Laser cooling and trapping III

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Cooling below the limits

Previous course:

Two different cooling mechanisms relying on polarization gradients that allow to beat the Doppler limit:

- The first one relying on a modulation of the ground states light shifts, leading to dipole forces and the so-called « Sysiphus » effect (lin perp lin polarisations)
- The second one on an atomic motion induced population difference between atomic ground states, leading to imbalanced radiation pressure ($\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-$ polarisations)

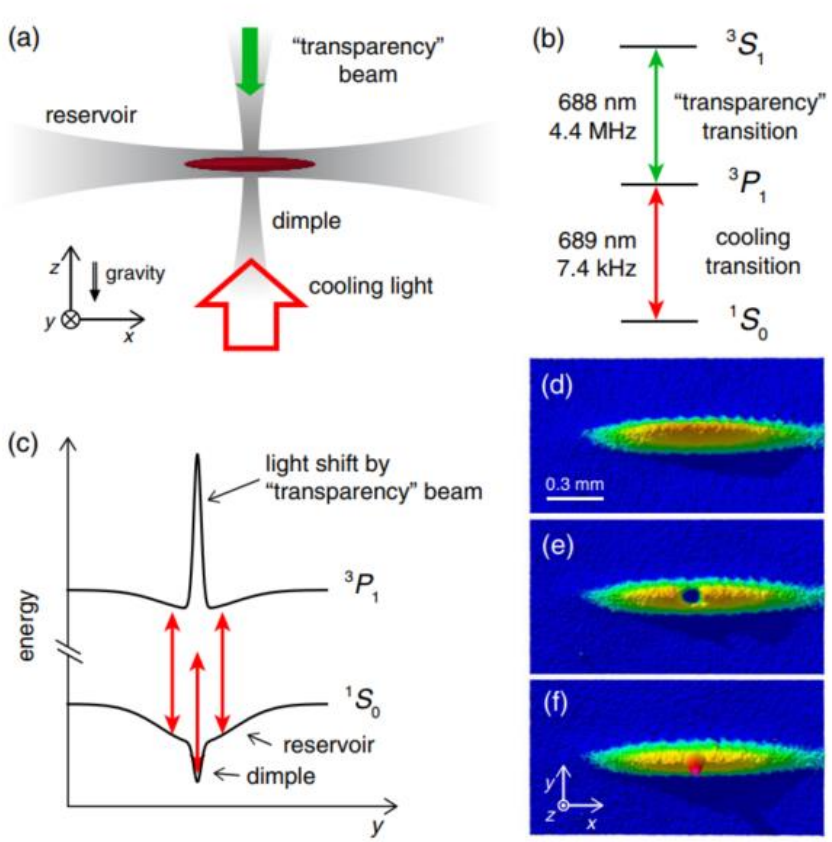
But in both cases, T_{limit} is of the order of a few E_r

The next question being : « can we beat this limit » ?

How ?

→ Prevent the slowest atoms from scattering photons

« Transparency » beam to induce a light shift
Stellmer at al, PRL 110, 263003 (2013)
→ scattering rate depends on position



Can we make the photon absorption rate velocity dependent ?

Organization of the lecture

1 : VSCPT cooling

2 : Raman cooling

3 : Grey molasses

Organization of the lecture

1 : VSCPT cooling

2 : Raman cooling

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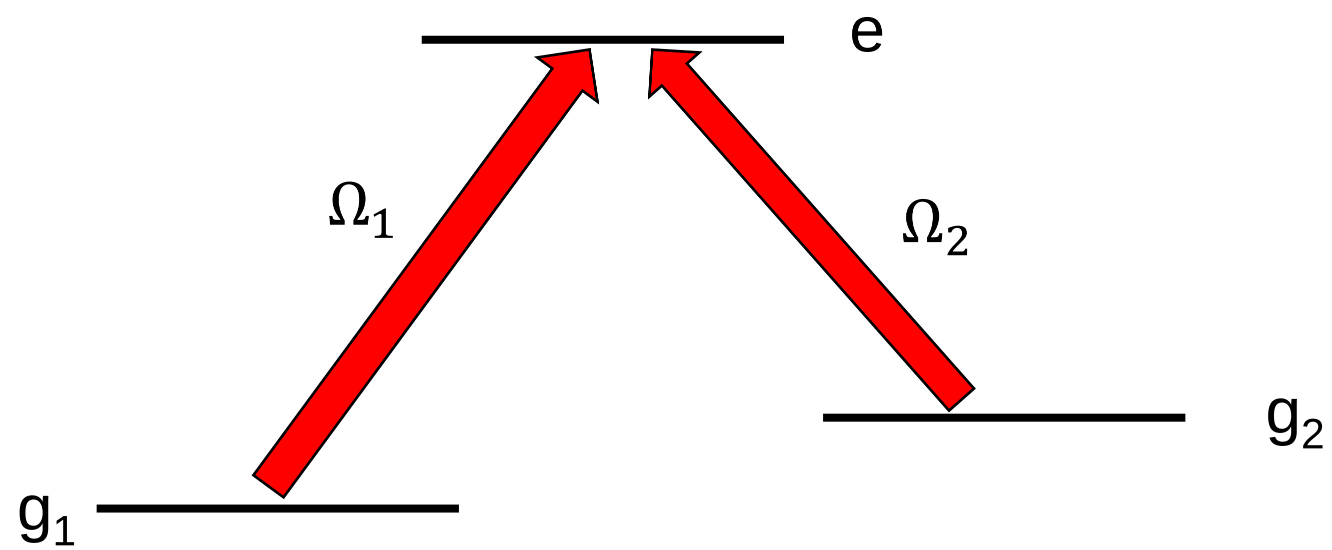
Coherent population trapping

Consider an atom with a Λ shape level structure

And two laser fields E_1 and E_2

which couple resp g_1 and g_2 to the excited state e, with Rabi frequencies Ω_1 and Ω_2



In such a configuration, there is a « dark » state, which is uncoupled to the light

This dark state is a superposition of g_1 and g_2

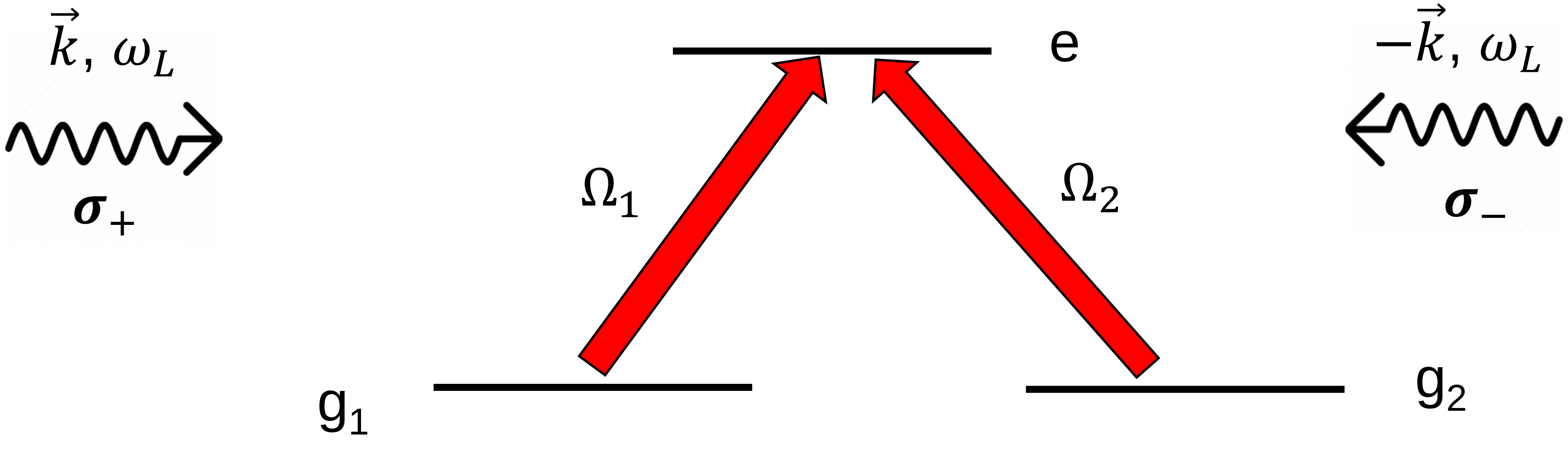
From that state, the excitation to e is not possible due to destructive interference between the two amplitudes of transition

Velocity selective coherent population trapping

Let us consider a simpler case, where the two ground states are degenerate

And the two laser fields are counterpropagating, with opposite circular polarisation but same frequency.

Example: g_1 and g_2 are Zeeman states m = -1 and +1, while e is a m = 0 state



The atom-laser coupling is given by

$$V = -\boldsymbol{D}.\left(\boldsymbol{E_1}(\vec{r},t) + \boldsymbol{E_2}(\vec{r},t)\right)$$
$$= \frac{\hbar\Omega_1}{2}|e\rangle\langle g_1|e^{-i\omega_L t - i\varphi_1(\vec{r})} + \frac{\hbar\Omega_2}{2}|e\rangle\langle g_2|e^{-i\omega_L t - i\varphi_2(\vec{r})} + h.c.$$

Velocity selective coherent population trapping

Since the two waves are counterpropagating, $\varphi_1(\vec{r}) = -kz$ and $\varphi_2(\vec{r}) = +kz$

$$V = \frac{\hbar\Omega_1}{2}|e\rangle\langle g_1|e^{-i\omega_L t + ikz} + \frac{\hbar\Omega_2}{2}|e\rangle\langle g_2|e^{-i\omega_L t - ikz} + h.c.$$

Let us consider an atom at rest a the position z,

and in the superposition state $|\Psi_{NC}\rangle \propto \Omega_2|g_1\rangle e^{-ikz} - \Omega_1|g_2\rangle e^{+ikz}$

We find $V|\Psi_{NC}\rangle = 0$

The state $|\Psi_{NC}\rangle$ is a dark state.
An atom in that state is not coupled to the light field and stays there for ever.

But if the atom is not at rest, the two waves are doppler shifted in opposite directions and the two amplitudes of transition do not interfere any more.

Velocity selective coherent population trapping

If we aim at describing cooling at recoil velocities and below, we shall treat laser phases $\varphi_1(\vec{r})$ and $\varphi_2(\vec{r})$, and thus z, as operators

Using $e^{\pm ikz} = \sum_p |p\rangle \langle p \mp \hbar k|$

The coupling writes

$$V = \sum_p \left[\frac{\hbar\Omega_1}{2} |e,p\rangle \langle g_1, p - \hbar k| + \frac{\hbar\Omega_2}{2} |e,p\rangle \left\langle g_{2,}, p + \hbar k \right| e^{-i\omega_L t} \right] e^{-i\omega_L t} + h.c.$$

The laser field introduces couplings between momenta states in momentum families

$$F_p = \left\{ |e,p\rangle,\ |g_1, p - \hbar k\rangle,\ \left|g_{2,}, p + \hbar k\right\rangle \right\}$$

As long as we ignore spontaneous emission, F_p is a closed family of states

Velocity selective coherent population trapping

In each family F_p, a state $|\Psi_{NC}(p)\rangle$ is now given by

$$|\Psi_{NC}(p)\rangle = \frac{1}{\sqrt{{\Omega_1}^2 + {\Omega_2}^2}} \big(\Omega_2 |g_1, p - \hbar k\rangle - \Omega_1 |g_{2,}, p + \hbar k\rangle\big)$$

We have indeed $\langle e, p|V|\Psi_{NC}(p)\rangle = 0$

Let us consider the state $|\Psi_C(p)\rangle$ is given by

$$|\Psi_C(p)\rangle = \frac{1}{\sqrt{{\Omega_1}^2 + {\Omega_2}^2}} \big(\Omega_1^* |g_1, p - \hbar k\rangle + \Omega_2^* |g_{2,}, p + \hbar k\rangle\big)$$

We have $\langle e, p|V|\Psi_C(p)\rangle = \frac{\hbar}{2}\sqrt{{\Omega_1}^2 + {\Omega_2}^2} e^{-i\omega_L t}$

$|\Psi_C(p)\rangle$ is coupled to the light

Velocity selective coherent population trapping

Let us assume that we have prepared an atom in the state $|\Psi_{NC}(p)\rangle$
and see how it will evolve

To calculate the evolution of the system in the family F_p , we have to add to the laser coupling V the free Hamiltonian $H = \frac{P^2}{2m} + \hbar\omega_e |e\rangle\langle e|$

Such calculations can be done using the formalism of the density matrix σ
(details in *A. Aspect et al, JOSA B 6, 2112 (1989)*)

We find

$$\frac{d}{dt}\langle\Psi_{NC}(p)|\sigma|\Psi_{NC}(p)\rangle = -ik\,\frac{p}{m}\frac{2\Omega_1\Omega_2}{{\Omega_1}^2+{\Omega_2}^2}\langle\Psi_{NC}(p)|\sigma|\Psi_C(p)\rangle + c.c.$$

For $p = 0$, the population of $\Psi_{NC}(p)$ does not evolve, even when taking account the free evolution (kinetic energy). This will still hold when taking into account spontaneous emission, as $\Psi_{NC}(p)$ is a superposition of stable ground states

But not when $p \neq 0$,
this because the kinetic energies of $|p + \hbar k\rangle$ and $|p - \hbar k\rangle$ differ by $\frac{2\hbar kp}{m}$

⇨ This coherent population trapping depends on p , it is thus velocity selective

Velocity selective coherent population trapping

Let us now consider the role of spontaneous emission

For resonant excitation ($\Delta = 0$) and weak excitation, $\Omega_1, \Omega_2 \ll \Gamma$,
the Rabi coupling $\Omega/\sqrt{2}$ (when $\Omega_1 = \Omega_2 = \Omega$) between $|\Psi_C(p)\rangle$ and $|e, p\rangle$ gives to the state $|\Psi_C(p)\rangle$ a finite width

$$\Gamma' = \frac{\Gamma}{2} s = \frac{\Gamma}{2} \frac{2\left(\frac{\Omega}{\sqrt{2}}\right)^2}{\Gamma^2} \Rightarrow \boxed{\Gamma' = \frac{\Omega^2}{2\Gamma}}$$

And the coupling $\frac{kp}{m}$ between $|\Psi_C(p)\rangle$ and $|\Psi_{NC}(p)\rangle$ gives to $|\Psi_{NC}(p)\rangle$ a width

$$\Gamma'' = \frac{\left(\frac{kp}{m}\right)^2}{\Gamma'} \Rightarrow \boxed{\Gamma'' = \frac{2\left(\frac{kp}{m}\right)^2 \Gamma}{\Omega^2}}$$

Γ'' is the probability per unit time for an atom to leave the state $|\Psi_{NC}(p)\rangle$

The smaller p, the longer an atom stays « trapped » in $|\Psi_{NC}(p)\rangle$

Equivalently, for an interaction time θ, only atoms with p such that $\Gamma''\theta < 1$,
or $\left(\frac{kp}{m}\right)^2 < \frac{\Omega^2}{2\theta\Gamma}$ can remain trapped in $|\Psi_{NC}(p)\rangle$

Velocity selective coherent population trapping

Role of spontaneous emission:

Allows jump from one family to an other, and eventually from a family F_p to the family $F_{p=0}$, where the atoms may be trapped in the state $|\Psi_{NC}(0)\rangle$

The mechanism for accumulating atoms in the « trapped » state is the diffusion in momentum space induced by spontaneous emission

But an atom falling into the $F_{p=0}$ family might fall into $|g_1, -\hbar k\rangle$ or $|g_{2,}, p + \hbar k\rangle$, or a linear combination of these, and not necessarity in $|\Psi_{NC}(0)\rangle$

Velocity selective coherent population trapping

Role of spontaneous emission:

Imagine that the atom falls for instance into the state $|g_1, -\hbar k\rangle$

$|g_1, -\hbar k\rangle = \frac{1}{\sqrt{2}}(|\Psi_{NC}(0)\rangle + |\Psi_C(0)\rangle)$ (here we consider $\Omega_1 = \Omega_2 = \Omega$)

$|\Psi_{NC}(0)\rangle$ is stable, but not $|\Psi_C(0)\rangle$, which means that it will excited by the lasers at a rate Γ'

After a time long with respect to $1/\Gamma'$, the atom will be either in $|\Psi_{NC}(0)\rangle$, where it will remain trapped, or will be involved in new fluorescence cycles

This state filtering process leaves half of the atoms in the trapped state, while the other half resume a sequence of fuorescence cycles

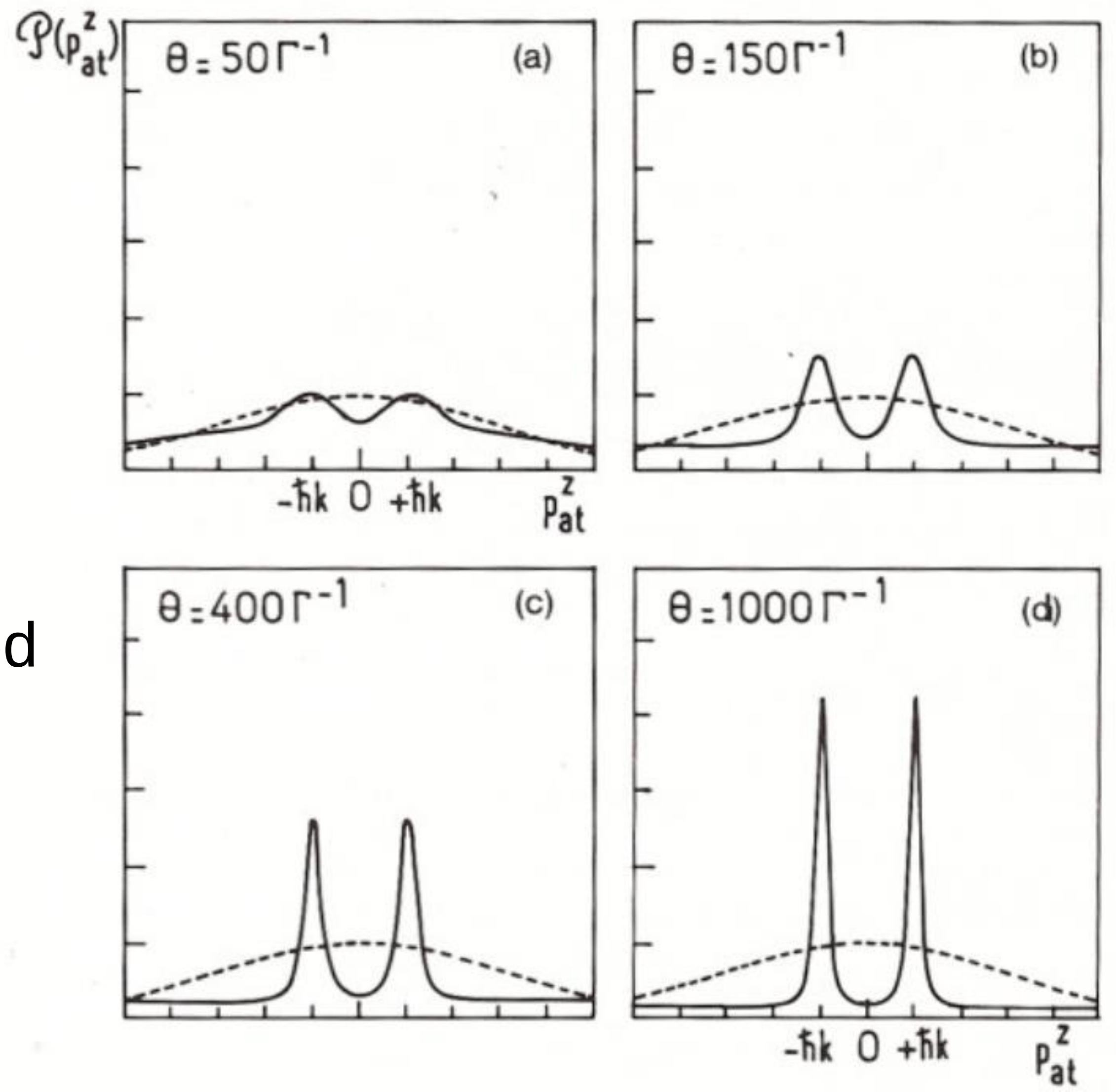
Velocity selective coherent population trapping

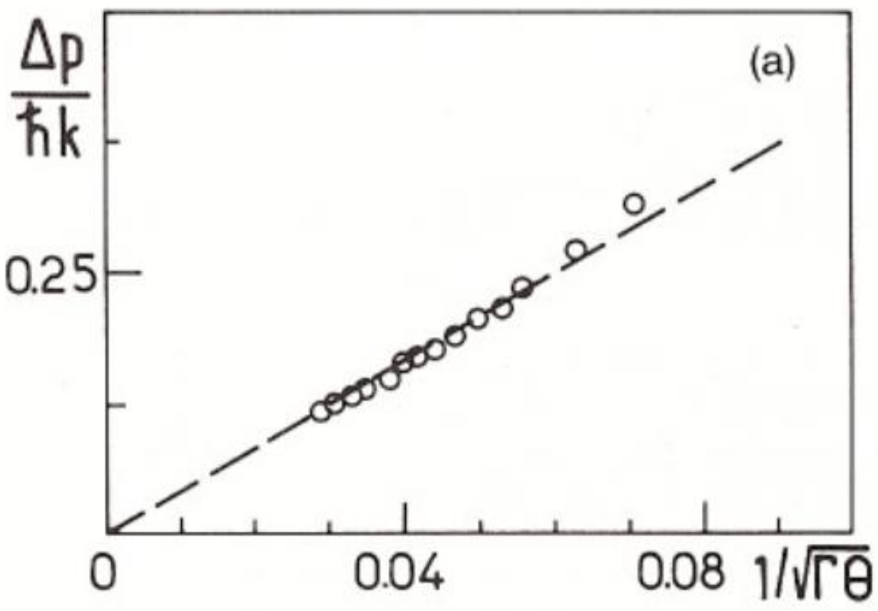
Time evolution of the momentum distibution can be obtained with numerical simulations

A. Aspect at al, JOSAB 6, 2112-2124 (1989)

We start with a Gaussian momentum distribution with a HWHM of $3\hbar k$

We observe the atoms accumulating in the trapped state (double subrecoil peak distribution) as the interaction time θ increases



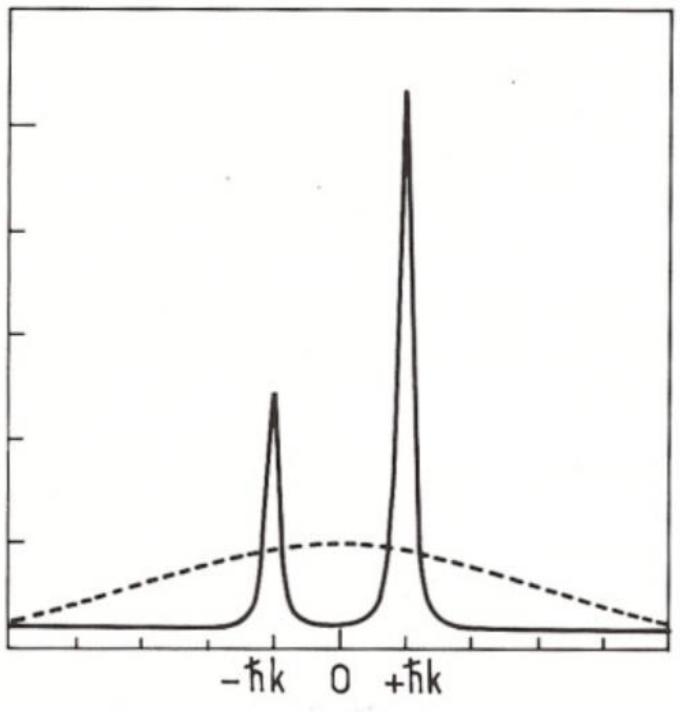


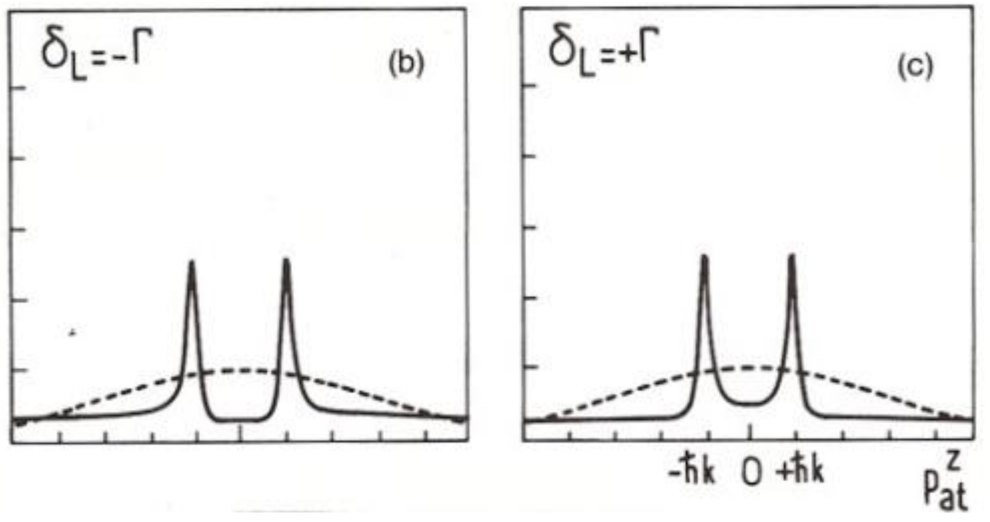
The width of the peaks decreases with the interaction time

Velocity selective coherent population trapping

For $\Omega_1 \neq \Omega_2$,
the momentum distribution is not symetric

$$|\Psi_{NC}(p)\rangle = \frac{1}{\sqrt{\Omega_1{}^2 + \Omega_2{}^2}} \left(\Omega_2 |g_1, p - \hbar k\rangle - \Omega_1 |g_{2,}, p + \hbar k\rangle\right)$$



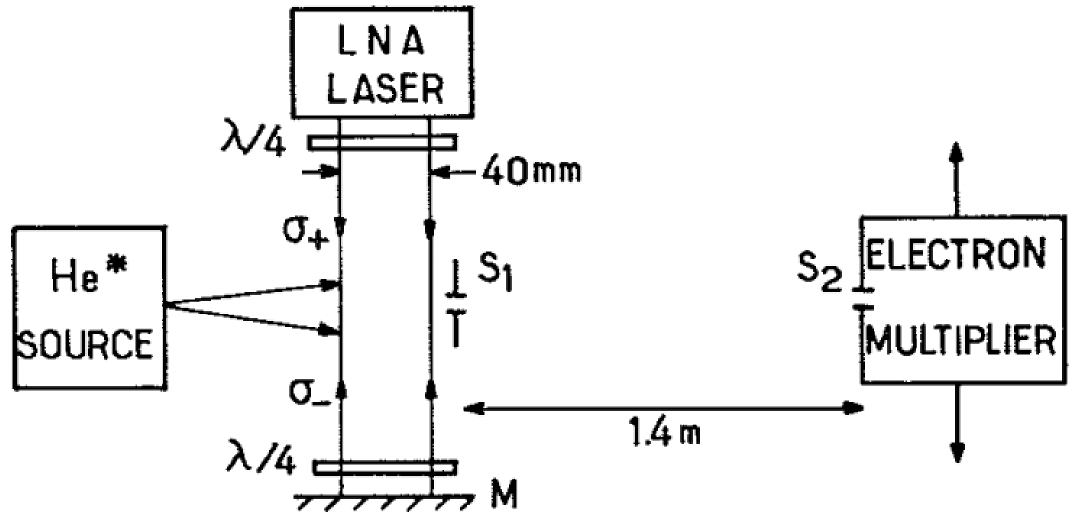


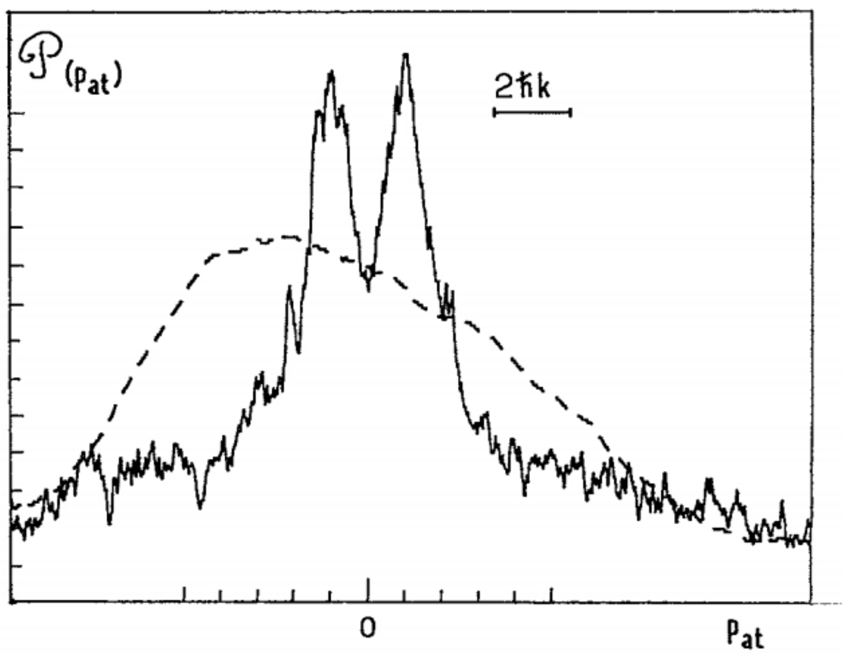
VSCPT works even when the lasers are detuned, and does not depend on the sign of the detuning

Velocity selective coherent population trapping

Experimental demonstrations:

Beam of metastable helium, ENS
Laser tuned on the $2^3S_1 \rightarrow 2^3P_1$ transition
A. Aspect et al, PRL 61, 826 (1988)



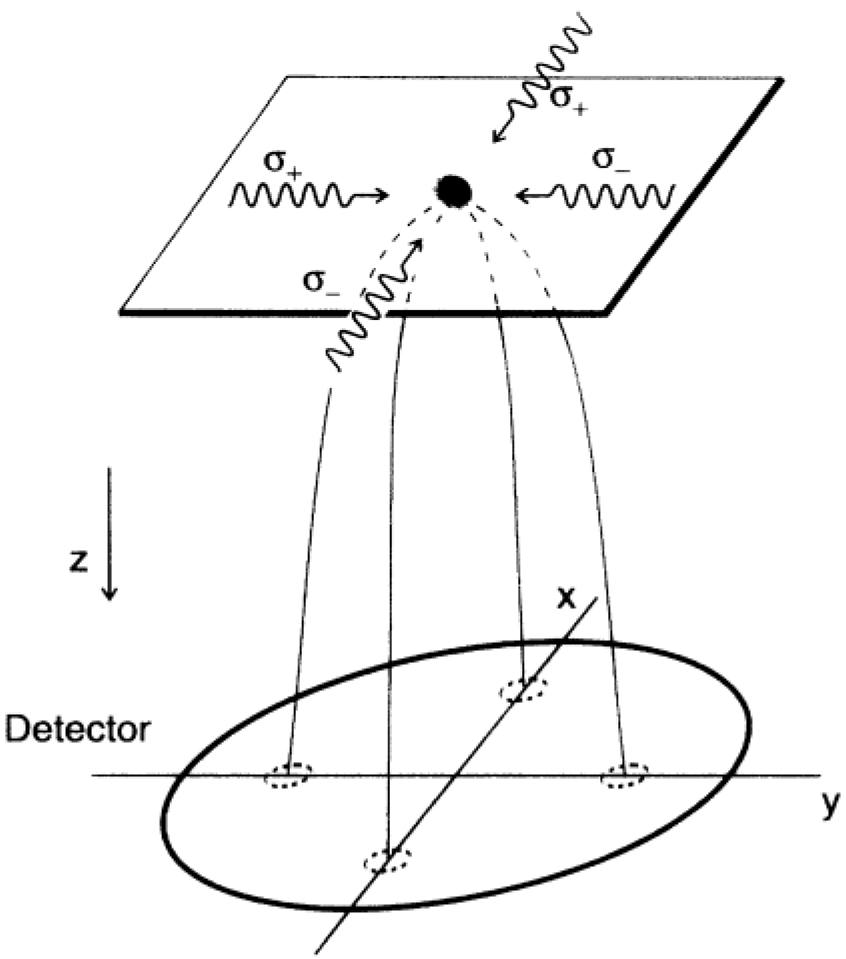


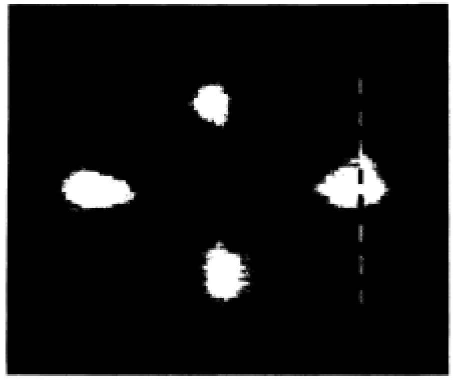
Double peak distribution with subrecoil widths

Temperatures as low as $T_R/800$ have been obtained
Measurement via interferometric method based on making the wavepackets overlap
Saubamea et al, PRL 79, 3146 (1997)

Velocity selective coherent population trapping

Extension at 2D:
Lawall et al, PRL 73, 3146 (1994)



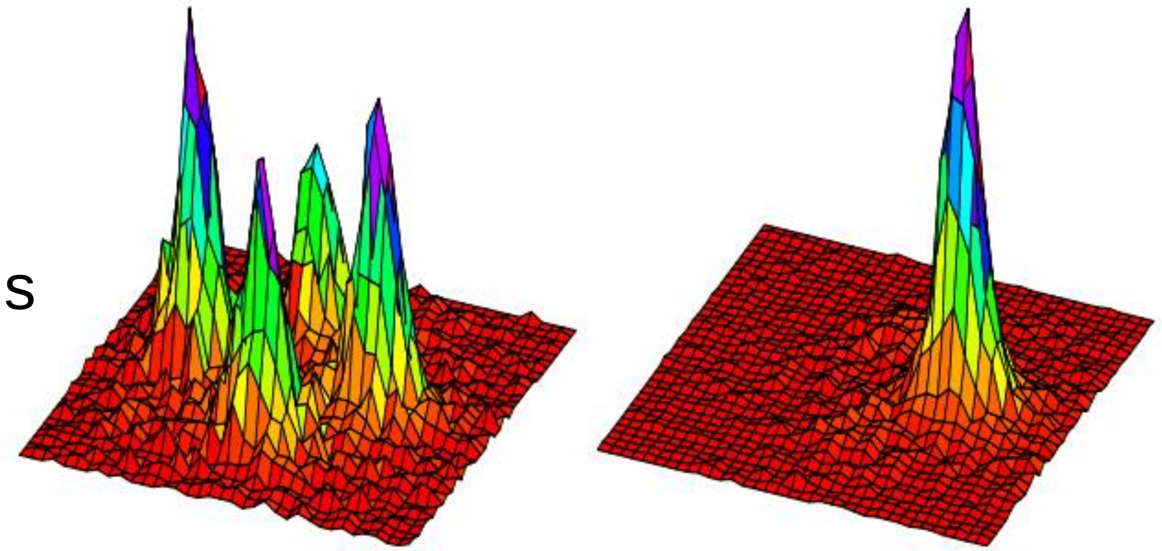


Four peaks

at $v_x = \pm v_r$ and $v_y = \pm v_r$

With subrecoil widths $\sim v_r/4$

Switching off adiabatically three of the four beams leave the atoms in a single subrecoil peak
S. Kulin, PRL 78,4185 (1997)



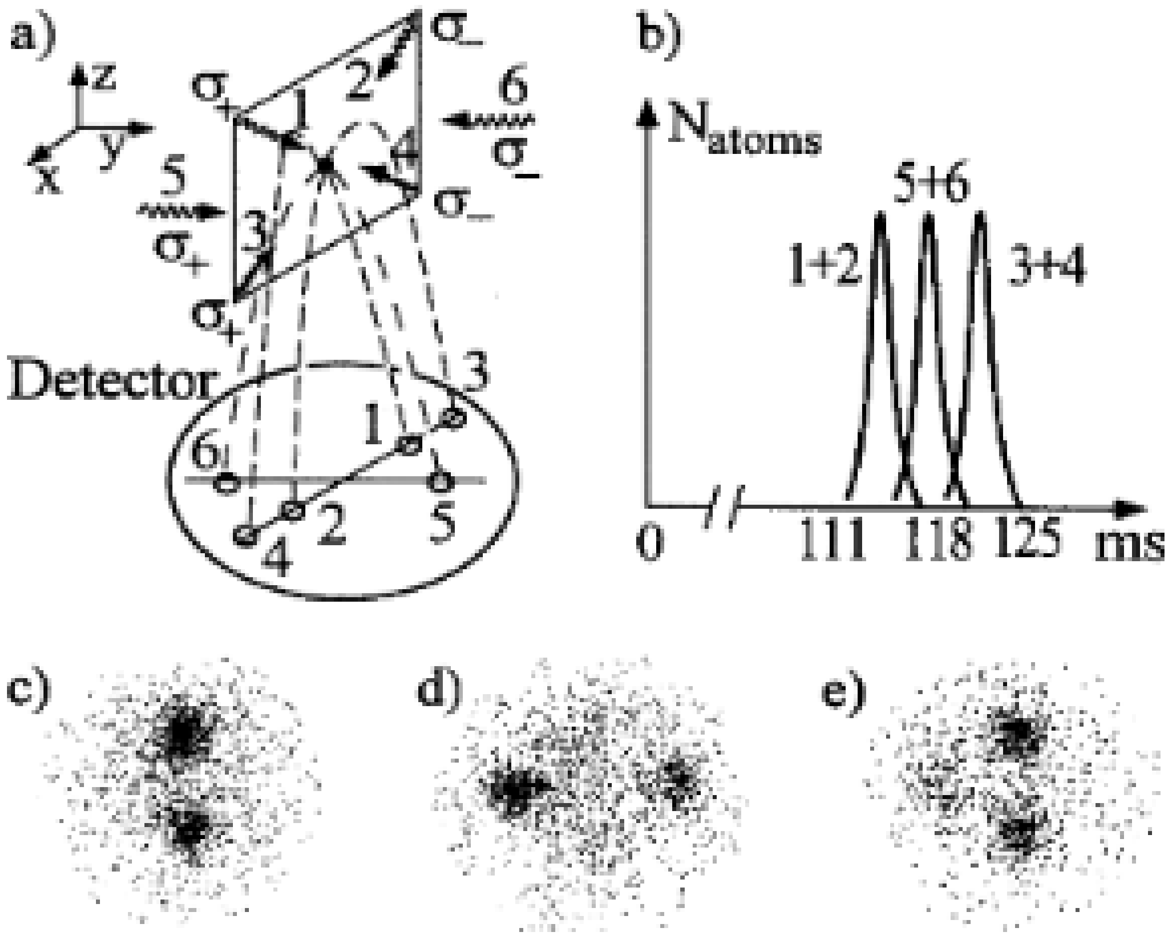
Velocity selective coherent population trapping

Extension at 3D:
Lawall et al, PRL 75, 4194 (1995)

4 beams at 45° in the (x,z) plane

2 beams in the y direction

This leads to the arrival of the atoms on the detector after different delays

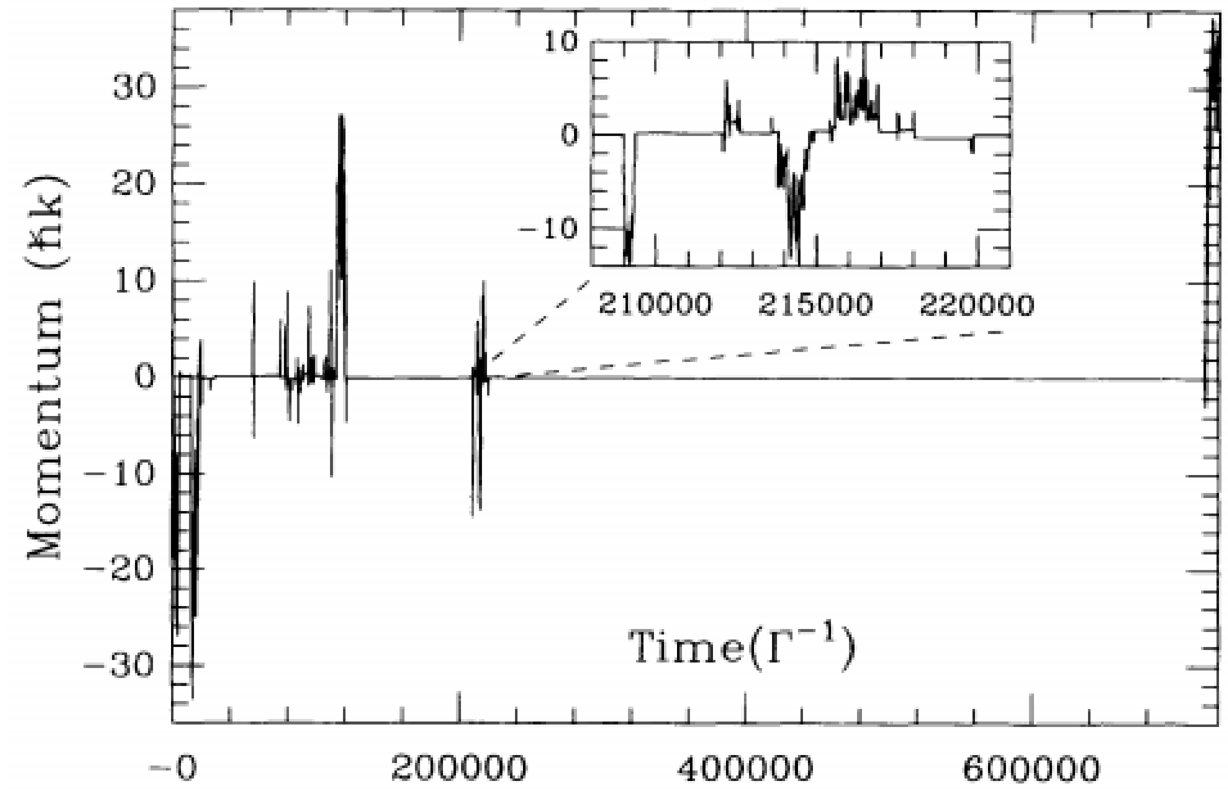


Velocity selective coherent population trapping

Quantum MC simulations:
Random walk in the momentum space dominated by rare events, whose duration is a significant fraction of the interaction time

The distribution of the trapping times τ in a small trapping volume around $p = 0$ scales as $f_\tau(\tau) \sim 1/\tau^{3/2}$ (1D)

$\langle\tau\rangle$ not defined (diverges)



F. Bardou et al, Phys. Rev. Lett. 72, 203 (1994)

Levy statistics are used to deal with such distributions

With this tool, one shows that

$$T \sim 1/\theta$$

$f(p) \sim \frac{1}{p^2}$ (Lorentzian-like rather than Gaussian)

Organization of the lecture

1 : VSCPT cooling

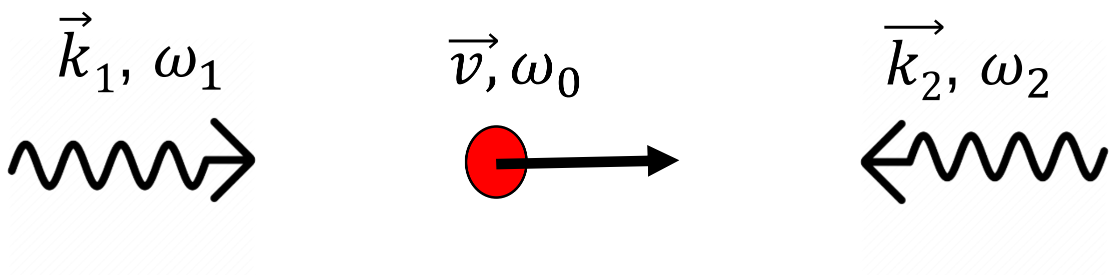
2 : Raman cooling

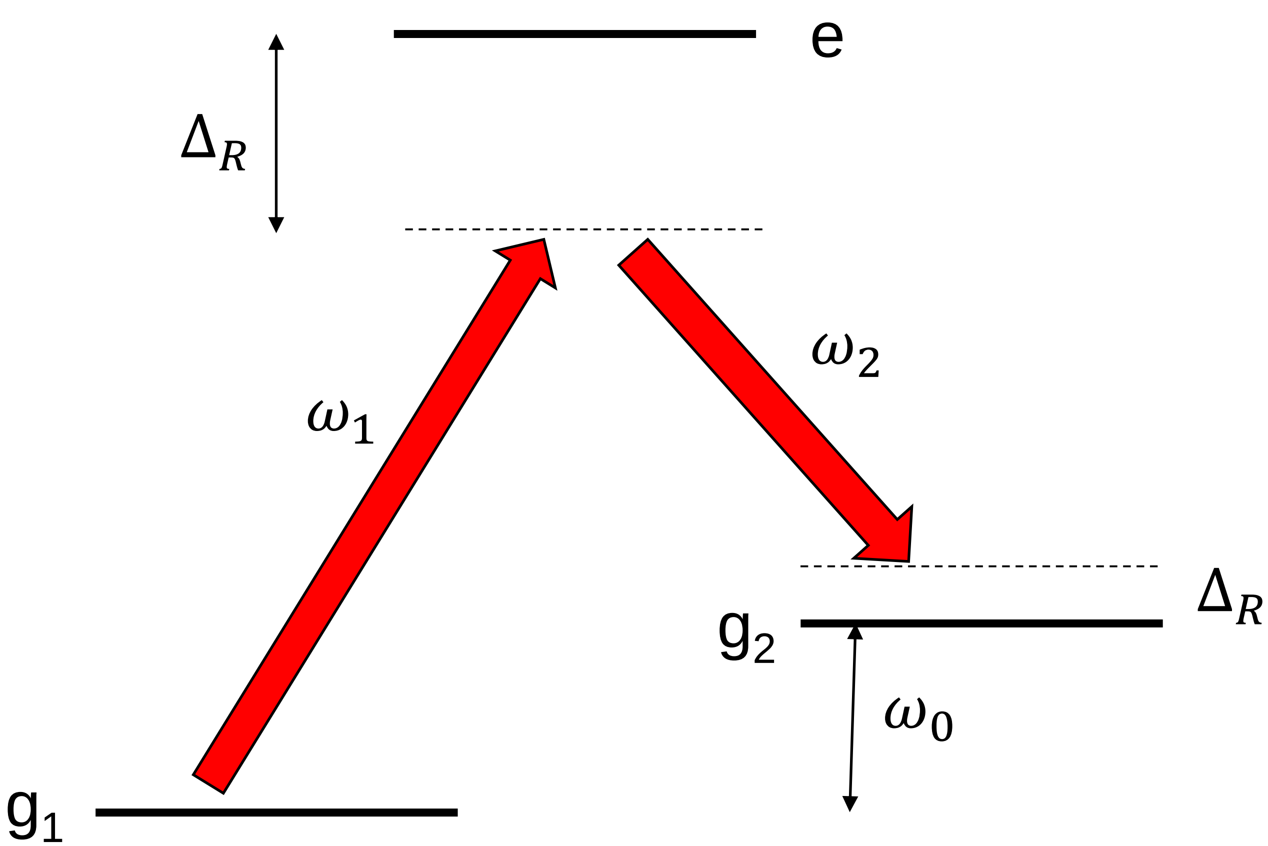
3 : Grey molasses

Stimulated Raman transition

Conter-propagating Raman transitions:

Two (counter-propagating) lasers L_1 and L_2, detuned from resonance with a frequency difference $\omega_1 - \omega_2$ that matches the energy difference ω_0 between g_1 and g_2





The atom initially in $|g_1, p\rangle$

Absorbs a photon in L_1

And reemits in a stimulated way in L_2

It ends up in $|g_2, p + \hbar k_1 - \hbar k_2\rangle$

$$k_{eff} = k_1 - k_2 \sim 2k$$

Resonance condition:

$$\omega_1 - \omega_2 = \omega_0 + \frac{k_{eff} p}{m} + \frac{\hbar k_{eff}^2}{2m}$$

Doppler shift
→ velocity selective process

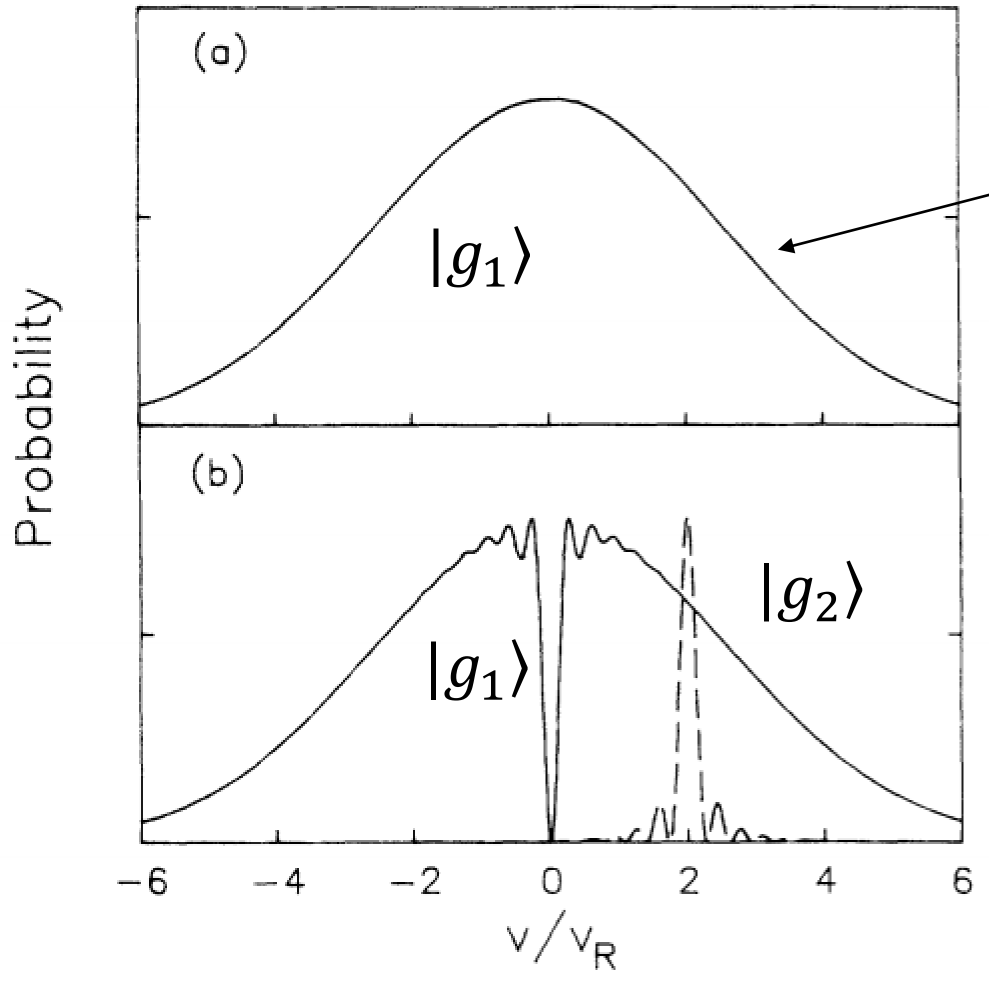
Stimulated Raman transitions

For large detunings $\Delta_R \sim$ a few GHz, one can neglect spontaneous emission

The system reduces to an effective two level atom

The coupling between $|g_1, p\rangle$ and $|g_2, p + \hbar k_{eff}\rangle$ leads to Rabi oscillations with a Rabi frequency

$$\Omega_R = \frac{\Omega_1 \Omega_2}{2\Delta_R}$$



Moler at al, PRA 45, 342 (1991)

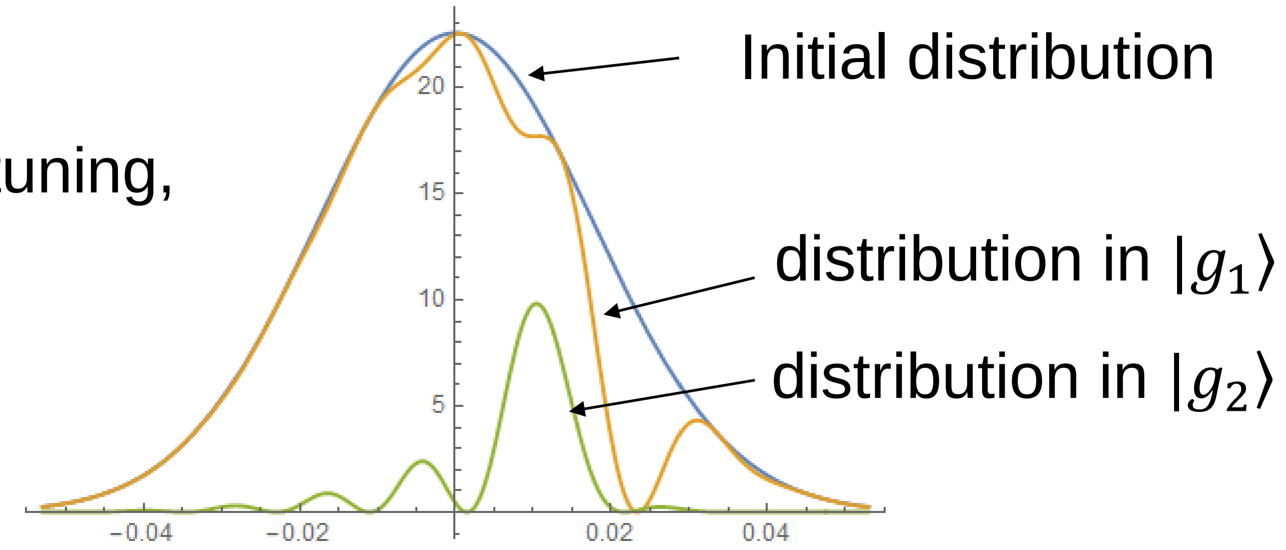
Initial velocity distribution in state $|g_1\rangle$, with rms width σ_v

Effect of a (square) Raman "π" pulse with $\Omega_R < k\sigma_v$

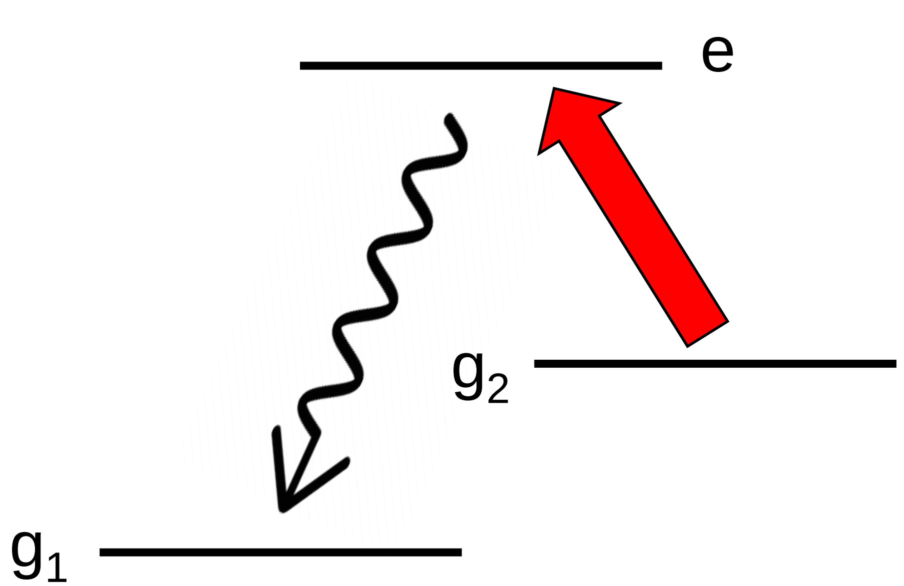
The resonant velocity class is transferred from state $|g_1\rangle$ to state $|g_2\rangle$

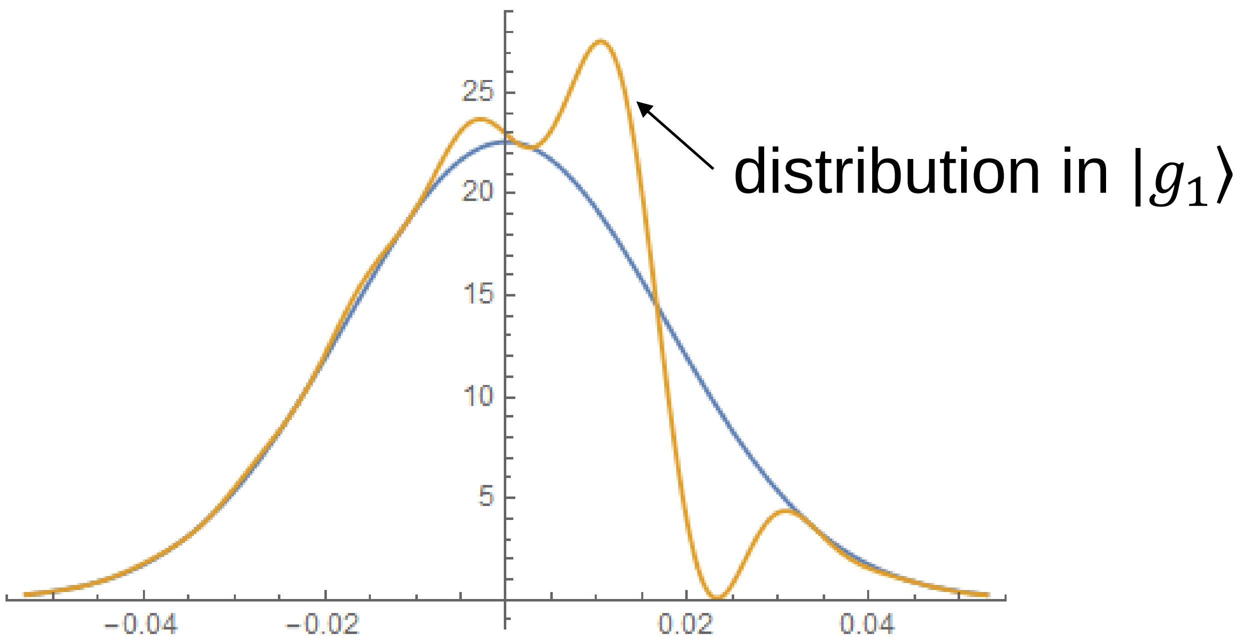
Raman cooling

1st step:
Realize such a "π" pulse, with a certain detuning, so as to reduce the velocity of (a certain velocity class) of the atoms



2nd step:
Apply a pulse of « repumper » to bring the velocity selected atoms back into g_1





Idea of Raman cooling:
Apply a sequence of such pulse with different Raman frequency differences in order to transfer all velocity classes one after the other

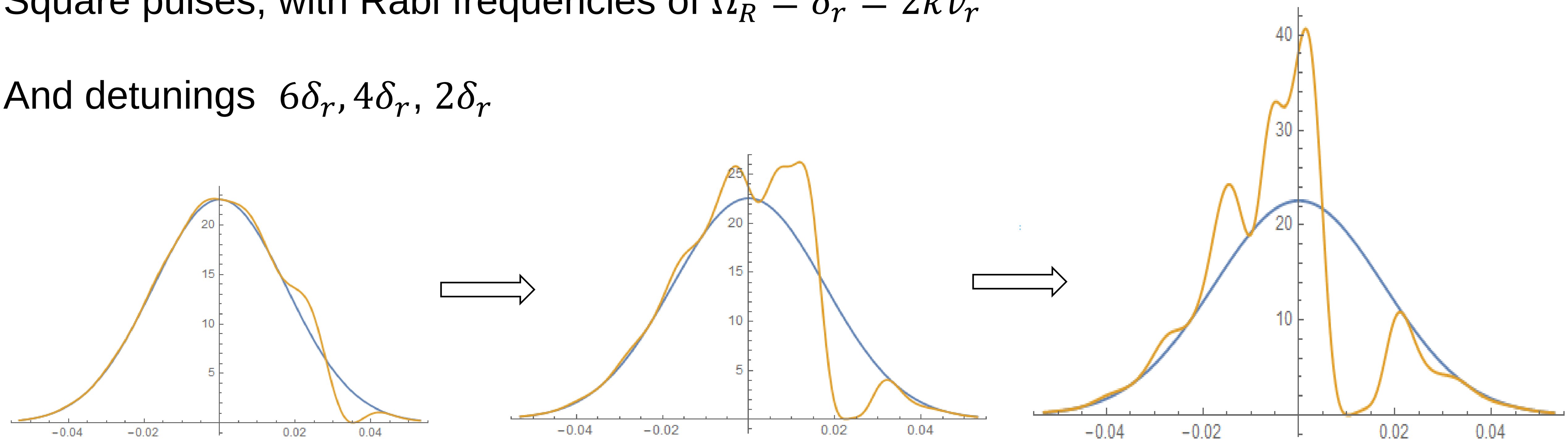
Raman cooling

Example:

Initial velocity distribution with rms width of $6v_r$

Square pulses, with Rabi frequencies of $\Omega_R = \delta_r = 2kv_r$

And detunings $6\delta_r, 4\delta_r, 2\delta_r$



Optimization requires:

- Transitions with momentum transfer in the other direction (swap directions of k_1 and k_2)
- Tailored sequences: pulse shape and detunings

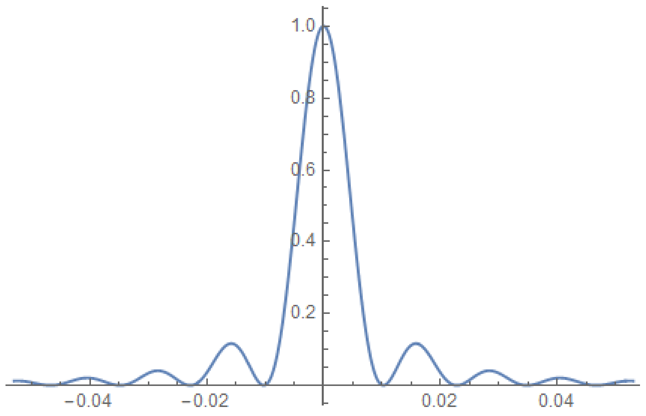
Raman cooling

How to improve the pulse velocity selectivity ?

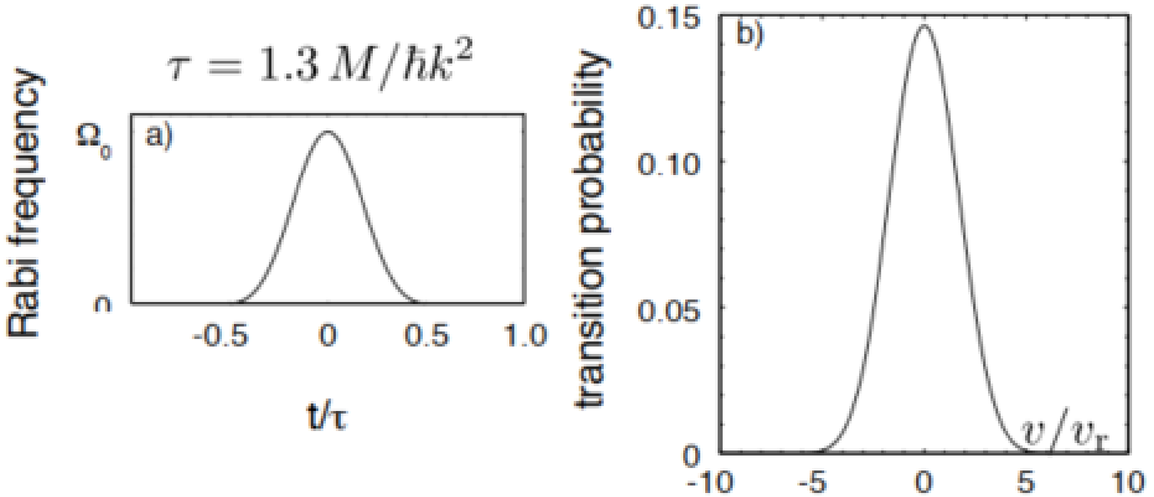
Ideal case would be a rectangular shape excitation profile,
that would transfer velocities in a given range, and nothing outside

- Square pulses: Ω_R constant

Sinus cardinal lineshape



- Blackman pulses: $\Omega_R(t) = 0.42 + 0.5\cos(2\pi t/\tau) + 0.08\cos(4\pi t/\tau)$, with τ the duration of the pulse

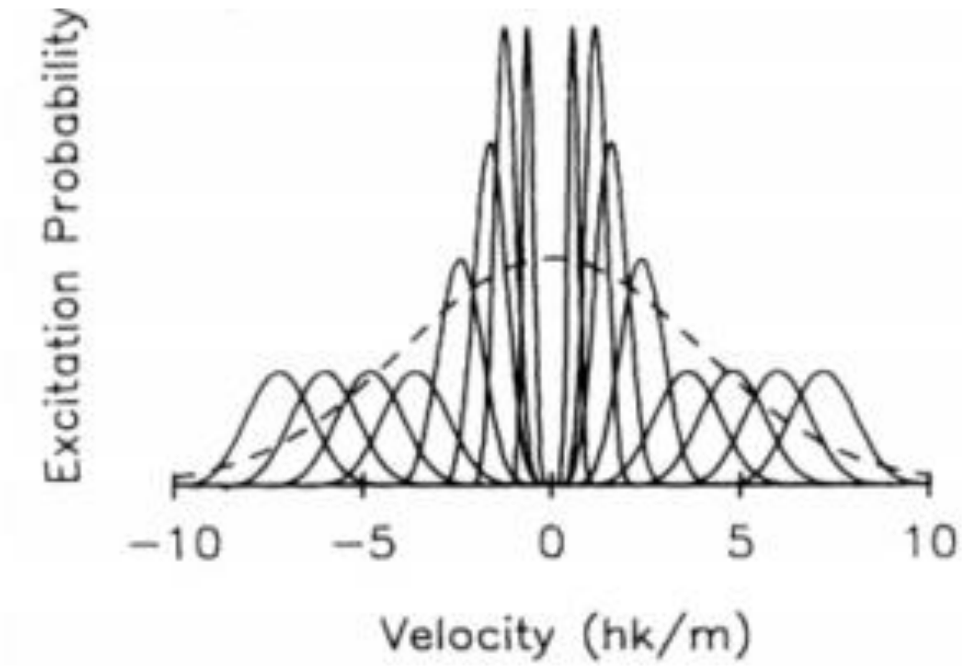


Not really rectangular
But better defined
No more ripples

Raman cooling

Optimization of the pulse sequence

Decreasing detunings and Rabi frequencies



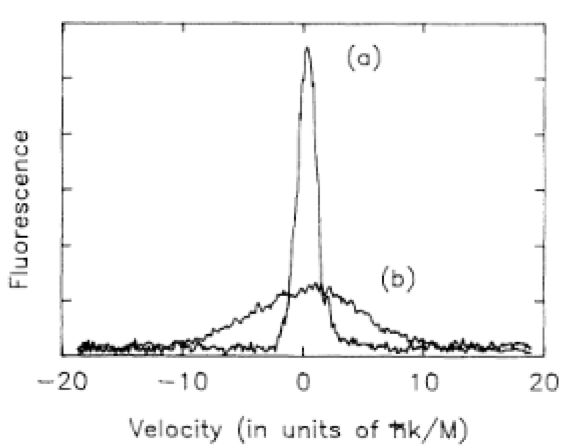
J. Reichel et al 1994 EPL 28 477

Experimental realizations:

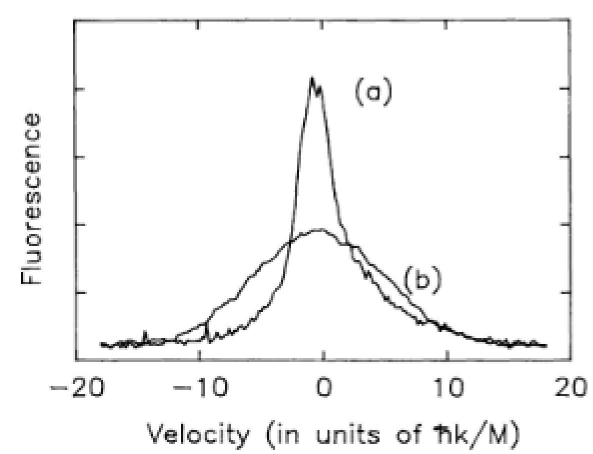
1D: $\sigma_v \sim 0.2 v_r$
Mark Kasevich and Steven Chu, PRL 69, 1741 (1992)

Extension to 2D: $\sigma_v \sim 1.2 v_r$ and 3D: $\sigma_v \sim 2.3 v_r$
Nir Davidson, Heun Jin Lee, Mark Kasevich, and Steven Chu, PRL 72, 3158 (1994)

2D



3D

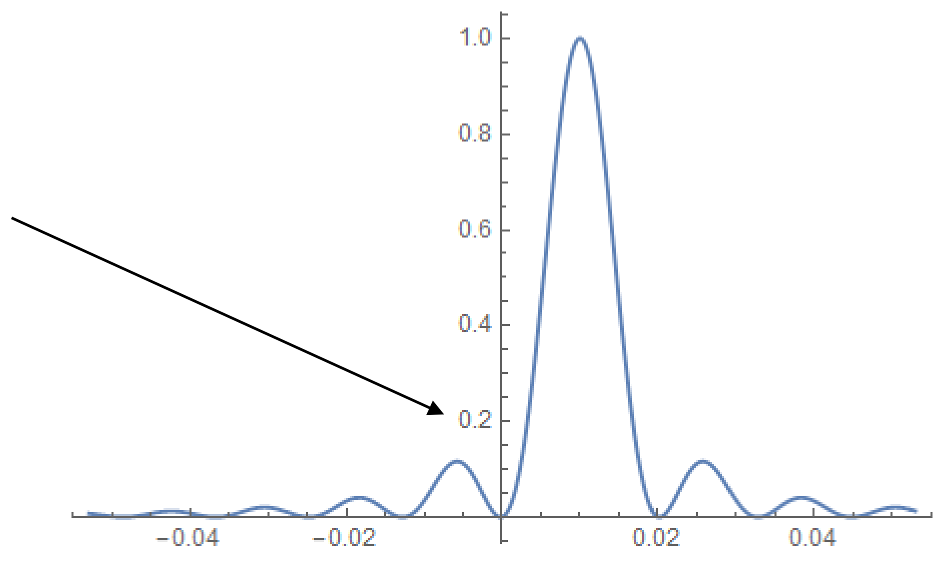


Raman cooling

Are square pulses that bad a choice ?

With a proper choice of parameters (δ, Ω_R), transfer efficiency is null for atoms at rest

$v = 0$ is a dark state



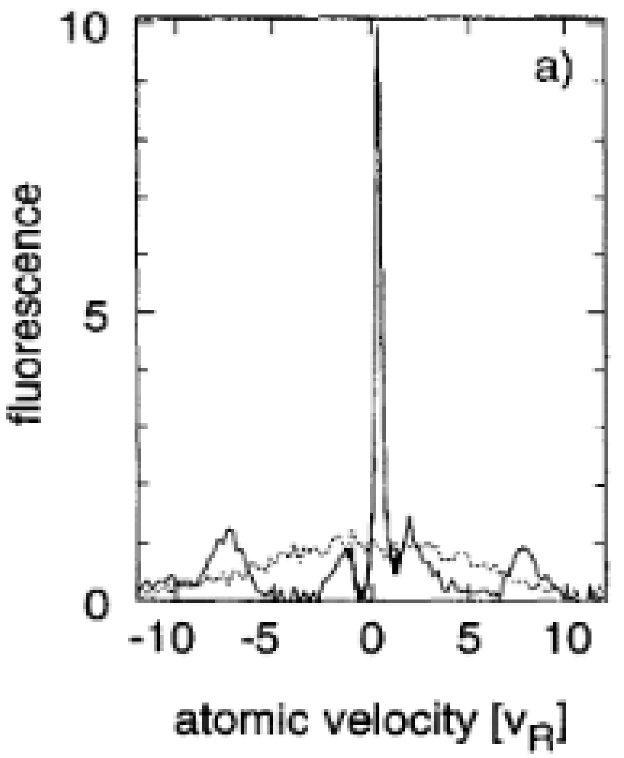
Experimental realizations:

1D: $\Delta v \sim 0.12 v_r$

J. Reichel, F. Bardou, M. Ben Dahan, E. Peik, S. Rand, C. Salomon, and C. Cohen-Tannoudji Raman Cooling of Cesium below 3 nK: New Approach Inspired by Lévy Flight Statistics PRL 75, 4575 (1995)

2D: $\Delta v \sim 0.39 v_r$

V. Boyer, L. J. Lising, S. L. Rolston, and W. D. Phillips PRA 70, 043405 (2004)

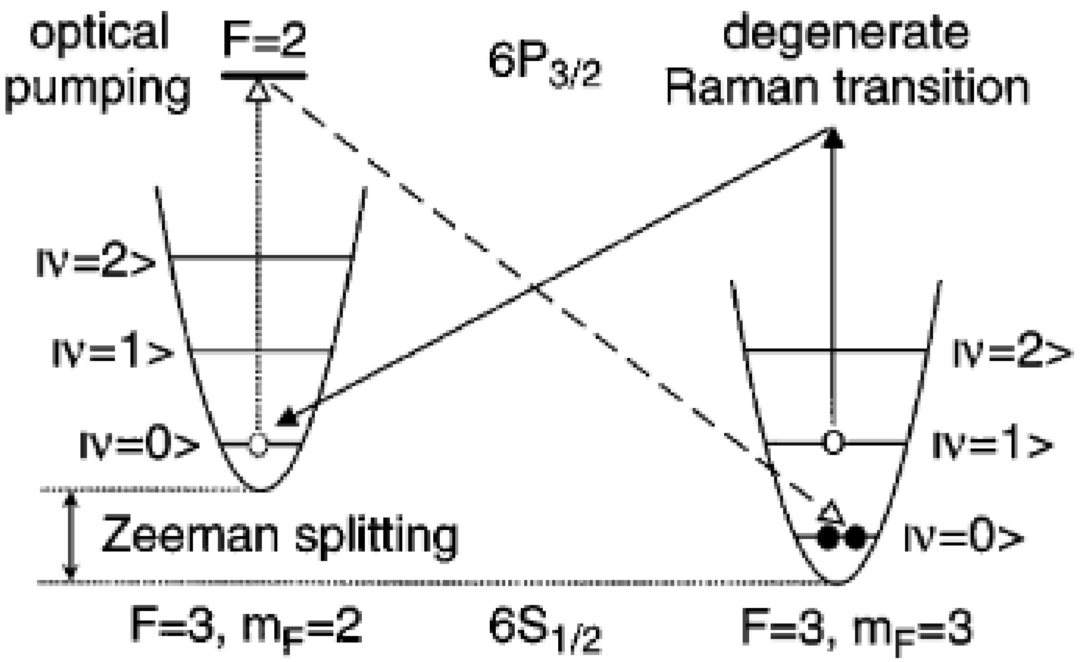


Raman sideband cooling

Extension of Raman cooling for atoms trapped in an optical lattice

Loading cold atoms out of MOT/molasses at temperature T ~ few µK and in a spin polarized state (F=3,m_F=+3 state for Cs atoms for instance) in an optical lattice

Atoms do occupy the different wells
in many vibrational states.
Typical vibrational frequency: 10-100 kHz range



- Apply a magnetic field to lift the degeneracy
- Make the energy levels $|F = 3, mF = 2, \nu\rangle$ and $|F = 3, mF = 3, \nu + 1\rangle$ degenerate

Vuletic et al, PRL 81, 5769 (1998)

Raman sideband cooling

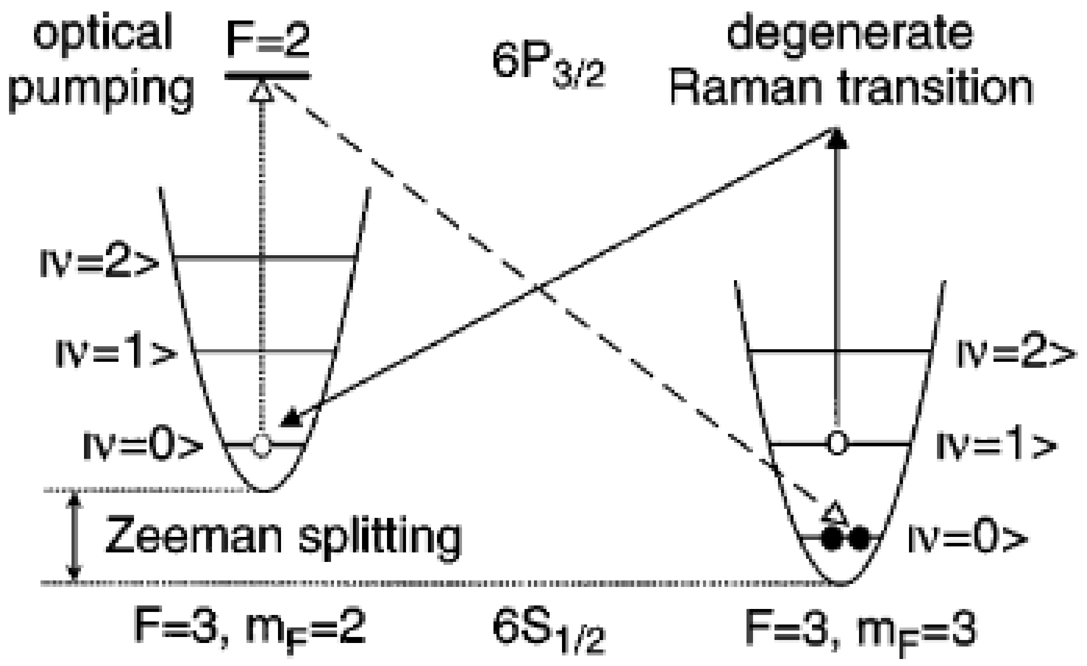
A cooling cycle consists in:

- a Raman pulse with $\pi - \boldsymbol{\sigma}_+$ polarisations

Transfer of the atoms from $|F = 3, mF = 3, \nu\rangle$ to $|F = 3, mF = 2, \nu - 1\rangle$

- a repumping pulse on the $|F = 2\rangle \rightarrow |F' = 2\rangle$

Optical pumping the atoms back into $|F = 3, mF = 3, \nu - 1\rangle$



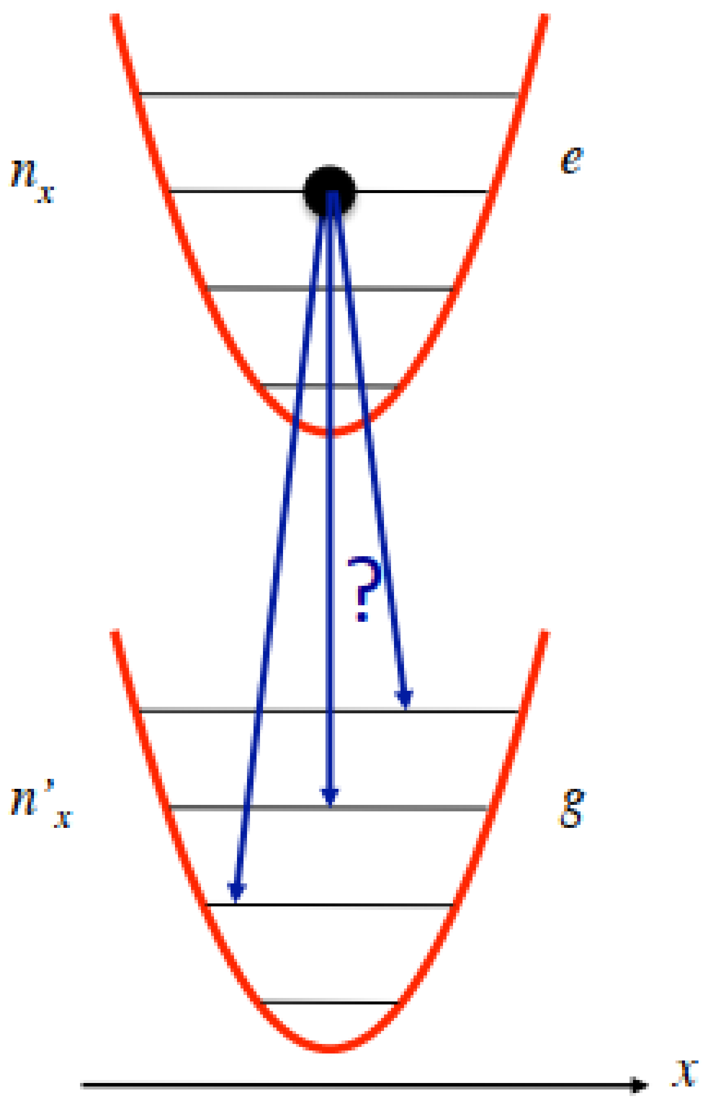
Atoms being tightly bound, the scattered photon do not change the vibrational state (Lamb Dicke regime)

At the end of the cycle, the atoms have lost one quantum of vibration

Raman sideband cooling

Lamb Dicke regime

Probability for the atom in state $|e, n_x\rangle$ to decay in $|g, n'_x\rangle \propto \left|\langle n_x|e^{ik\hat{x}}|n'_x\rangle\right|^2$



Spatial extension of a state $|n_x\rangle \approx \sqrt{n_x}a_0$

With $a_0 = \sqrt{\frac{\hbar}{m\omega}}$, the extension of the ground state

For small n_x, $kr \sim ka_0 \sim \sqrt{\frac{\hbar k^2}{2m\omega}} = \eta$

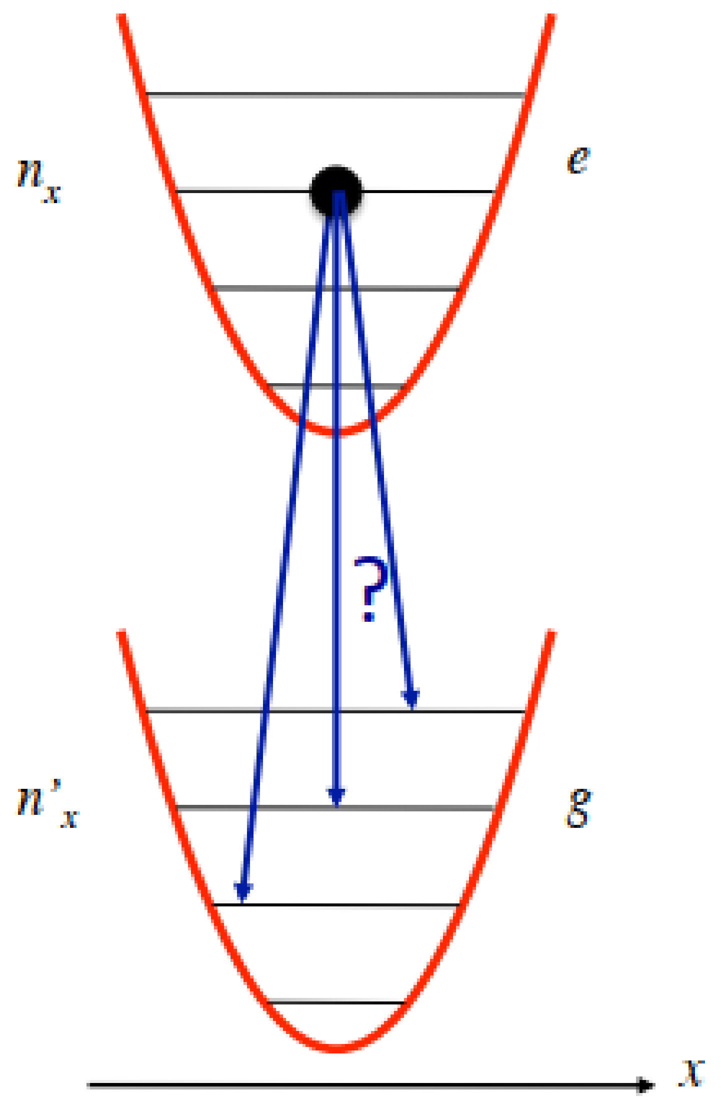
Lamb Dicke parameter
$\eta \ll 1$ for large enough ω

Credit:Jean Dalibard

Raman sideband cooling

Lamb Dicke regime

Probability for the atom in state $|e, n_x\rangle$ to decay in $|g, n'_x\rangle \propto \left|\langle n_x | e^{ik\hat{x}} | n'_x \rangle\right|^2$



For tight confinements,

$kr \ll 1,\ \eta \ll 1,\ e^{ik\hat{x}} = 1 + k\hat{x}$

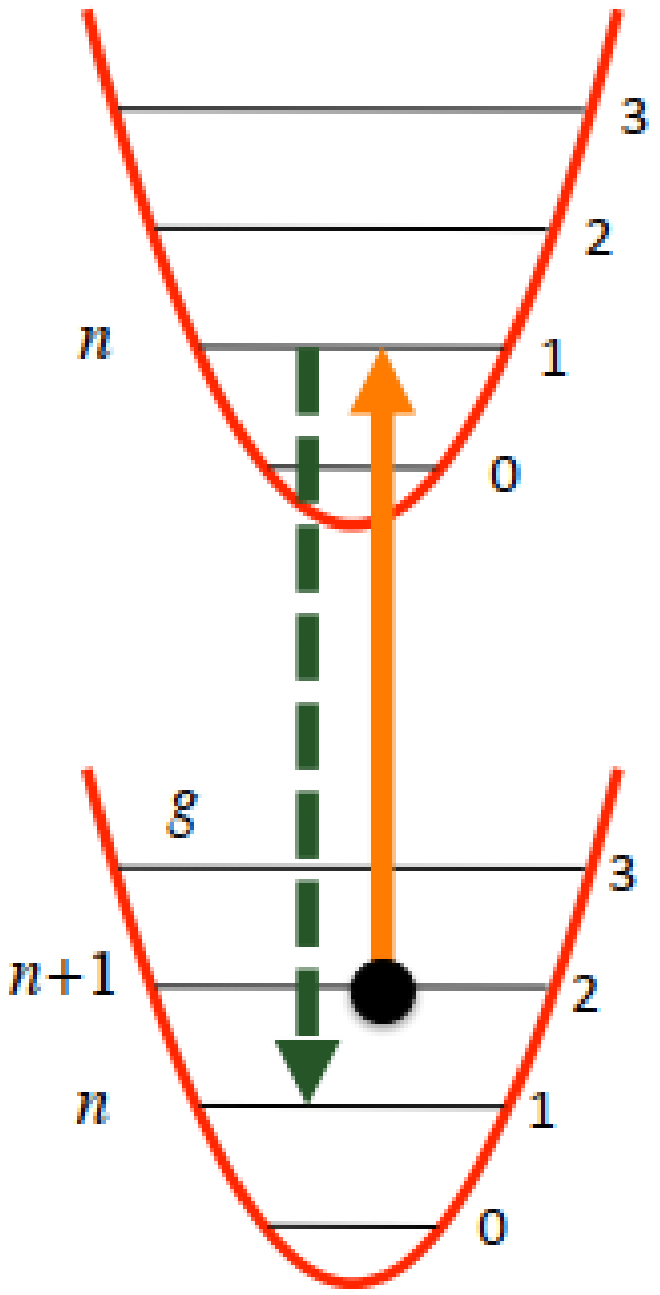
with $\hat{x} = \frac{a_0}{\sqrt{2}}(\hat{a}^+ + \hat{a})$

$$\langle n_x | e^{ik\hat{x}} | n'_x \rangle \approx \delta_{n_x, n'_x} + \frac{k a_0}{\sqrt{2}} (\delta_{n_x, n'_x+1} + \delta_{n_x, n'_x-1})$$

$\sim \eta \ll 1$

Decay predominantly towards $n'_x = n_x$

Raman sideband cooling



Credit:Jean Dalibard

Method employed in ion traps

Laser tuned on the red sideband
$|g, n+1\rangle \rightarrow |e, n\rangle$

When deexciting the ions returns preferentially in $|g, n\rangle$

At each absorption/fluorescence cycle, the ion looses on quantum of energy $\hbar\omega$

Raman sideband cooling

Let us come back to the cooling of neutral atoms

A sequence of such cooling cycles allow to reduce the vibration quantum number ν to 0

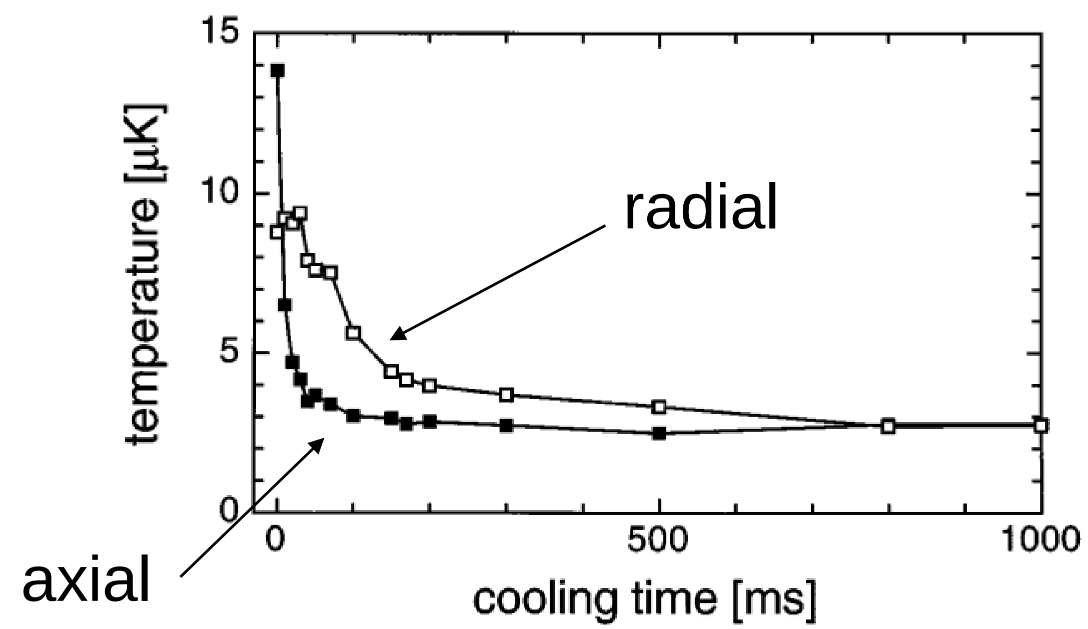
Atoms in the vibrational ground state are in a « dark » state, with respect to both the Raman transitions and the optical pumping

Experimental realizations:

Vuletic et al, PRL 81, 5769 (1998)
1D lattice, Cs atoms

3D (!) temperature: 2.8 µK
80% in the vibrational ground state
(in the steepest direction)

Collisional coupling allows 3D cooling



Notably : phase space density $\sim 1/180$ and density of $1.4\ 10^{13}$ at/cm^3

This is 4-5 orders of magnitude better than with molasses

Raman sideband cooling

Extension to 2D and 3D lattices

S.E. Hamman et al, PRL 80, 4149 (1998)

2D lattice with Cs atoms

- $\overline{n_x} = \overline{n_y} \sim 0.024$
- $> 95\%$ in the vibrational ground state

Kerman et al, PRL 84, 440 (2000)

3D lattice with Cs atoms

- Cooling time 10 ms
- 80% in the vibrational ground state
- 290 nK (after adiabatic release)

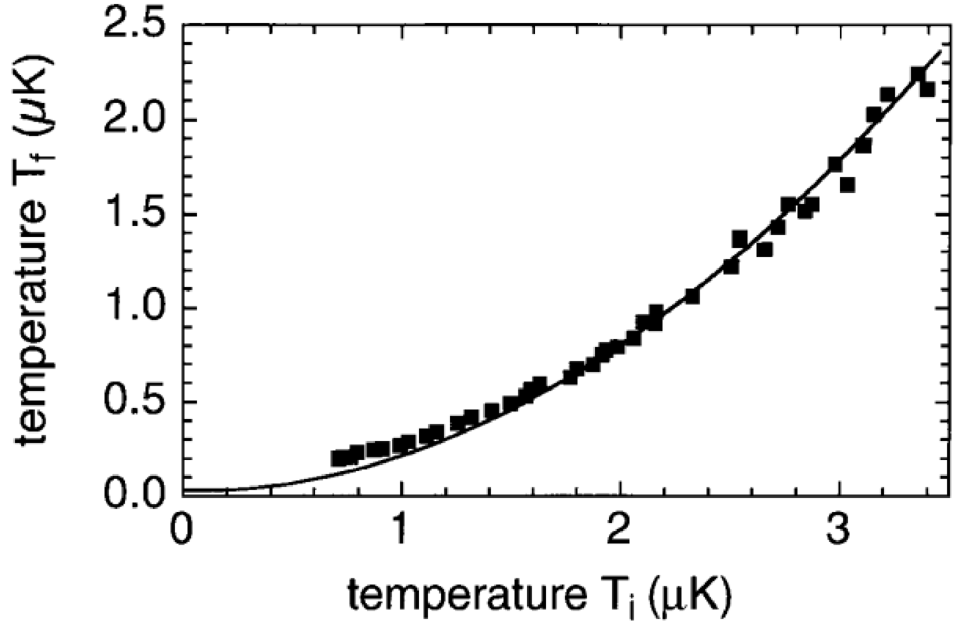
phase space density $\sim \frac{1}{500}$

density of 1.1 10^{11} at/cm^3

Adiabatic cooling

Decrease of the lattice depth over 1ms

$U(t) = U_0(1 + t/t_0)$, with $t_0 = 100$ µs



Jiazhong Hu at al., Science 358, 1078 (2017)

2D lattice Rb87

Cycles of cloud compression + Raman degenerate cooling + far detuned pumping beam

→ BEC reached with 1400 atoms in 300 ms

Organization of the lecture

1 : VSCPT cooling

2 : Raman cooling

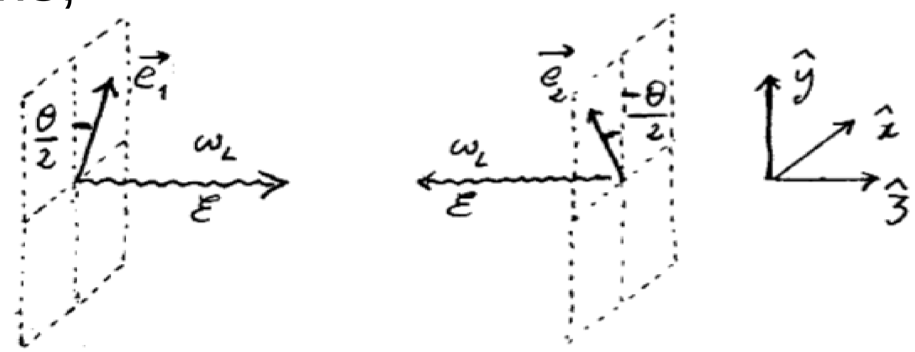
3 : Grey molasses

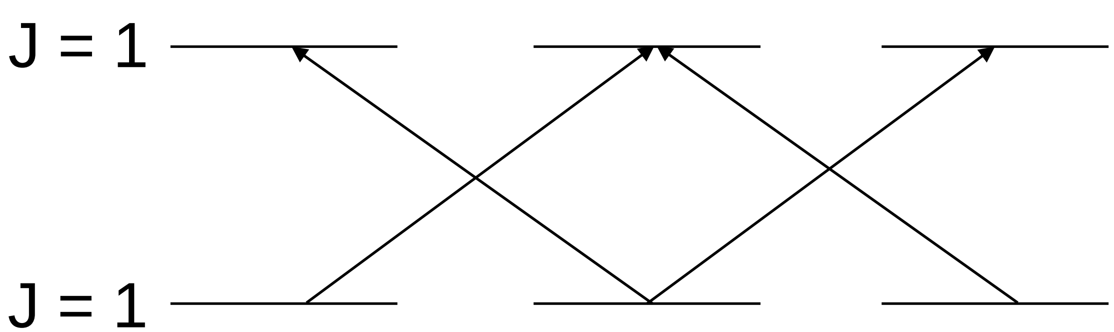
Grey molasses

Idea: combine dark states and Sisyphus-like cooling

Model transition: J=1 → J=1 transition, for which a dark state exists

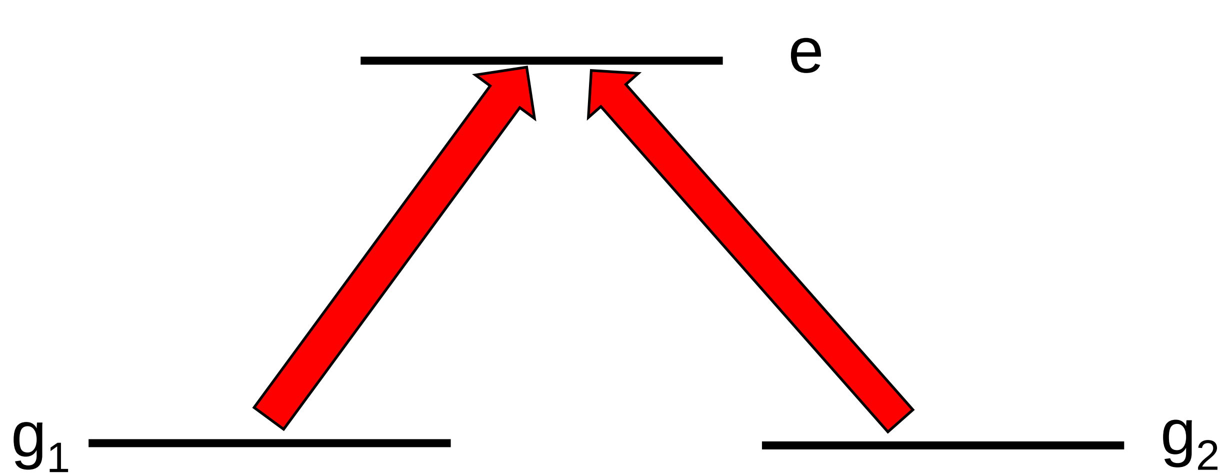
Configuration « lin θ lin »: two counterpropagating laser beams, with linear polarisations, with an angle θ between them





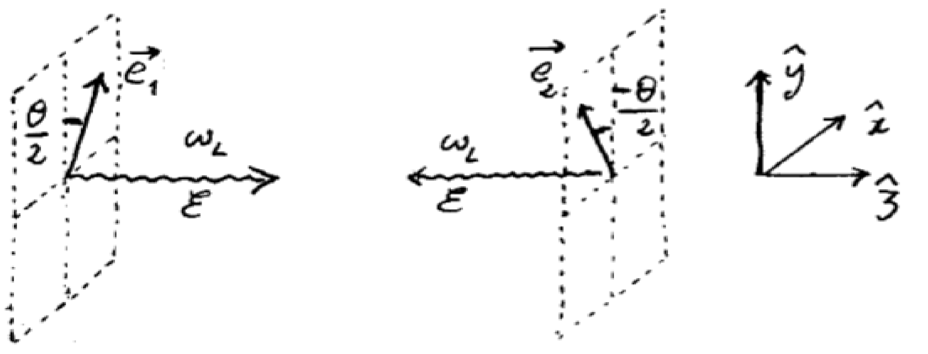
Since the coupling 0-0 is null,
the evolution is restricted after a few cycles
to the states $|e, m = 0\rangle$, $|g, m = -1\rangle$, $|g, m = +1\rangle$

Lambda system
$|g_1\rangle = |g, m = -1\rangle$
$|g_2\rangle = |g, m = +1\rangle$
$|e\rangle = |e, m = 0\rangle$



Grey molasses

The total electric field is $E(z,t) = \mathcal{E}^{+}(z)\exp(-i\omega t) + c.c.$



with $\mathcal{E}^{+}(z) = -\mathrm{i}\frac{\mathcal{E}}{2}(\epsilon_1 \exp(ikz) + \epsilon_2 \exp(-ikz))$
and $\epsilon_1 = \epsilon_x \sin\frac{\theta}{2} + \epsilon_y \cos\frac{\theta}{2}$ and $\epsilon_2 = -\,\epsilon_x \sin\frac{\theta}{2} + \epsilon_y \cos\frac{\theta}{2}$

This also writes

$$\mathcal{E}^{+}(z) = \frac{\mathcal{E}}{\sqrt{2}}\left(\epsilon_{+}\cos\left(kz + \frac{\theta}{2}\right) + \epsilon_{-}\cos\left(kz - \frac{\theta}{2}\right)\right) \quad \text{with} \begin{cases} \epsilon_{+} = -\frac{1}{\sqrt{2}}(\epsilon_x + i\epsilon_y) \\ \epsilon_{-} = \frac{1}{\sqrt{2}}(\epsilon_x - i\epsilon_y) \end{cases}$$

The polarisation is equivalent to two phase-shited $\boldsymbol{\sigma}_{+}$ and $\boldsymbol{\sigma}_{-}$ standing waves

Since the two $\boldsymbol{\sigma}_{+}$ and $\boldsymbol{\sigma}_{-}$ fields are orthogonal, the intensity is given by

$$I(z) = \frac{\mathcal{E}^2}{2}\left(\cos\left(kz - \frac{\theta}{2}\right)^2 + \cos\left(kz - \frac{\theta}{2}\right)^2\right) = \frac{\mathcal{E}^2}{2}(1 + \cos(2kz)\cos\theta) = \frac{\mathcal{E}^2}{2}D(z)$$

The intensity is modulated with a spatial period of $\frac{\lambda}{2}$

Grey molasses

The laser coupling now writes as

$$V = \frac{\hbar\Omega_1}{2}\left(-\cos\left(kz + \frac{\theta}{2}\right)|e\rangle\langle g_1| + |e\rangle\langle g_2|\cos\left(kz - \frac{\theta}{2}\right)\right)e^{-i\omega_L t} + h.c.$$

We now introduce the states $|\Psi_{NC}(z)\rangle$ and $|\Psi_{NC}(z)\rangle$

$$|\Psi_{NC}(z)\rangle = \frac{1}{\sqrt{D(z)}}\left(\cos\left(kz - \frac{\theta}{2}\right)|g_1\rangle - \cos\left(kz + \frac{\theta}{2}\right)|g_2\rangle\right)$$

$$|\Psi_{C}(z)\rangle = \frac{1}{\sqrt{D(z)}}\left(\cos\left(kz + \frac{\theta}{2}\right)|g_1\rangle + \cos\left(kz - \frac{\theta}{2}\right)|g_2\rangle\right)$$

The laser coupling is then given by $V = \frac{\hbar\Omega(z)}{2}|e\rangle\langle\Psi_C(z)|e^{-i\omega_L t} + h.c.$

with $\Omega(z) = \Omega_1 D(z)$

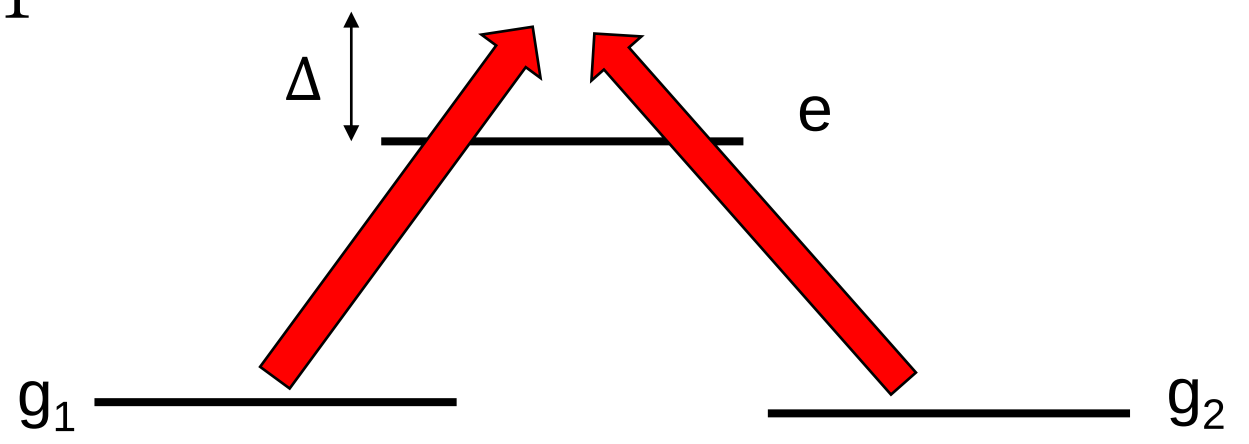
$|\Psi_C(z)\rangle$ is coupled to the light field, $|\Psi_{NC}(z)\rangle$ is not

Grey molasses

Let us now consider the weak excitation regime,

$$s = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} \ll 1$$

and a detuning $\Delta > 0$



$|\Psi_C(z)\rangle$ being coupled to the light acquires

- a light shift $\hbar\Delta' = \hbar\frac{\Delta}{2}s = \hbar\frac{\Delta}{4}\frac{\Omega(z)^2}{\Delta^2+\frac{\Gamma^2}{4}} = \delta' D(z)$ with $\delta' = \frac{\Delta}{4}\frac{{\Omega_1}^2}{\Delta^2+\frac{\Gamma^2}{4}}$ independent of z

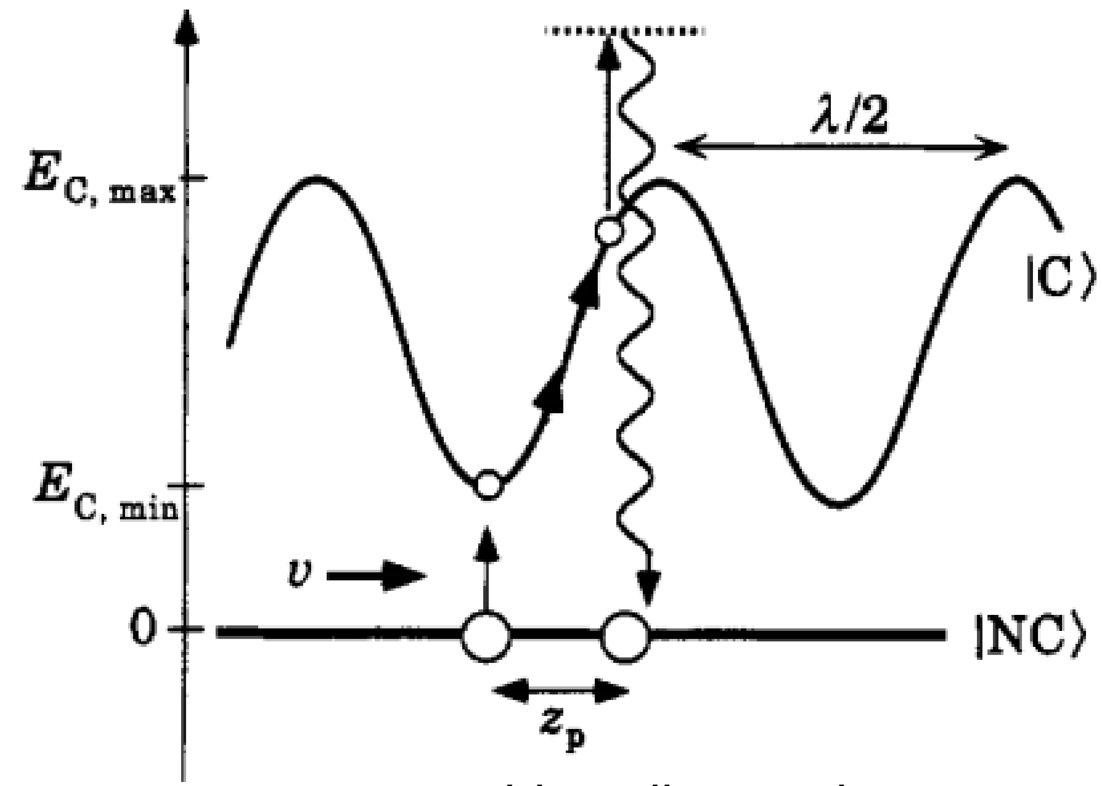
- a radiative decay rate $\Gamma' = \frac{\Gamma}{2}s = \frac{\Gamma}{2}\frac{\Omega(z)^2}{\Delta^2+\frac{\Gamma^2}{4}} = \gamma' D(z)$ with $\gamma' = \frac{\Gamma}{4}\frac{{\Omega_1}^2}{\Delta^2+\frac{\Gamma^2}{4}}$ independent of z

$|\Psi_{NC}(z)\rangle$ being not coupled to the light does not get shifted and does not decay

Grey molasses

Since $\Delta > 0$, $|\Psi_C(z)\rangle$ lies above $|\Psi_{NC}(z)\rangle$

Its light shift, being proportional to $D(z)$, is modulated with a spatial period of $\frac{\lambda}{2}$



Weidemüller, at al.
Europhysics Letters 27 (2), 109–114 (1994)

Cooling mechanism

Consider an atom
initially in $|\Psi_{NC}(z_0)\rangle$ at initial position z_0
moving with a velocity v

At a later time and position z_1, $|\Psi_{NC}(z_0)\rangle$ is no longer a dark state, since at the position z_1, it differs from the dark state $|\Psi_{NC}(z_1)\rangle$

$|\Psi_{NC}(z_0)\rangle$ is thus contaminated by $|\Psi_C(z_1)\rangle$.
It thus acquires a radiative lifetime, and can scatter photons.

In the above picture, the atom is transferred to $|\Psi_C\rangle$,
then climbs a hill, scatters a photon and returns in $|\Psi_{NC}\rangle$

→ Sisyphus-like cooling

Grey molasses

Restricting ourselves to the ground states,
we can write the state $|\Psi\rangle$ in the $\{|\Psi_C(z)\rangle, |\Psi_{NC}(z)\rangle\}$ basis

$$|\Psi\rangle = a(t)|\Psi_{NC}(z)\rangle + b(t)|\Psi_C(z)\rangle$$

with $z = vt$ for an atom moving a velocity v

At $t = 0$, the atom is at the position z_0 in the state $|\Psi_{NC}(z_0)\rangle$, so that $a(0) = 1$ and $b(0) = 0$

The evolution of $|\Psi\rangle$ follows the Schrodinger equation $i\hbar \frac{d}{dt}|\Psi\rangle = \mathrm{H}|\Psi\rangle = (H_{eff} + V)|\Psi\rangle$,
with $V = \frac{\hbar\Omega(z)}{2}|e\rangle\langle\Psi_C(z)|e^{-i\omega_L t} + h.c.$ and $H_{eff} = \hbar\begin{pmatrix} \Delta' - i\frac{\Gamma'}{2} & 0 \\ 0 & 0 \end{pmatrix}$

Since $\frac{d}{dt}|\Psi\rangle = \frac{d}{dt}\left(a(t)|\Psi_{NC}(z)\rangle + b(t)|\Psi_C(z)\rangle\right)$

$$= \dot{a}(t)|\Psi_{NC}(z)\rangle + a(t)v\frac{d}{dz}|\Psi_{NC}(z)\rangle + \dot{b}(t)|\Psi_C(z)\rangle + b(t)v\frac{d}{dz}|\Psi_C(z)\rangle$$

And projecting over, $\langle\Psi_C(z)|$, we get

$$i\dot{b}(t) = b(t)(\Delta' - i\frac{\Gamma'}{2}) - ia(t)v\langle\Psi_C(z)|\frac{d}{dz}|\Psi_{NC}(z)\rangle$$

Grey molasses

Using the expression of $|\Psi_{NC}(z)\rangle$ and $|\Psi_C(z)\rangle$, we find $\langle\Psi_C(z)|\frac{d}{dz}|\Psi_{NC}(z)\rangle = -\frac{k\ \sin\theta}{D(z)}$

So that $i\dot{b}(t) = b(t)\left(\Delta' - i\frac{\Gamma'}{2}\right) + ia(t)v\frac{k\ \sin\theta}{D(z)}$

For slow velocities, and at steady state is, $a \approx 1$ and $b \approx \frac{-i}{\left(\Delta' - i\frac{\Gamma'}{2}\right)}\frac{kv\ \sin\theta}{D(z)}$

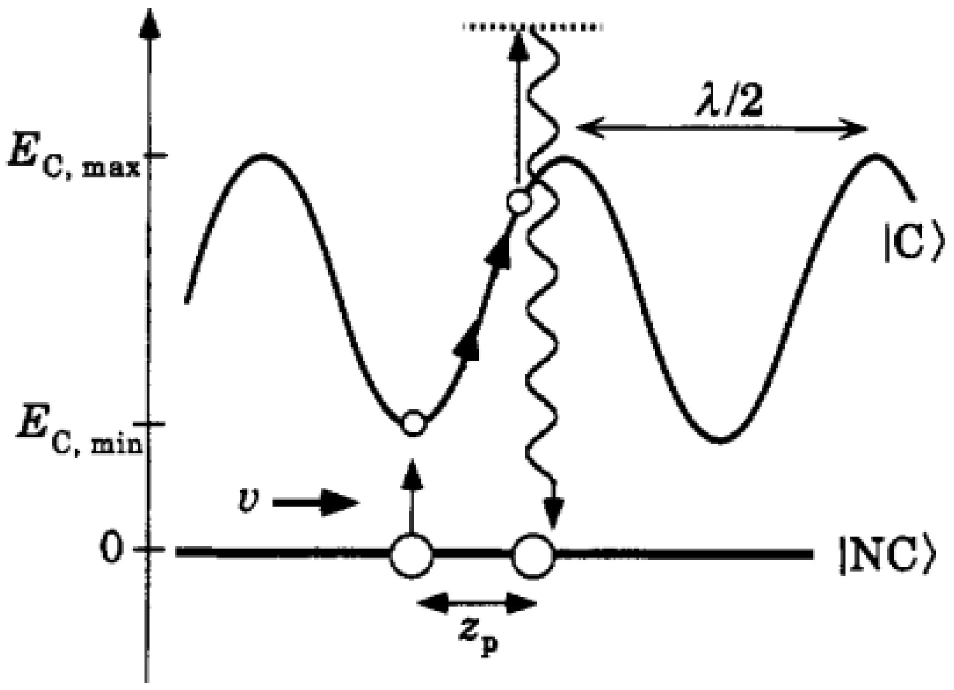
$$|\Psi(t)\rangle = |\Psi_{NC}(z)\rangle + \frac{-i}{\left(\Delta' - i\frac{\Gamma'}{2}\right)}\frac{kv\ \sin\theta}{D(z)}|\Psi_C(z)\rangle$$

The coupling to $|\Psi_C(z)\rangle$ is a « motional » coupling, related to the time evolution of the wavefunctions of $|\Psi_{NC}\rangle$ and $|\Psi_C\rangle$

It is maximal when $D(z)$ is minimal,
which corresponds to the minimum of the potential

Atoms are thus preferentially transfered to $|\Psi_C(z)\rangle$ at the bottom of the hill

But they absorb photons preferentially at the top,
since this is where the intensity is maximal.



Grey molasses

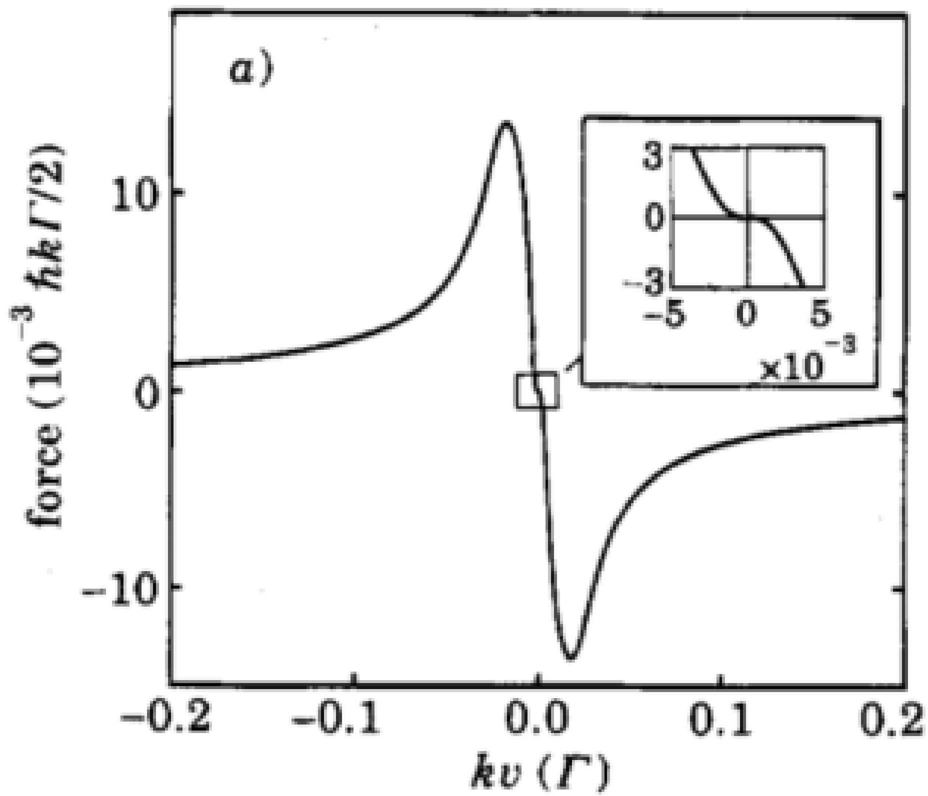
The force is given by

$$F = -128\,\hbar k\,\frac{\Delta}{\Gamma}\left(\Delta^2 + \frac{5}{4}\Gamma^2\right)\frac{(kv)^3}{{\Omega_1}^4}A(\theta)$$

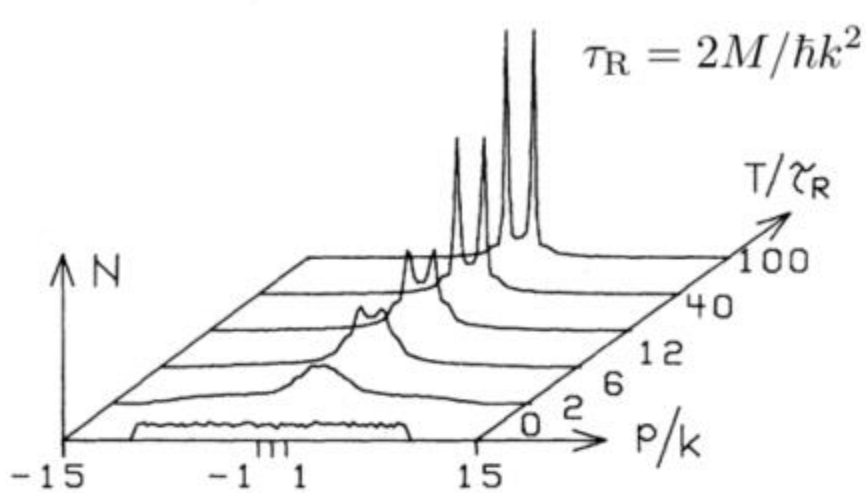
With $A(\theta) = \frac{8}{5\pi}\sin^2\theta\cos\theta\left|\int_0^{2\pi} du\,\frac{\cos u}{(1+\cos\theta\,\cos u)^5}\right|$

It scales as v^3, and depends on the angle θ

For $\theta = \pi/2$, it vanishes since the intensity is constant and this is no light shift modulation



Time evolution of the momentum distribution calculated with quantized momenta
Shahriar et al., PRA 48, R4035 (1993)



At short times, Sisyphus cooling
At long times, accumulation in the dark state

Grey molasses

(Early) experimental demonstrations

C. Valentin et al EPL 17 133 (1992)
Observation of sub-Doppler cooling on a J → J-1 transition, with a blue detuning

D. Boiron, C. Triché, D. R. Meacher, P. Verkerk, and G. Grynberg
Phys. Rev. A 52, R3425(R) (1995)
3D cooling in a 4 beam molasses tuned on a $F_g=3 \rightarrow F_e=2$ transition of Cs
T < 5 µK in 1 ms

D. Boiron et al. Phys. Rev. A 53, R3734(R) (1996)
6 beam molasses on the blue side of the $F_g=3 \rightarrow F_e=2$ transition of Cs
Minimum temperature 1.1 µK

Sides effects:
Temperature deviates from theory for large detunings
This heating is due to parasitic excitation to $F_g=3 \rightarrow F_e=3$
Temperature depends on density ~ 0.6µK / 10^{10}at/cm^3
This effect is attributed to photon multiple scattering

Grey molasses

More recent experimental demonstrations

Grey molasses have gained a renewed interest in recent years for cooling K, Li but also Rb

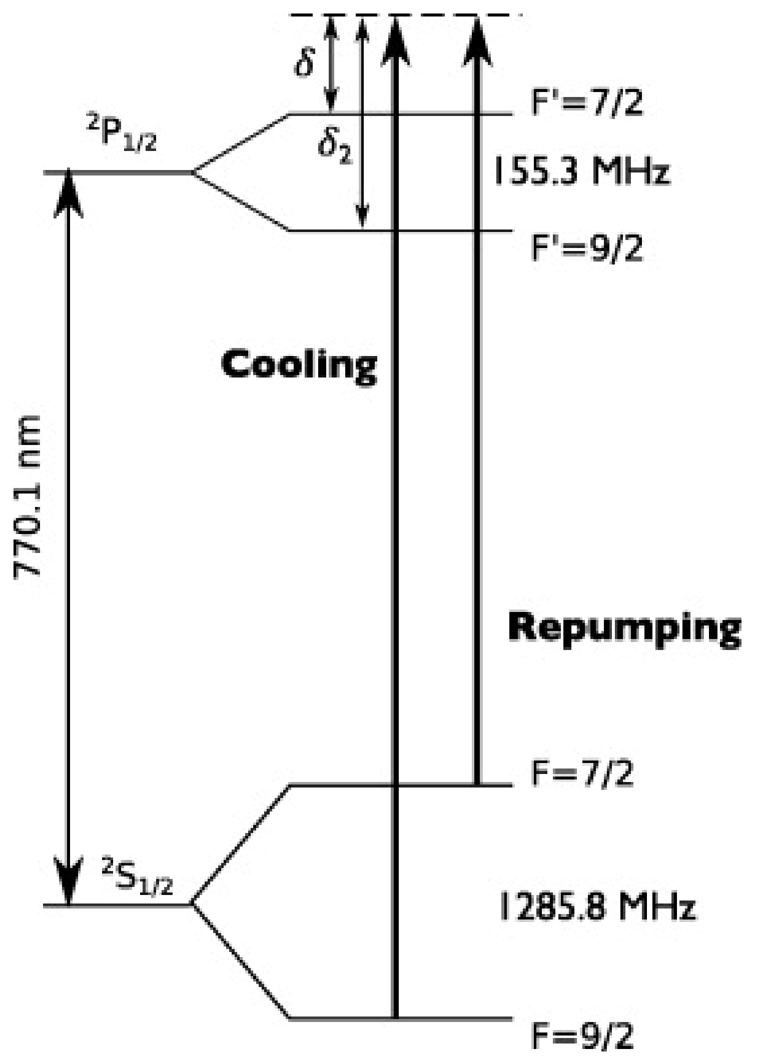
Difficulty with K and Li:

Small (of order of a few Γ), and eventually inverted, hyperfine structure
$\rightarrow$ compromizes the efficiency of red detuned sub-Doppler cooling (especially when increasing the detuning)

Implementation of grey molasses on the D1 line (instead of the D2):

- less hyperfine states,
- hyperfine states better separated

D. Rio Fernandes, EPL 100 63001 (2012)
6.5 10^8 atoms of 40K cooled down to 20 µK in 8 ms

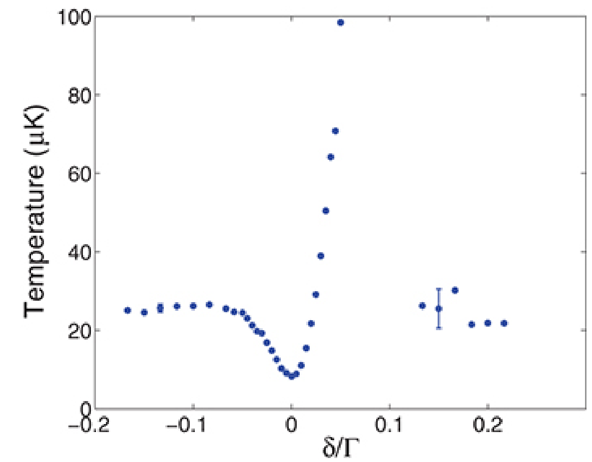


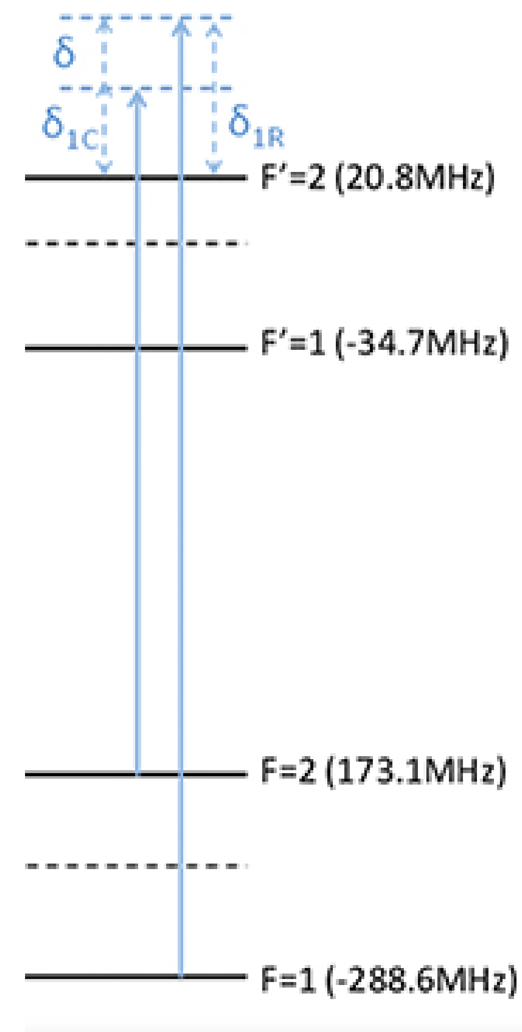
Grey molasses

Same scheme applied to:

- 7Li (*A.T. Grier at al., PRA 87, 063411 (2013)*)
- 39K (*G. Salomon et al, EPL* ***104*** *63002 2013*)

Temperature as low as 6 µK

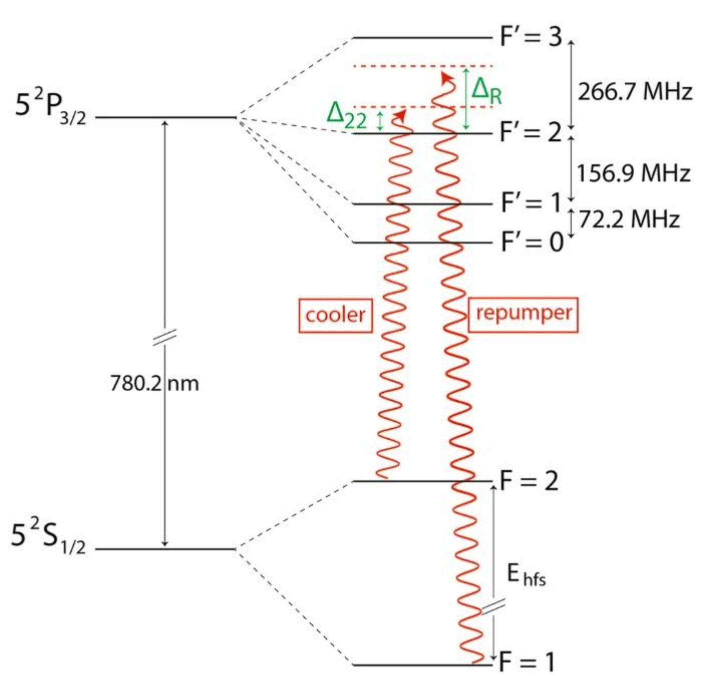


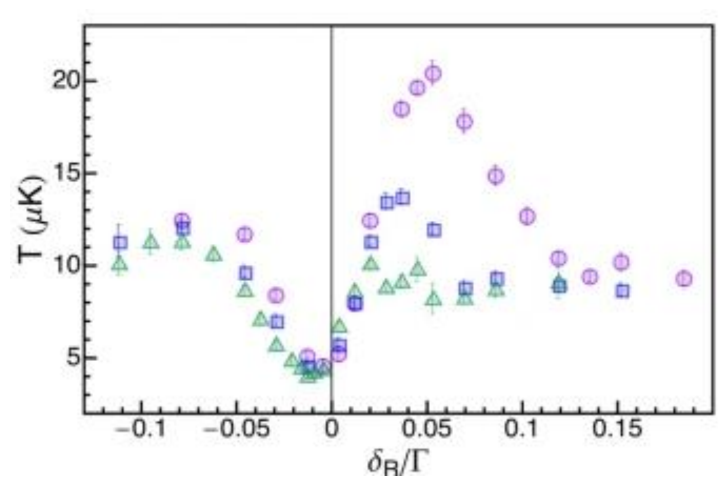


Narrow minimum when exactly
at the Raman resonance condition

But with 87Rb, it works fine on the D2 line as well

Rosi et al.. Sci Rep 8, 1301 (2018)





$T = (4.0 \pm 0.3)\,\mu K$

But a gain of ~10 in PSD with respect to ordinary laser sub-Doppler cooling