 2kv \sin 2kz/\gamma_0}{1+4\left(\frac{kv}{\gamma_0}\right)^2}\right)$

Sub-Doppler cooling

The force (averaged over one wavelength) is $\boxed{F = \frac{-\alpha v}{1+\left(\frac{v}{v_c}\right)^2}}$

With $\alpha = -3\hbar k^2 \frac{\Delta}{\Gamma}$ and $kv_c = \frac{1}{2}\gamma_0$

Note : the expression is valid for $kv \ll \Gamma$



Sub-Doppler cooling

Temperature ?

It is given by $k_B T = \frac{D}{\alpha}$

Three contributions to the diffusion coefficient D:

- Fluctuations of the momentum carried away by fluorescence photons
- Fluctuations in the difference among the number of photons absorbed in each of the two laser waves
- Fluctuations of the instantaneous dipole force oscillating back and forth between $F_+(z)$ and $F_-(z)$

Two first contributions are already present for a J = 0 → J = 1 transition

It is then given by $\mathrm{D} = \frac{7}{10}(\hbar k)^2 \Gamma s_0$

The third is specific to atoms with several ground states

$$\mathrm{D}' = \frac{3}{4}(\hbar k)^2 \frac{\Delta^2}{\Gamma} s_0$$

Dominates over the two first contributions when $\Delta \gg \Gamma$

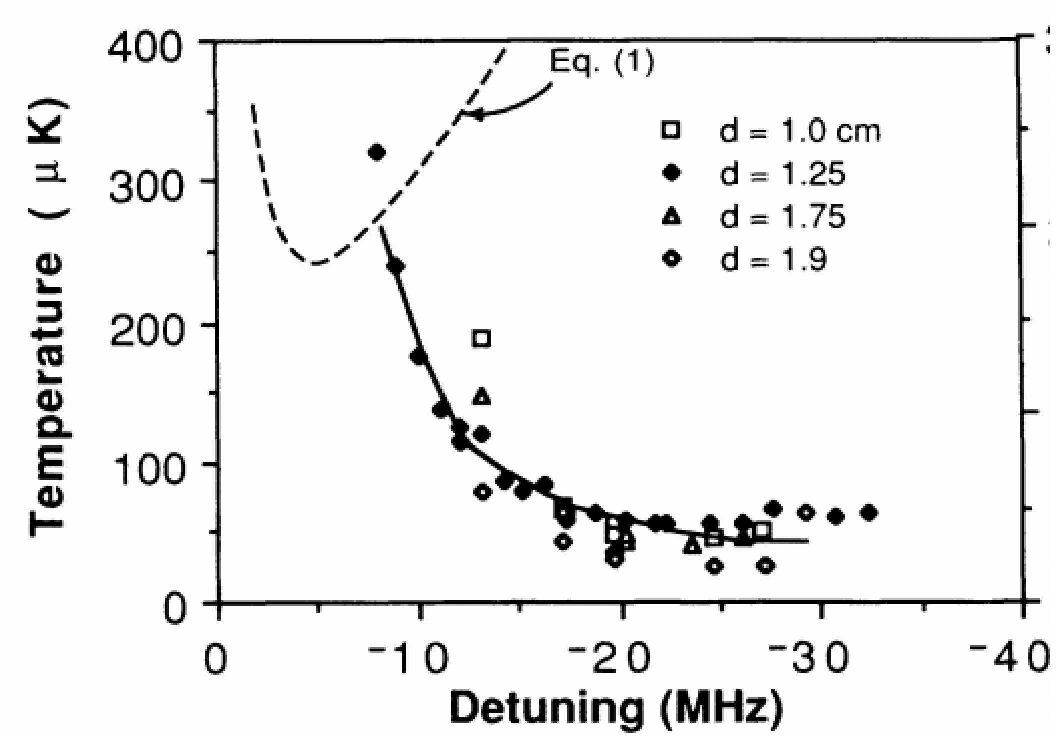
Sub-Doppler cooling

For $\Delta \gg \Gamma$, $k_B T = \frac{D}{\alpha}$

$$k_B T = \frac{\frac{3}{4}(\hbar k)^2 \frac{\Delta^2}{\Gamma} s_0}{-3\hbar k^2 \frac{\Delta}{\Gamma}} = \frac{1}{4}\hbar \Delta s_0 \sim \delta U$$

For $\Delta \gg \Gamma$, $s_0 = \frac{{\Omega_0}^2/2}{\Delta^2 + \Gamma^2/4} \approx \frac{{\Omega_0}^2}{2\Delta^2}$

$$\boxed{k_B T = \frac{\hbar {\Omega_0}^2}{8|\Delta|} \sim \frac{I_0}{|\Delta|}}$$



Temperature decreases when
- Increasing the detuning
- Decreasing the laser intensity

Sub-Doppler cooling

What is the limit temperature ?

Force is linear versus v only for $v \ll v_c$

$$\sigma_v \ll v_c \Leftrightarrow \sigma_v \ll \frac{1}{2k}\gamma_0 \Leftrightarrow \sigma_v \ll \frac{1}{2k}\frac{2}{9}\Gamma s_0 \Leftrightarrow \sigma_v \ll \frac{1}{18}\frac{\Gamma {\Omega_0}^2}{k\Delta^2}$$

With $\sigma_v = \sqrt{\frac{k_B T}{m}} = \sqrt{\frac{\hbar {\Omega_0}^2}{8|\Delta| m}}$, we finally get the constraint that

$$\Omega_0 \gg \frac{18}{\sqrt{8}}\sqrt{\frac{\hbar k^2}{m}}\frac{\Delta^{3/2}}{\Gamma}$$

And thus that

$$\sigma_v \gg \sqrt{\frac{\hbar}{8|\Delta| m}}\frac{18}{\sqrt{8}}\sqrt{\frac{\hbar k^2}{m}}\frac{\Delta^{3/2}}{\Gamma} \gg \frac{18}{8}\frac{\hbar k}{m}\frac{|\Delta|}{\Gamma}$$

The velocity distribution remains larger than the recoil velocity $v_r = \frac{\hbar k}{m}$

Sub-Doppler cooling

What is the limit temperature ?

At the lowest temperatures, some of our approximations break …

- When $k_B T \sim \delta U$, the atoms tend to be trapped in the potential well,

while our expression of the force assumes atoms moving at constant velocities

- When $\sigma_v \sim v_r$, the semiclassical approach fails.

One needs to a quantum treatment (eg MC simulation)

Sub-Doppler cooling

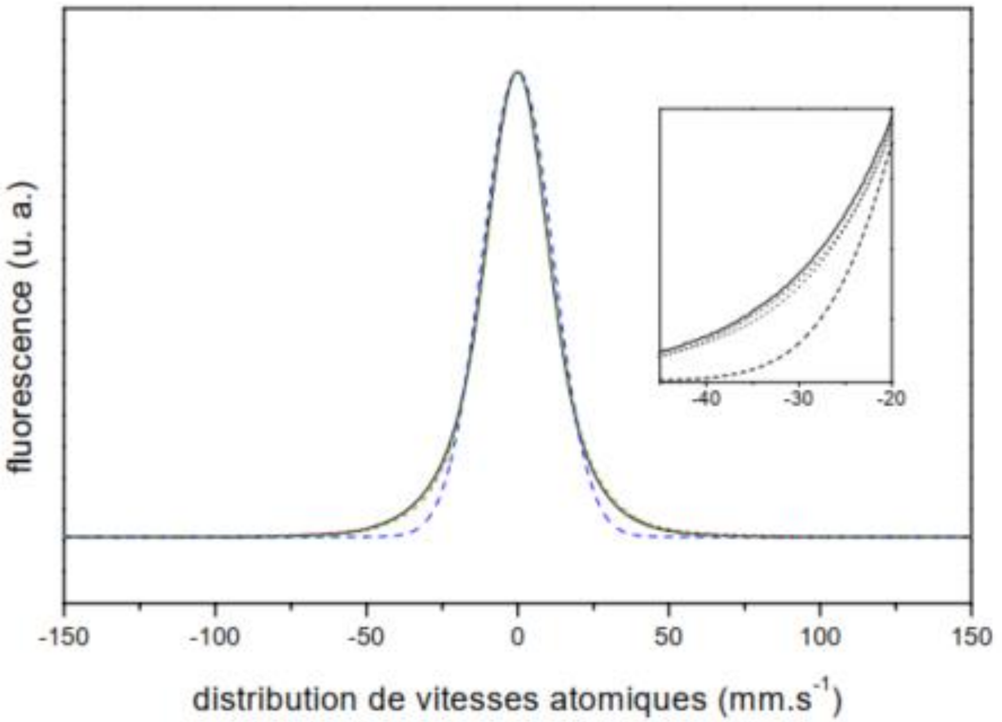
What is the limit temperature ?

MC simulations confirm limits of a few v_r

Yvan Castin and Klaus Mølmer, Phys. Rev. Lett. 74, 3772 (1995)

Experimentally, one finds velocity with HWHM as low as $\sim 2v_r$

And non Gaussian velocity distributions, but rather Lorentzian b : $f(v) \propto \dfrac{1}{(1+\left(\frac{v}{v_0}\right)^2)^b}$



Yvan Sortais, PhD thesis

Rb fountain clock

$v_0 \sim 3v_r$ and $b \sim 2.1$

Sub-Doppler cooling

Polarisation configuration for MOTs (and subsequent molasses) is $\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-$ (not « lin perp lin »)

But subDoppler temperatures also observed for $\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-$ configuration !

The total electric field is given by $E(z,t) = \mathcal{E}_+(z)\exp(-i\omega t) + c.c.$

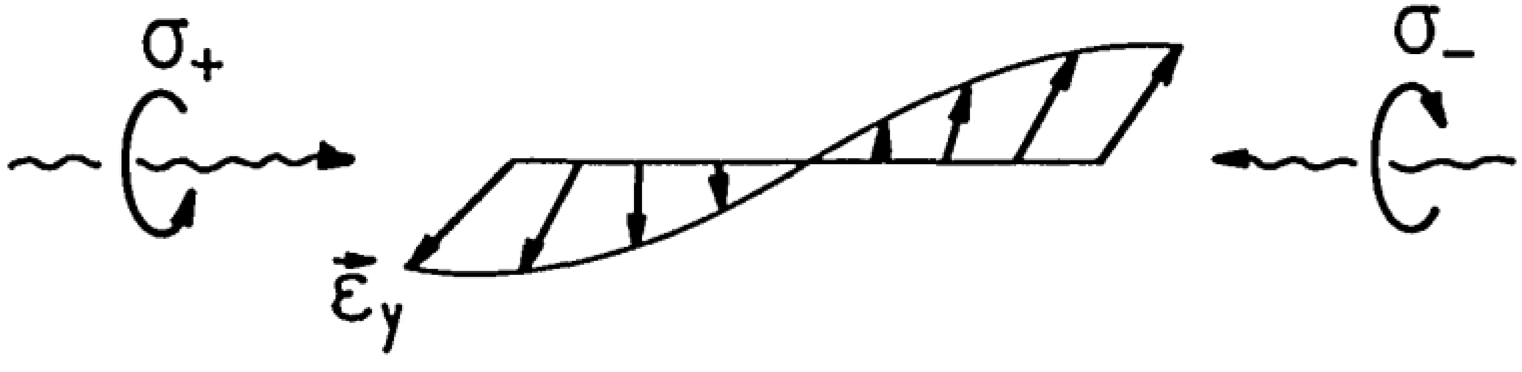
With $\mathcal{E}^+(z) = \mathcal{E}_0\, \epsilon\, \exp(ikz) + \mathcal{E}_0'\, \epsilon'\, \exp(ikz)$

And $\epsilon = \epsilon_+ = -\frac{1}{\sqrt{2}}\left(\epsilon_x + i\epsilon_y\right)$ and $\epsilon' = \epsilon_- = \frac{1}{\sqrt{2}}\left(\epsilon_x - i\epsilon_y\right)$

When the intensities are identical ($\mathcal{E}_0 = \mathcal{E}_0$'), we get

$$\mathcal{E}^+(z) = -\mathrm{i}\sqrt{2}\mathcal{E}_0(\epsilon_x \sin(kz) + \epsilon_y \cos(kz)) = -\,\mathrm{i}\sqrt{2}\mathcal{E}_0\epsilon_Y$$

The field has a linear polarisation which rotates, describing an helix with a pitch λ



Sub-Doppler cooling

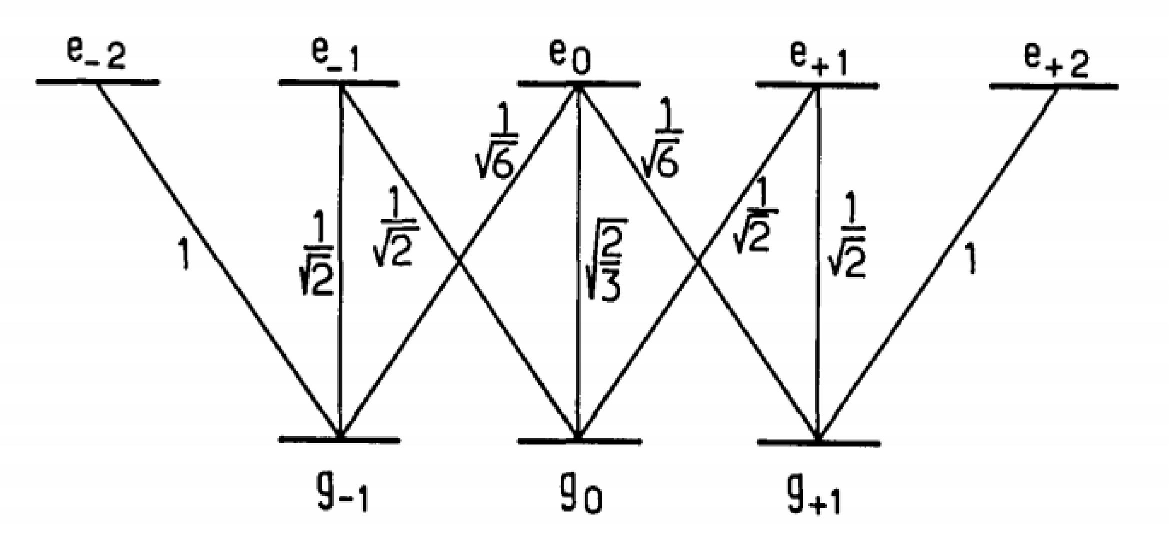
The polarisation being linear at any position,
the ground states light shifts are independent of z, whatever the angular momentum J

→ No dipole force (no light shift gradients)

But there is an atomic orientation along z appearing in the ground state,
even at very low velocity

Because of this motion induced atomic orientation, there will be an imbalance between the absorption in the two counterpropagating beams, leading to a friction force

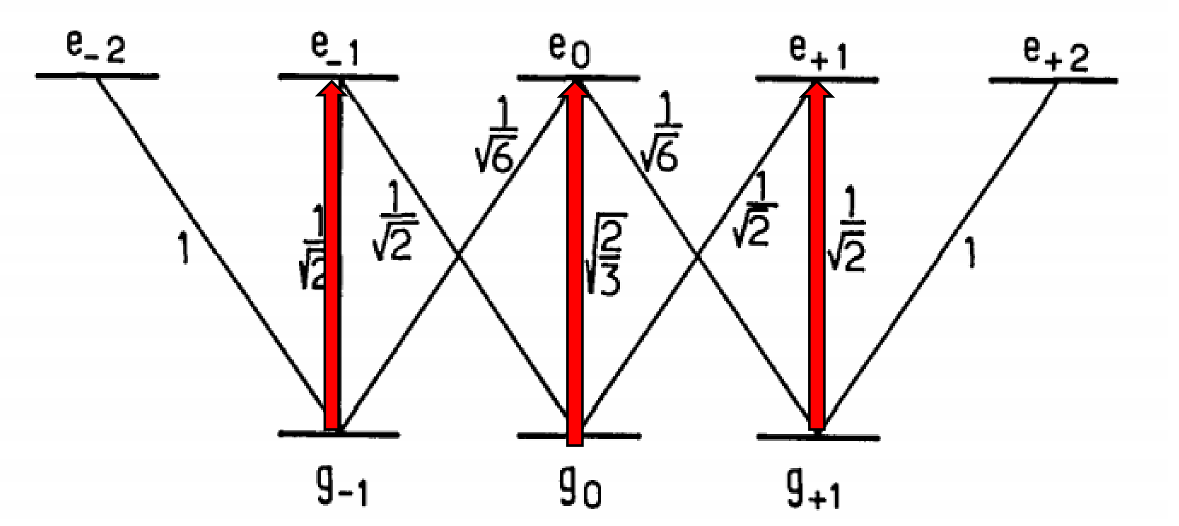
As a model, we consider a J=1 → J=2 transition
(this is actually the simplest posible atomic transition for such a scheme)



Sub-Doppler cooling

Let us start with an atom at rest at z=0

We choose the quantization axis along the linear polarisation of the light, which is ϵ_y



The light field couples g_i to e_i (π transitions)

Optical pumping rates are unbalanced :

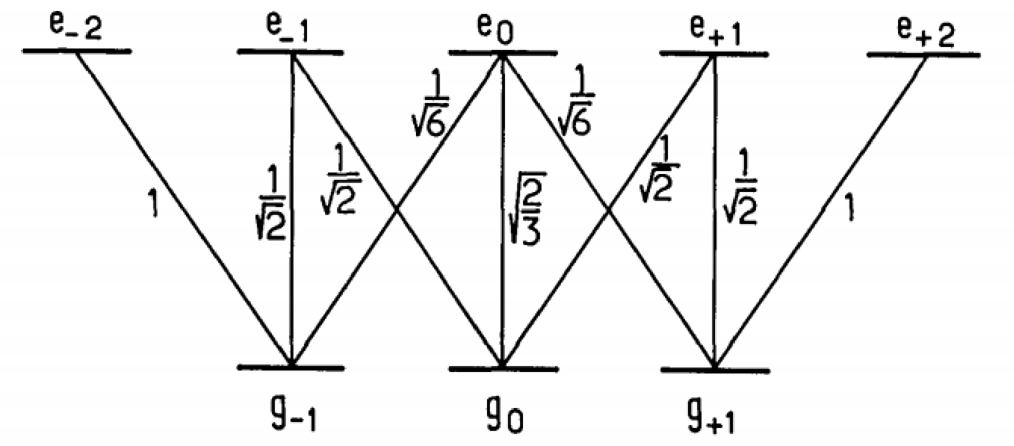
- From g_{+1} or g_{-1} to g_0 , it is proportional to $(\frac{1}{\sqrt{2}})^2(\frac{1}{\sqrt{2}})^2 = \frac{1}{4}$
- From g_0 to g_{+1} or g_{-1} to g_0 , it is proportional to $(\sqrt{\frac{2}{3}})^2\ (\sqrt{\frac{1}{6}})^2 = \frac{1}{9}$

This will concentrate atoms in the g_0 state

Steady state populations are 4/17, 9/17 and 4/17 for the states g_{-1}, g_0 and g_{+1} resp.

Sub-Doppler cooling

For a red detuning, the ground states are light shifted by different amounts as the coupling g_i to e_i are different

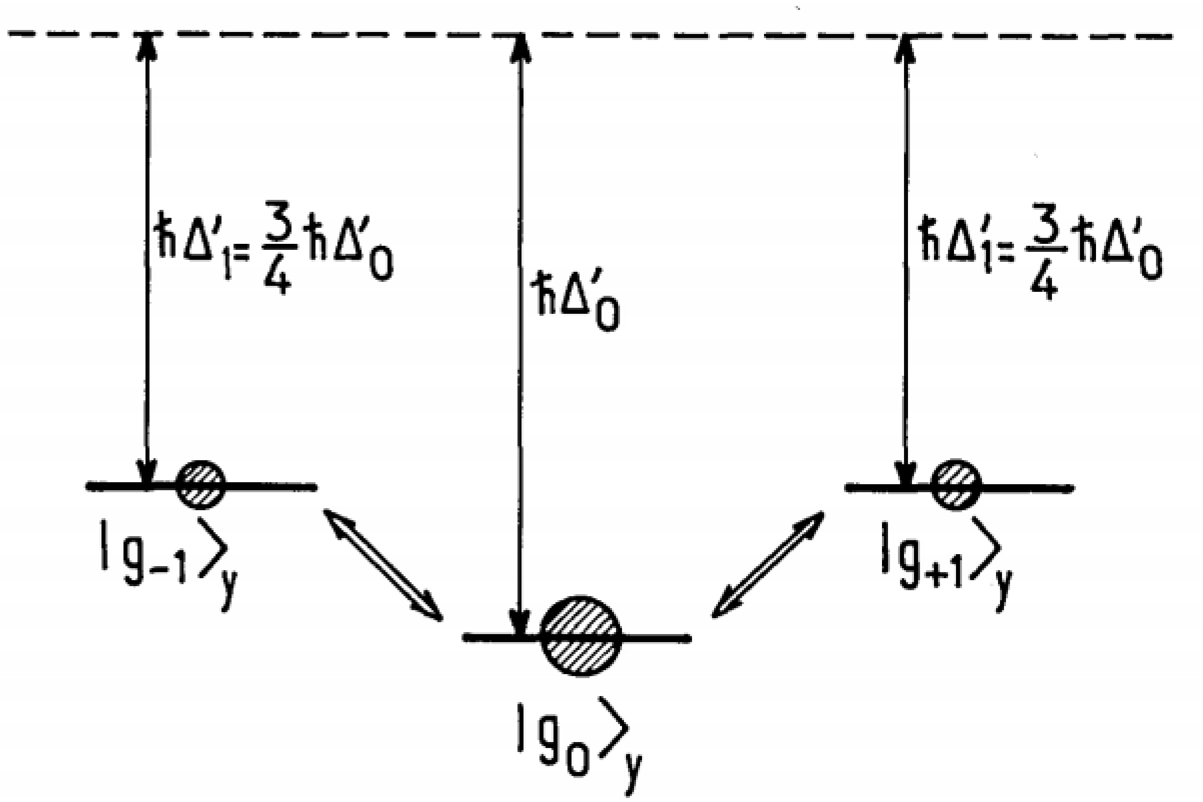


$$\Delta E_i \propto {C_{i,i}}^2$$

$$\Delta E_{\pm 1} = \hbar \Delta'_{\pm 1} \propto \frac{1}{2}$$

$$\Delta E_0 = \hbar \Delta'_0 \propto \frac{2}{3}$$

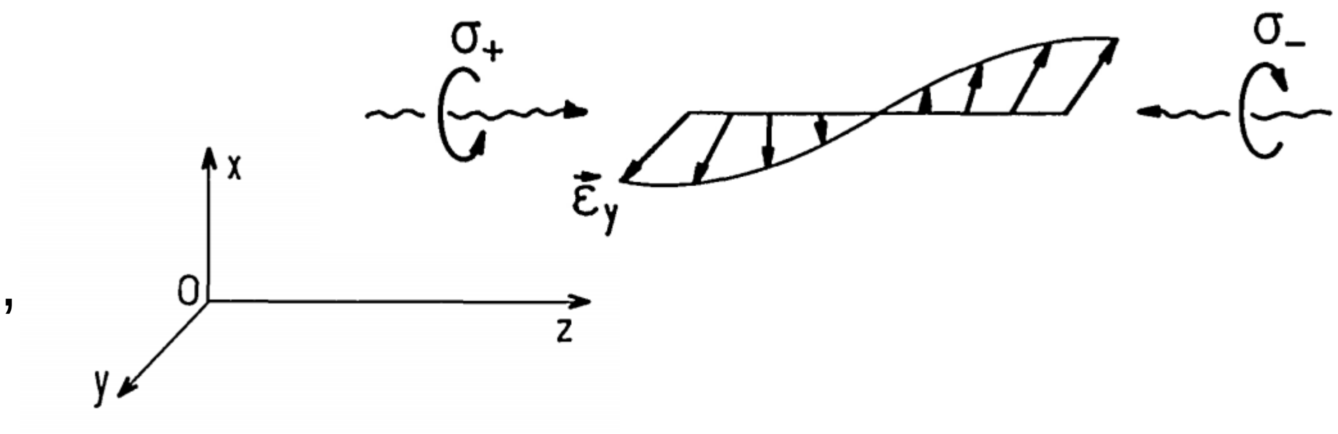
$$\Rightarrow \frac{\hbar \Delta'_{\pm 1}}{\hbar \Delta'_0} = \frac{1/2}{2/3} = \frac{3}{4}$$



Sub-Doppler cooling

Let us now consider an atom at velocity v along the z axis

In its rest frame, the atom sees a linear polarisation $\epsilon_Y = (\epsilon_x \sin(kz) + \epsilon_y \cos(kz))$, which rotates in the plane (Oxy) around z, making an angle $\varphi = kz = kvt$



Let us introduce a rotating frame in the rest frame of the atom so that the direction of the polarisation points in a constant direction, and work in this frame

Such a frame transformation leads to an inertial field, which looks like a fictitious magnetic field along z. The amplitude of this equivalent field is such that the Larmor frequency corresponds to the rotation velocity kv.

The corresponding coupling is $\mathrm{V} = kvJ_z$

This term introduces couplings proportional to kv between $|g_0\rangle_{\mathrm{y}}$ to $|g_{\pm 1}\rangle_{\mathrm{y}}$

Sub-Doppler cooling

This term leads to perturbed eigenstates (first order perturbation theory for $kv \ll \Delta'$)

$$\overline{|g_0\rangle_{\mathrm{y}}} = |g_0\rangle_y + \frac{kv}{\sqrt{2}({\Delta'}_0 - {\Delta'}_1)}(|g_1\rangle_y + |g_1\rangle_y)$$

$$\overline{|g_1\rangle_{\mathrm{y}}} = |g_1\rangle_y - \frac{kv}{\sqrt{2}({\Delta'}_0 - {\Delta'}_1)}|g_0\rangle_y$$

$$\overline{|g_{-1}\rangle_{\mathrm{y}}} = |g_{-1}\rangle_y - \frac{kv}{\sqrt{2}({\Delta'}_0 - {\Delta'}_1)}|g_0\rangle_y$$

Note that, to first order, energies and steady state populations are identical to those of the unperturbed states. Only the eigenstates are different.

But we will show that the populations of the eigenstates $|g_{+1}\rangle_z$ and $|g_{-1}\rangle_z$ of J_z are now imbalanced (while they are when the atom is at rest)

This population imbalance is proportional to the steady state value of J_z,

$$\langle J_z\rangle = \hbar(\Pi_{+1} - \Pi_{-1})$$

Sub-Doppler cooling

We now calculate the average value of J_z, but in the basis $\overline{|g_m\rangle_{\mathrm{y}}}$

We find: $$\overline{{}_{\mathrm{y}}\langle g_0|}\, J_z\, \overline{|g_0\rangle_{\mathrm{y}}} = \frac{2\hbar k v}{\Delta'_0 - \Delta'_1}$$

and $$\overline{{}_{\mathrm{y}}\langle g_{+1}|}\, J_z\, \overline{|g_{+1}\rangle_{\mathrm{y}}} = \overline{{}_{\mathrm{y}}\langle g_{-1}|}\, J_z\, \overline{|g_{-1}\rangle_{\mathrm{y}}} = -\frac{\hbar k v}{\Delta'_0 - \Delta'_1}$$

Weighting by the populations and summing over all states, we get

$$\langle J_z\rangle = \frac{2\hbar k v}{\Delta'_0 - \Delta'_1}\left(\frac{9}{17} - \frac{2}{17} - \frac{2}{17}\right) = \frac{40}{17}\frac{\hbar k v}{\Delta'_0}$$

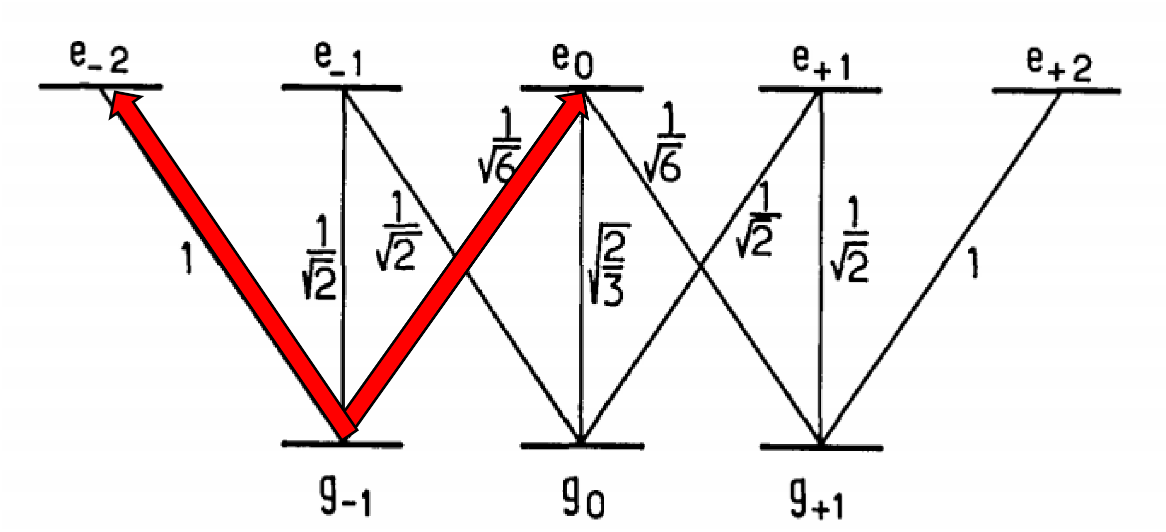
The corresponding population imbalance is thus

$$\Pi_{+1} - \Pi_{-1} = \frac{40}{17}\frac{k v}{\Delta'_0}$$

Sub-Doppler cooling

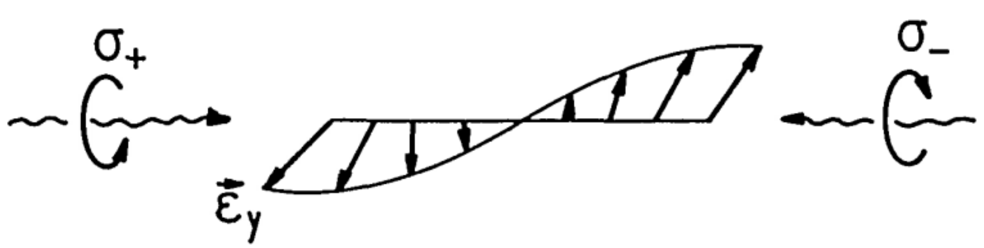
Let us consider a red detuning $(\Rightarrow \Delta'_0 < 0)$ and $v > 0$, $\Pi_{+1} < \Pi_{-1}$

This population difference leads to a force imbalance



An atom in $|g_{-1}\rangle_z$ has a probability to absorb a $\boldsymbol{\sigma}_-$ photon 6 times higher than a $\boldsymbol{\sigma}_+$ one

An atom in $|g_{+1}\rangle_z$ has a probability to absorb a $\boldsymbol{\sigma}_+$ photon 6 times higher than a $\boldsymbol{\sigma}_-$ one



Since $\Pi_{+1} < \Pi_{-1}$, the radiation pressures exerted by the two waves are imbalanced

The atom will scatter more counterpropagating photons ($\boldsymbol{\sigma}_-$) than copropagating ones

The net force is opposed to the motion

Sub-Doppler cooling

Order of magnitude of the force:

The difference between the number of scattered photons per unit time is $\sim \Gamma'(\Pi_{+1} - \Pi_{-1})$, where Γ' is the scattering rate in the ground state

Each $\boldsymbol{\sigma}_+$ (resp. $\boldsymbol{\sigma}_-$) photon imparts a momentum $\hbar k$ (resp. $-\hbar k$), this leads to a force

$$F \sim \Gamma' \frac{kv}{\Delta'_0} \hbar k$$

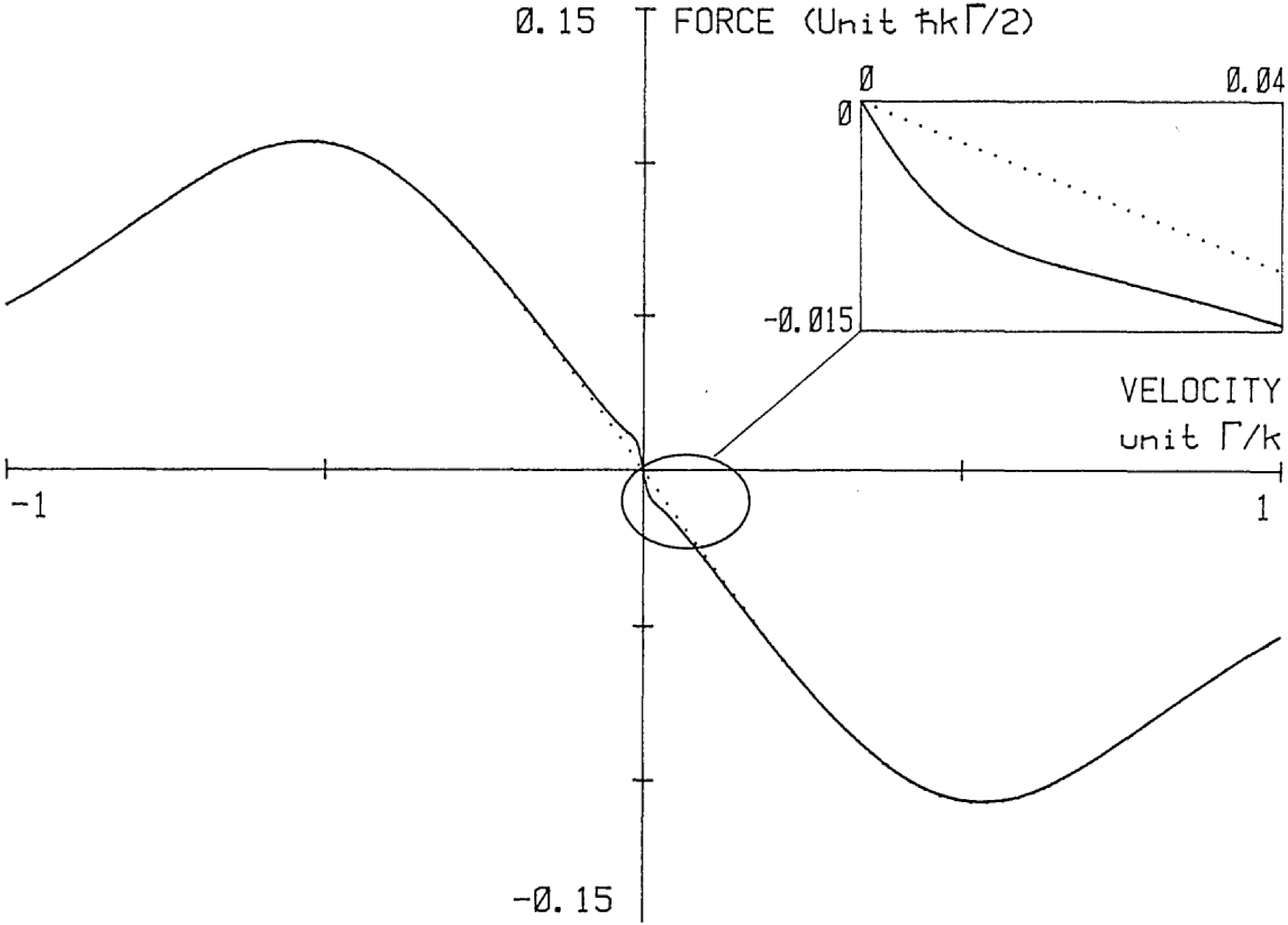
Since $\Gamma' \sim \Omega^2\ \Gamma/\Delta^2$ and $\Delta'_0 \sim \Omega^2/\Delta$, we get $\boxed{F \sim \hbar k^2 \frac{\Gamma}{\Delta} v}$

The friction coefficient is $\boxed{\alpha \sim -\hbar k^2 \frac{\Gamma}{\Delta}}$

NOTE : as for Sysiphus cooling (lin perp lin configuration)
The friction is independent from the laser intensity

But for lin perp lin cooling, $\alpha \sim -\hbar k^2 \frac{\Delta}{\Gamma} \gg -\hbar k^2 \frac{\Gamma}{\Delta}$, since $\Delta \gg \Gamma$

Sub-Doppler cooling



Sub-Doppler cooling

The force is much weaker than in the lin perp lin configuration

BUT the diffusion coefficient is also reduced

With more detailed calculation, we find $\alpha = -\frac{120}{17}\frac{\Delta\Gamma}{5\Gamma^2 + 4\Delta^2}\hbar k^2$

And two contributions to the diffusion:

- D_1 corresponding to fluctuations of the momentum carried away by fluorescence photons

$$\mathrm{D}_1 = \frac{18}{170}(\hbar k)^2\Gamma s_0$$

- D_2 corresponding to fluctuations of the difference of photons absorbed in the two waves

$$\mathrm{D}_2 = \left(\frac{36}{17}\frac{1}{1 + 4\Delta^2/5\Gamma^2} + \frac{4}{17}\right)(\hbar k)^2\Gamma s_0$$

Sub-Doppler cooling

For $\Delta \gg \Gamma$, D_1 and D_2 are of the same order of magnitude

$$\mathrm{D} = \mathrm{D}_1 + \mathrm{D}_2 = \frac{418}{170}(\hbar k)^2 \Gamma s_0$$

And the temperature is given by

$$\boxed{k_B T = \frac{\mathrm{D}}{\alpha} \sim \frac{1}{10}\frac{\hbar {\Omega_0}^2}{|\Delta|} \sim \frac{I_0}{|\Delta|}}$$

The scaling with laser intensity and detuning is similar to the lin perp lin case.

The above expression for the temperature is valid for $kv \ll \Delta'_0$ (linearity of the force)

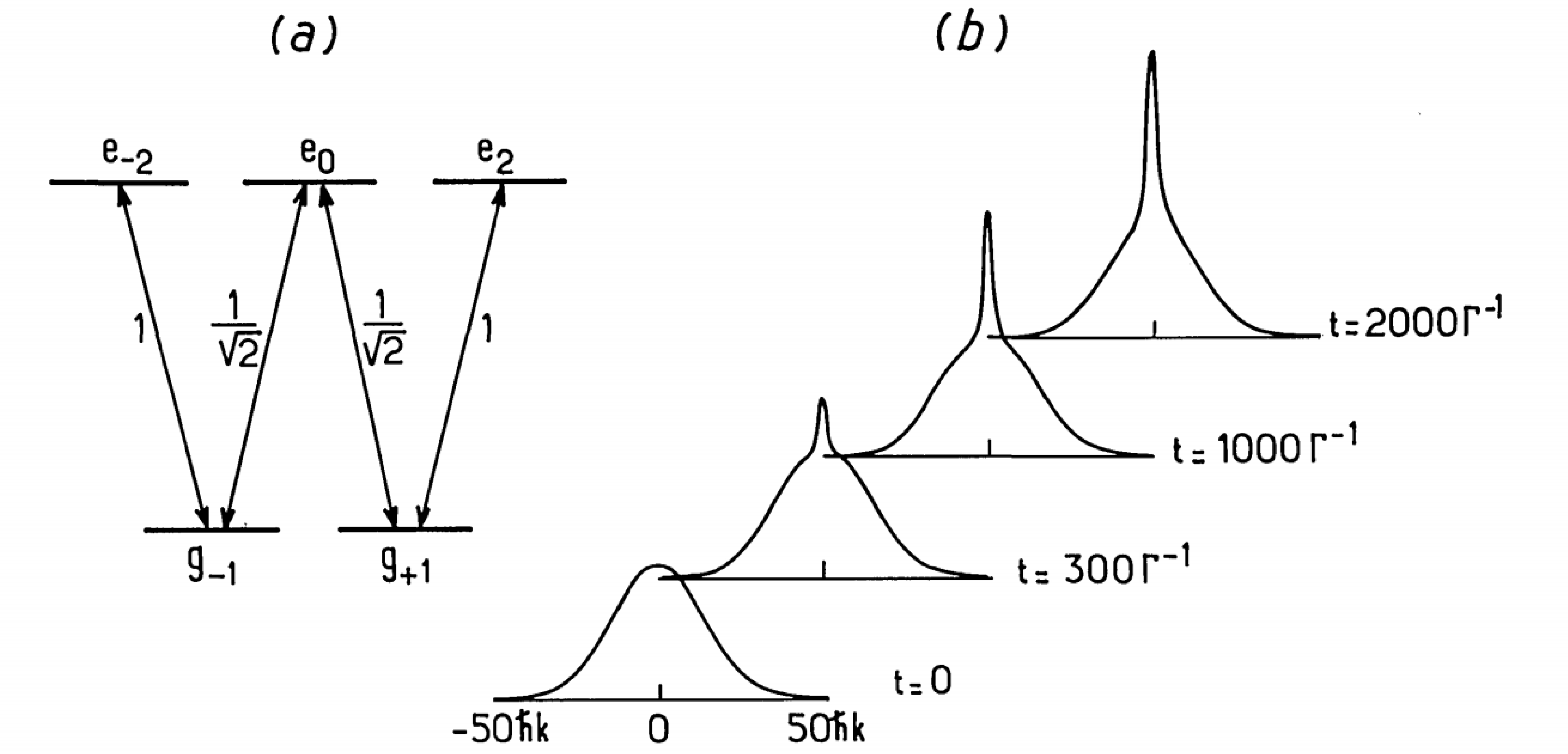
This demands that $\Omega_0 \gg \sqrt{\frac{\hbar k^2}{m}\Delta}$

and thus that $\sigma_v \gg \frac{\hbar k}{m}$

Such a limit is smaller than the one we found for lin perp lin cooling : $\sigma_v \gg \frac{18}{8}\frac{\hbar k}{m}\frac{|\Delta|}{\Gamma}$

Sub-Doppler cooling

When the cooling limit becomes of the order of the recoil velocity, one needs a full quantum treatment



Example : an initial velocity distribution, initially gaussian, evolves towards a distribution characterized by a double structure, with a narrow peak whose HWHM is of order of 2vr

Experimentally, one finds non Gaussian velocity distributions, but rather Lorentzian b as in the lin perp lin case

$$f(v) \propto \frac{1}{(1+\left(\frac{v}{v_0}\right)^2)^b}$$

Conclusion

Cooling below the Doppler cooling limit is possible for atoms with adequate internal structure

Different mechanisms depending on atomic struture and polarisation configurations:

- for Lin perp Lin polarisation, polarisation gradients lead to dipole forces and the so-called « Sysiphus » effect

- for $\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-$ polarisations, atomic motion induces a population difference between atomic ground states, leading to imbalanced radiation pressure

But in both cases,

- the force is a cooling (friction) force for $\Delta < 0$
- the temperature scales as $\frac{I_0}{|\Delta|}$
- T_{limit} is of the order of a few Er

The next question being can one beat this limit ?