BOSE EINSTEIN CONDENSATION

Quantum Fluids of Light

from superfluid light to Mott insulators and fractional quantum Hall liquids

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PROVINCIA AUTONOMA DI TRENTO

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European Commission

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PhoQuS Photons for Quantum Simulation

MIR-BOSE

Why not hydrodynamics of light ?

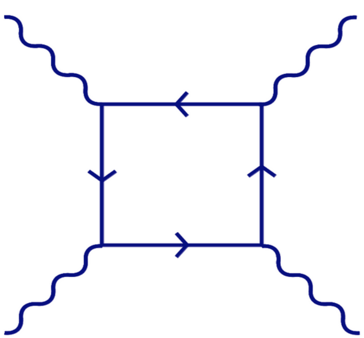
Light field/beam composed by a huge number of photons

- *in vacuo* photons travel along straight line at c
- (practically) do not interact with each other
- in standard cavity, thermalization via walls and absorption/emission

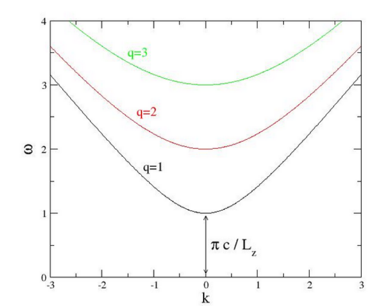
optics in vacuo typically dominated by single-particle physics

In suitable photonic structures:

- spatial confinement → effective photon mass
- $\chi^{(3)}$ nonlinearity → photon-photon interactions



Collective behaviour of *quantum fluid of light*



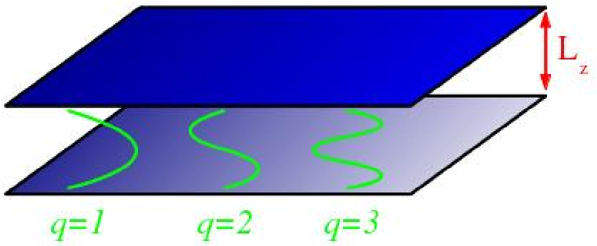
IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

What about mass?

In vacuo: photons massless, dispersion $\omega = c\,|k|$

In planar cavity → confinement along *z*, free propagation along *x,y*

Quantization along *z*: $k_z^{(q)} = q\,\pi / L_z$

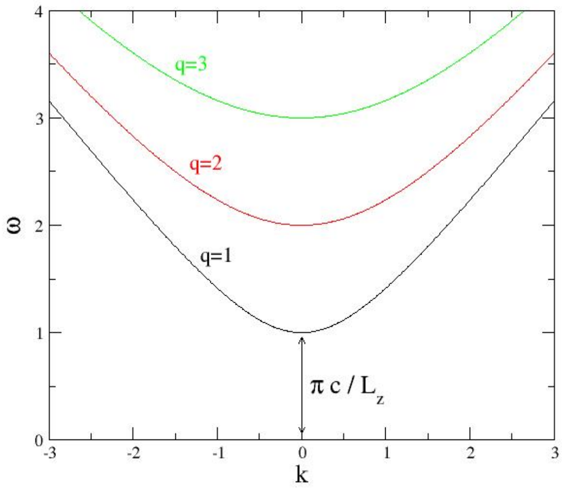


Massive dispersion along *x,y*:

$$\omega^{(q)}(\mathbf{k}_\parallel) = c\sqrt{[k_z^{(q)}]^2 + \mathbf{k}_\parallel^2} = c\sqrt{\left(\frac{q\pi}{L_z}\right)^2 + \mathbf{k}_\parallel^2} \simeq ck_z^{(q)} + \frac{c}{2k_z^{(q)}}\mathbf{k}_\parallel^2$$

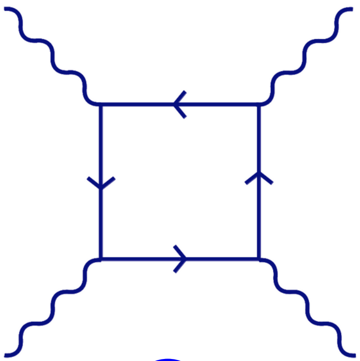
Confinement gives effective photon mass $m_{ph}c^2 = \hbar c k_z^o$

- Rest mass → cut-off in the dispersion
- Inertial mass → curvature of dispersion



IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

What about interactions?



Photon-photon interactions exist in QED:

Heisenberg-Euler processes via electron-positron exchange

… but cross section ridiculously small for visible light

(recent experiment in accelerator → Nat. Phys. 2017)

$$\sigma \sim \alpha^4 \left(\frac{\hbar^2}{m^2 c^2}\right)\left(\frac{\hbar \omega}{mc^2}\right)^6$$

Compton λ → pm range

How to enhance it ?

Replace electron-positron pair (E~1MeV) with
electron-hole pair (E~1eV) → gain factor $(10^6)^6 = 10^{36}$!!

In optical language:

- $\chi^{(3)}$ nonlinearity ↔ local photon-photon interactions
- typical material → spatially local (or quasi-local) $\chi^{(3)}$

Modern exceptional media:

- Rydberg atoms
  - Ultra-large, long-range nonlinearity in Rydberg-EIT config.
- Superconducting circuits
  - Strong coupling to macroscopic oscillation mode of superconductor device

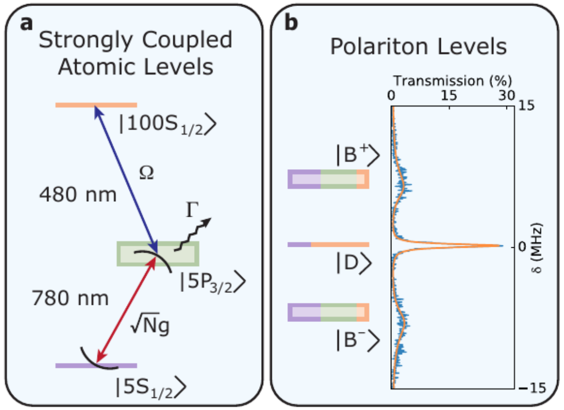
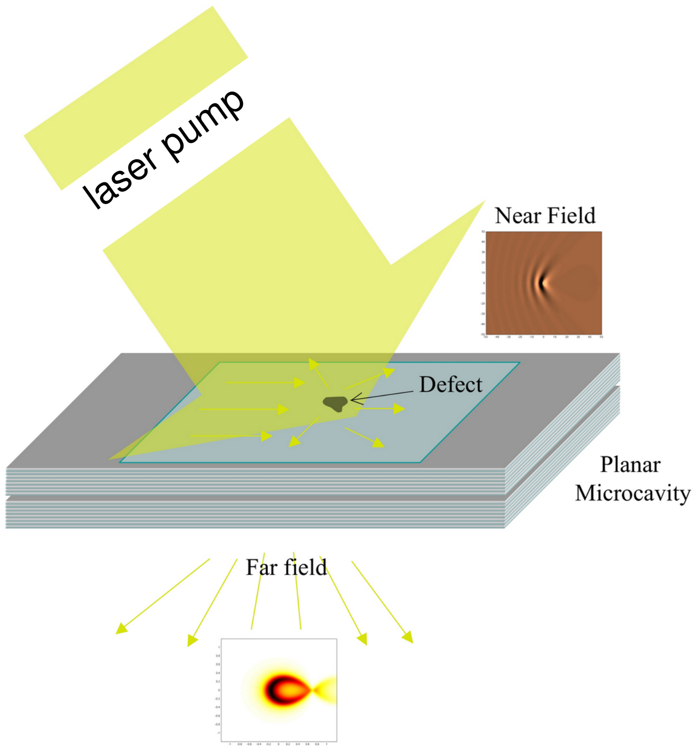


Figure: J. Simon's group @ U. Chicago

How to create and detect the photon gas?



Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

- Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
  spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation

IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

Part 2:

Weakly interacting fluids of light

Superfluid light &

Non-Equilibrium Statistical Mechanics

Mean-field theory: generalized GPE

$$i\frac{d\psi}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|\psi|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|\psi|^2}-\gamma\right)\right\}\psi+F_{ext}$$

Time-evolution of macroscopic wavefunction ψ of photon/polariton condensate

- standard terms: kinetic energy, external potential V_{ext}, interactions g, losses γ
- under coherent pump: forcing term
- under incoherent pump: polariton-polariton scattering from thermal component give saturable amplification term as in semiclassical theory of laser

→ a sort of Complex Landau-Ginzburg equation

To go beyond mean-field theory:

- Exact diagonalization, Wigner representation, Keldysh diagrams, ...

IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

2006 - Photon Bose-Einstein condensation

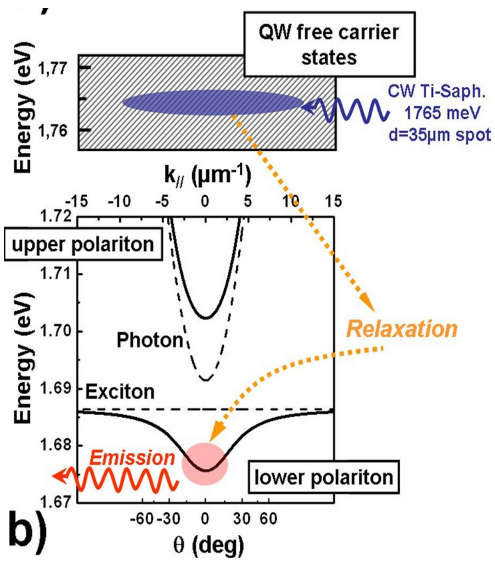
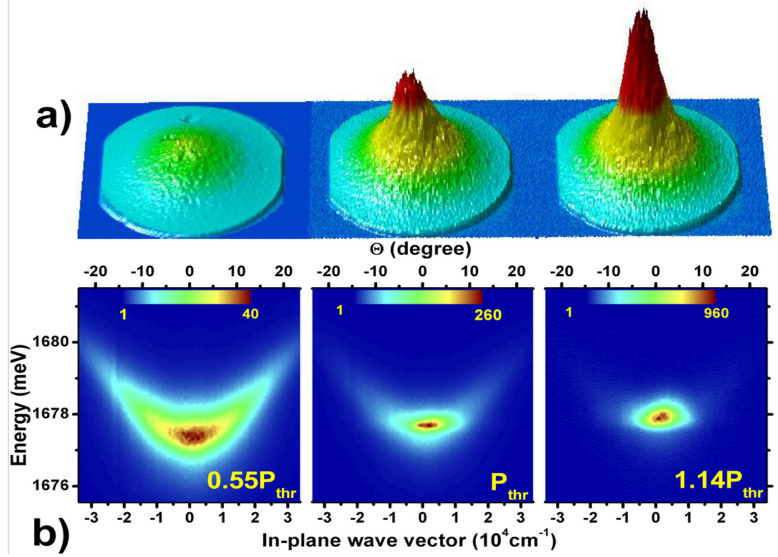
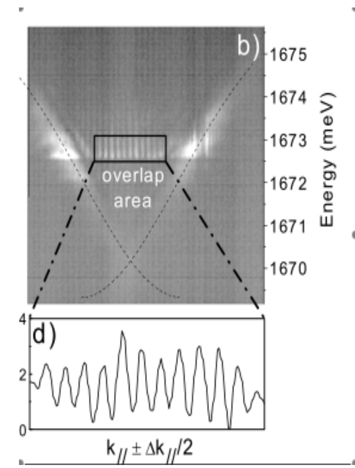


Figure from Kasprzak et al., Nature 2006



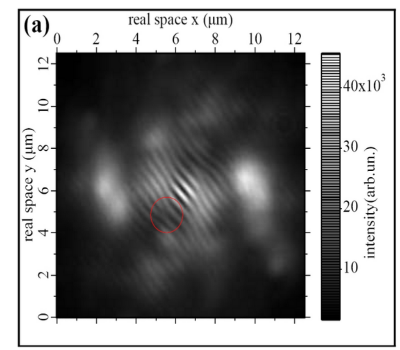
Momentum distribution

Kasprzak et al., Nature **443**, 409 (2006)



Interference

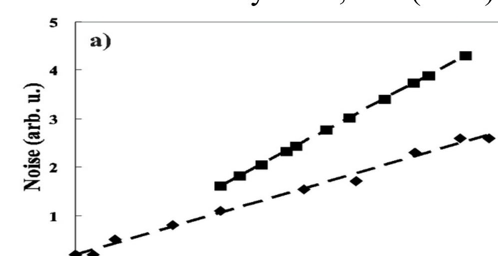
Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

K. Lagoudakis *et al.*

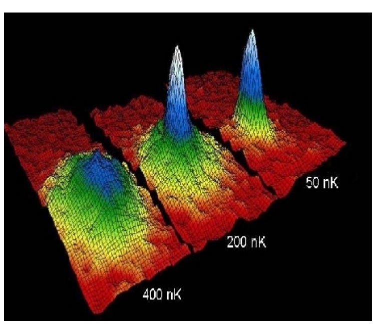
Nature Physics 4, 706 (2008).



Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

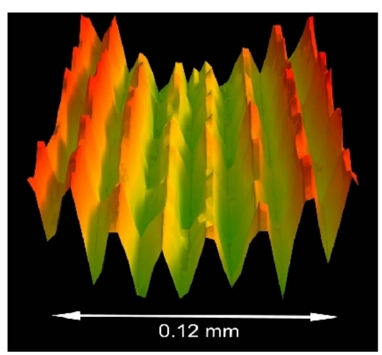
Many features very similar to atomic BEC



The first atomic BEC

M. H. Anderson et al.

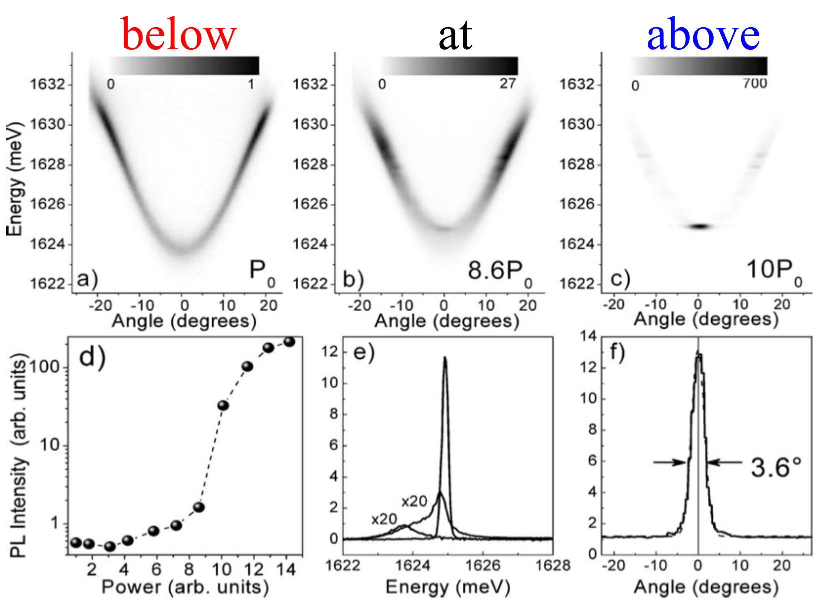
Science **269**, 198 (1995)



Interference pattern of two expanding atomic BECs

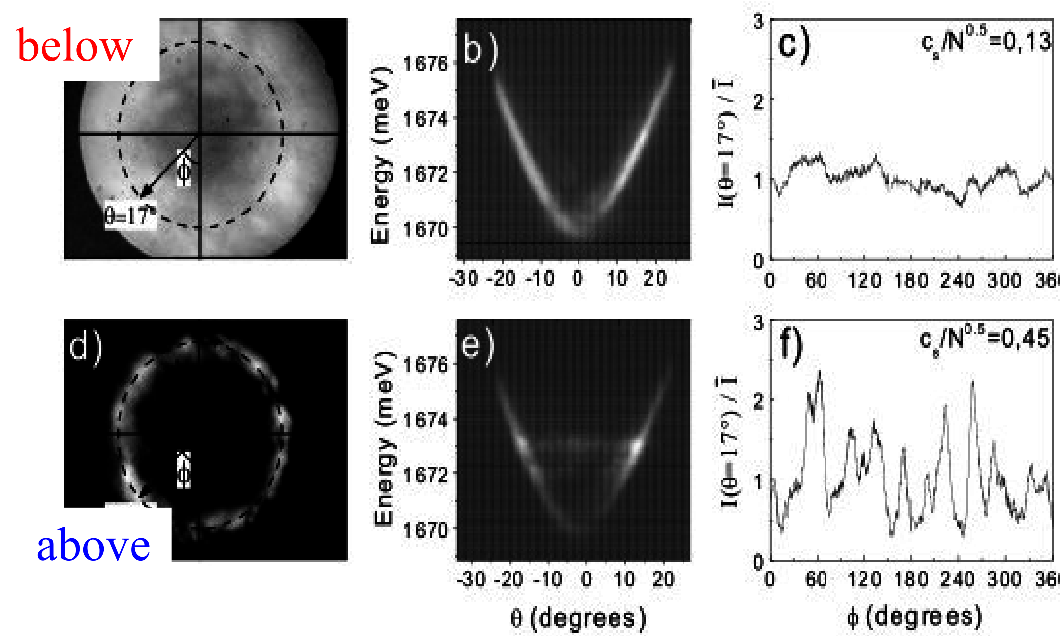
M. R. Andrews, Science **275**, 637 (1995)

A crucial difference: the shape of a non-equilibrium BEC



wide pump spot: 20 μm

M. Richard et al., PRB **72**, 201301 (2005)



narrow pump spot: 3 μm

M. Richard et al., PRL **94**, 187401 (2005)

Experimental observations under non-resonant pump:

- condensate shape depends on pump spot size
  - wide pump spot: condensation at k=0
  - narrow pump spot: condensation on a ring of modes at finite |k|

Analogous experiments in J. Bloch's group @ LPN in 1D geometry: Wertz et al. Nat. Phys 2008

Physical interpretation of condensation at k≠0

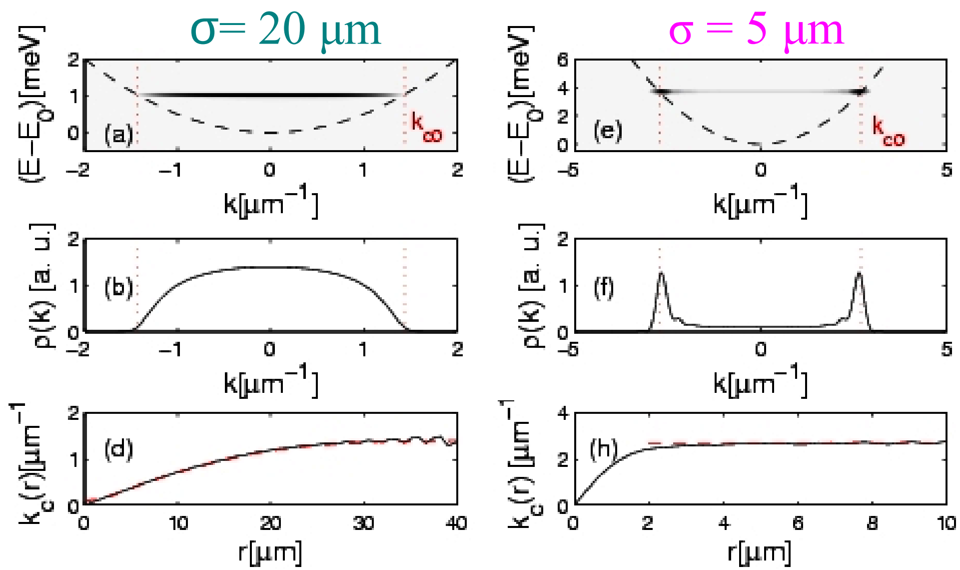
Repulsive interactions

- outward radial acceleration
- energy conservation

$$E = k^2/2m + U_{int}(r)$$

→ radially increasing flow velocity

→ coherent ballistic flow

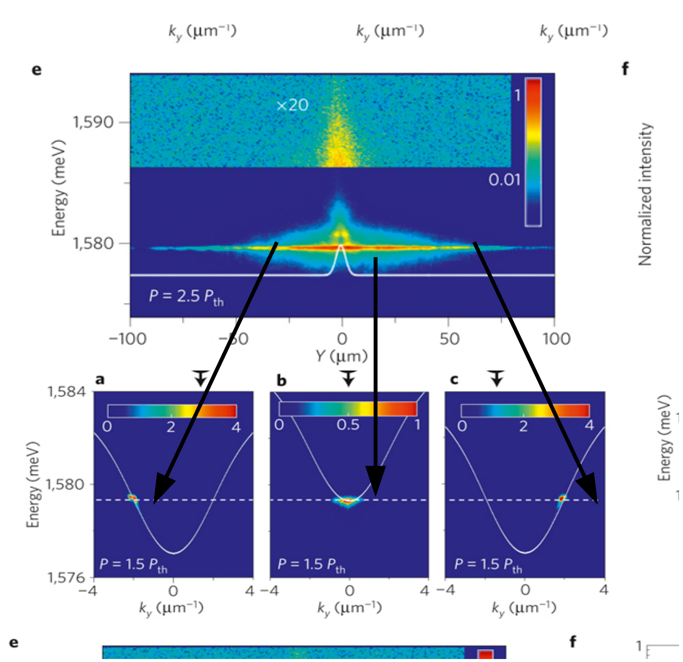


M. Wouters, IC, and C. Ciuti, PRB **77**, 115340 (2008)

Narrow spot:

- ballistic free flight outside pump spot $U_{int}(r)=0$
- emission mostly on free particle dispersion

Later expts confirm mechanism →



Wertz et al., Nat. Phys. 6, 860 (2010)

T-reversal breaking:

- allowed by non-equilibrium
- allows for non-zero current
- also visible as n(k)≠n(-k)

Quasi-condensation features

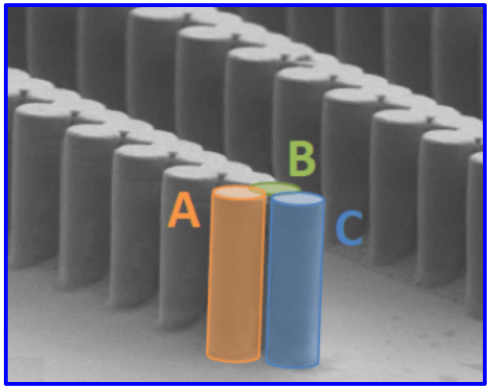
Hohenberg-Mermin-Wagner theorem:

- at equilibrium, no BEC in d<3

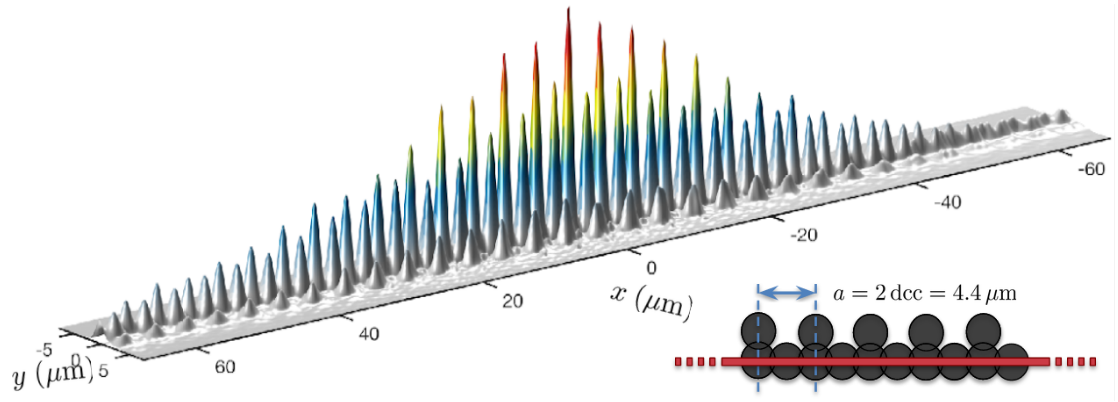
Non-equilibrium condensation or lasing:

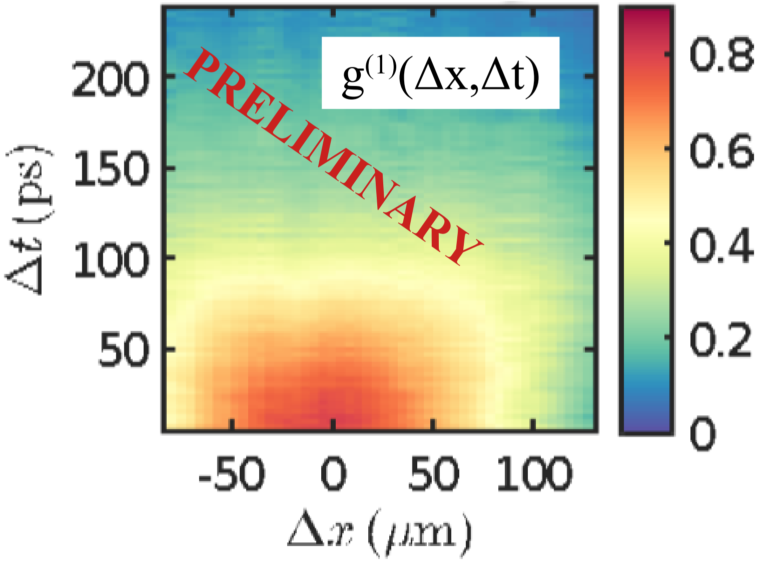
- no BEC in 1D: exponential decay of $g^{(1)}(x)$
  (Graham-Haken, 1970; Wouters-IC, PRB 2006)

- debate in 2D: KPZ nonlinearities destroy BKT transition?
  (Dagvadorj et al., PRX 2015; Altman et al., PRX 2015; Zamora et al., PRX 2017)



Experiment: 1D lattice with array of semiconductor micropillars



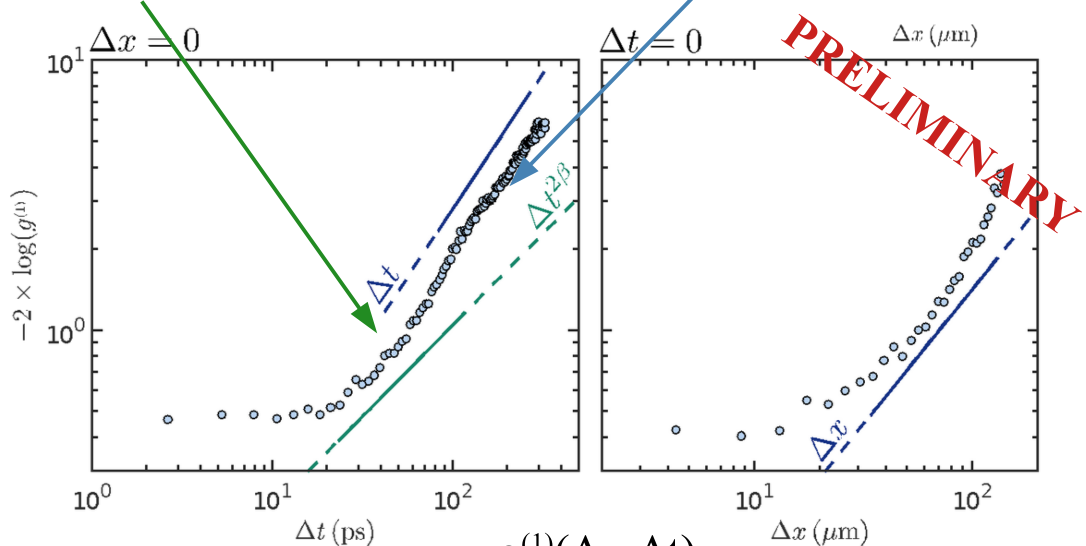


Expt data: Q. Fontaine at C2N – group of J. Bloch, S. Ravets, A. Amo

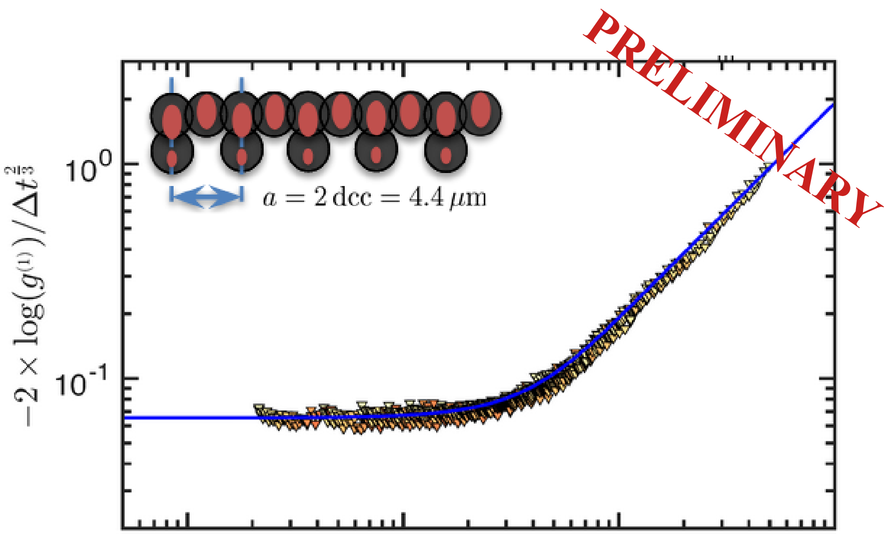
Non-equilibrium KPZ features

KPZ features in $g^{(1)}(x,t)$ → signature of non-equilibrium quasi-condensation process

Power laws in the decay of $g^{(1)}(\Delta x,\Delta t)$



$g^{(1)}(\Delta x,\Delta t)$



Collapse of cuts of $g^{(1)}(\Delta x,\Delta t)$
on Kardar-Parisi-Zhang universal curve

Expt data: Q. Fontaine at C2N – group of J. Bloch, S. Ravets, A. Amo

2008 - Superfluid light (under coherent pump)

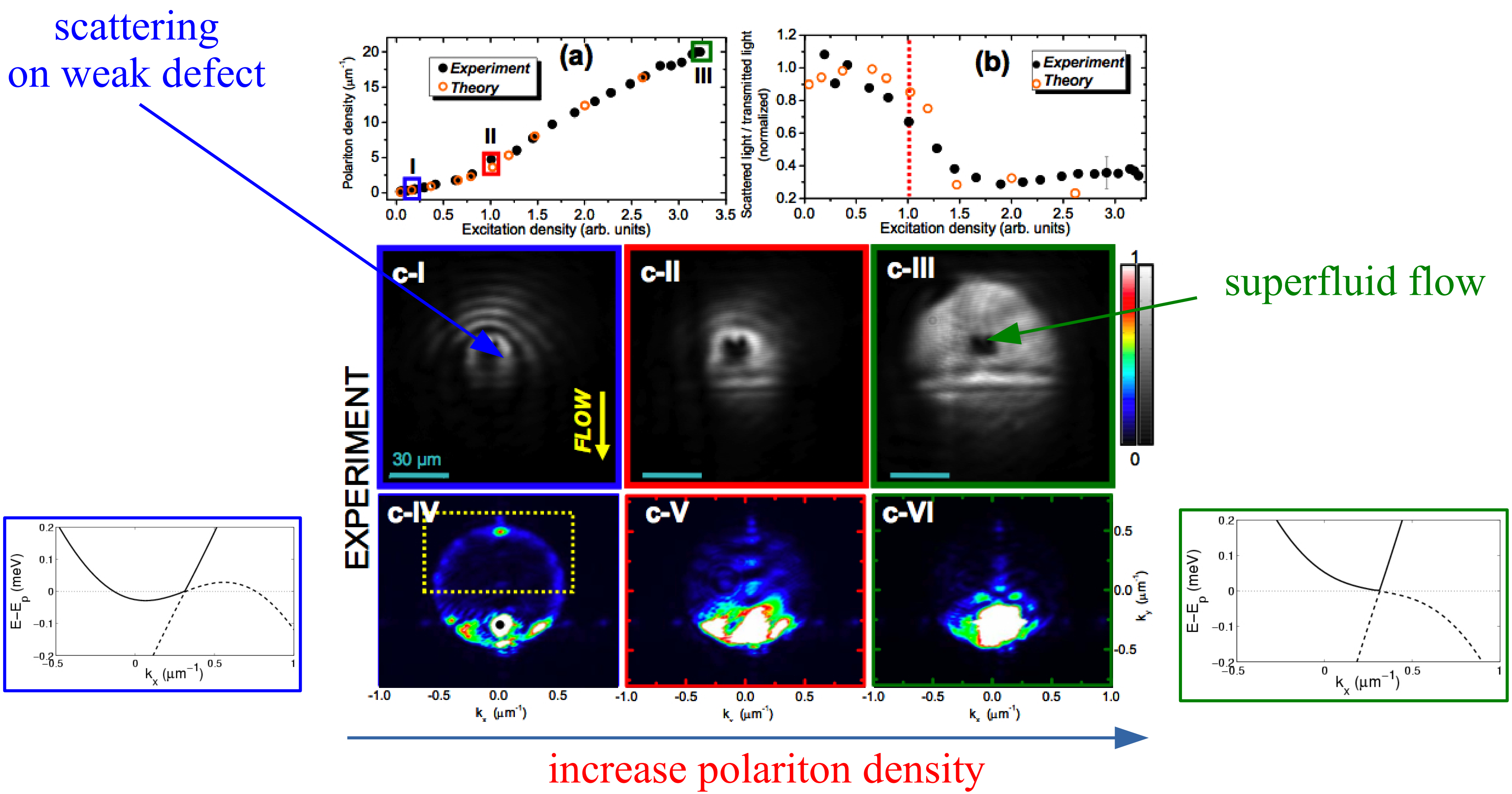


Figure from LKB-P6 group:
A.Amo, J. Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

Theory: IC and C. Ciuti, PRL **93**, 166401 (2004).

Sound in photon BECs: non-equilibrium effects

Polariton BEC regime under incoherent pump (i.e. polariton lasing)

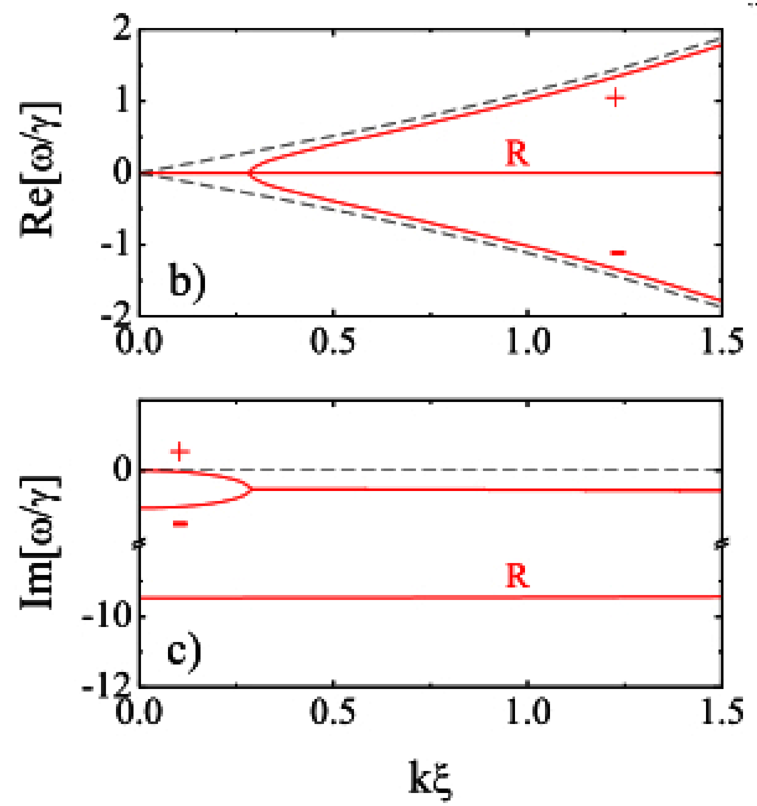
Linearize GPE around steady state

- Reservoir R mode at $-i\,\gamma_R$
- Condensate density and phase modes at:

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - \frac{\Gamma^2}{4}}$$

with:

$$\omega_{Bog}(k) = \sqrt{\frac{\hbar k^2}{2m_{LP}}\left(\frac{\hbar k^2}{2m_{LP}} + 2\,\mu\right)}$$

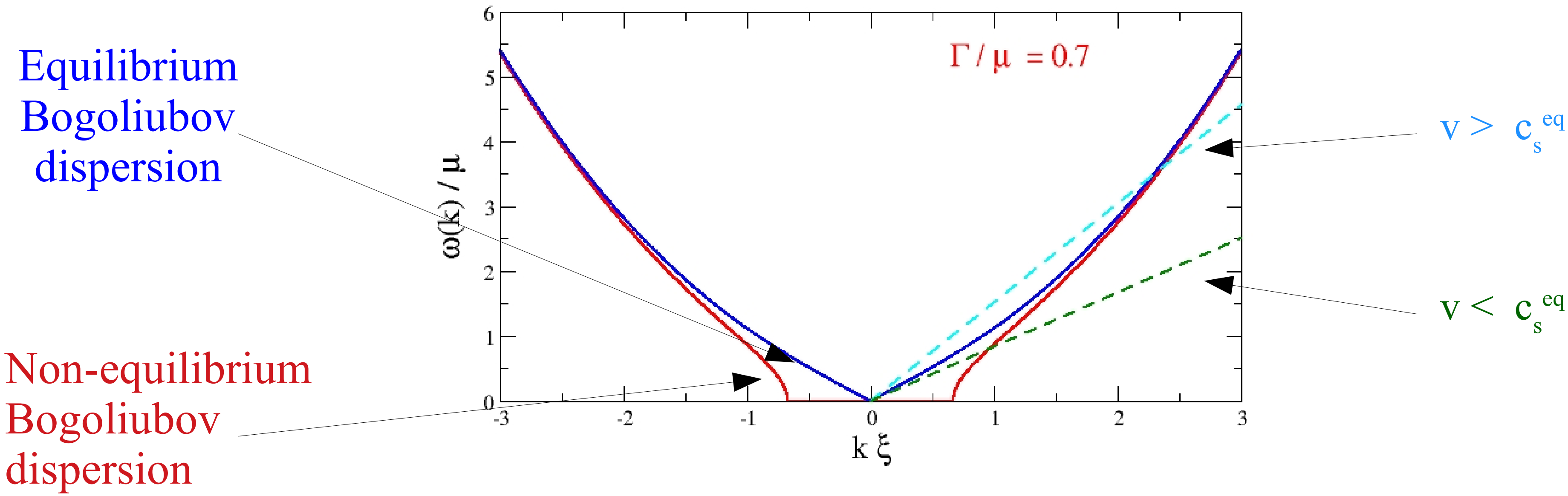


- density (-) and phase (+) oscillations decoupled around k=0
- Goldstone phase (+) mode is diffusive

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. **99**, 140402 (2007)
Similar results in: M. H. Szymanska, J. Keeling, P. B. Littlewood, PRL **96**, 230602 (2006)

Not yet experimentally observed

Consequences on superfluidity



Long-range coherence → metastability of supercurrents (mode stability of ring lasers)

Interaction with defect: naïf Landau argument

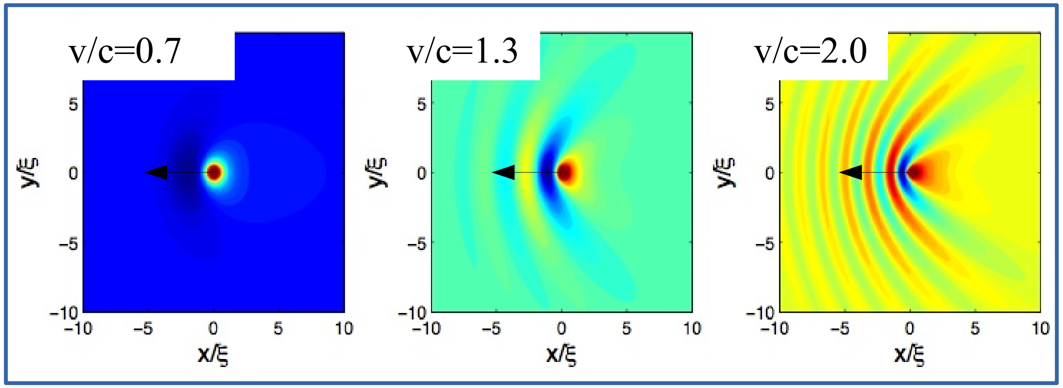
- Landau critical velocity $v_L = \min_k[\omega(k) / k] = 0$ at non-equilibrium BEC
- Any moving defect expected to emit phonons

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. **99**, 140402 (2007)
Similar results in: M. H. Szymanska, J. Keeling, P. B. Littlewood, PRL **96**, 230602 (2006)

But nature is always richer than expected...

Steady-state → well defined ω
Defect → k not a good quantum number

(Complex) k vs.(real) ω dispersion

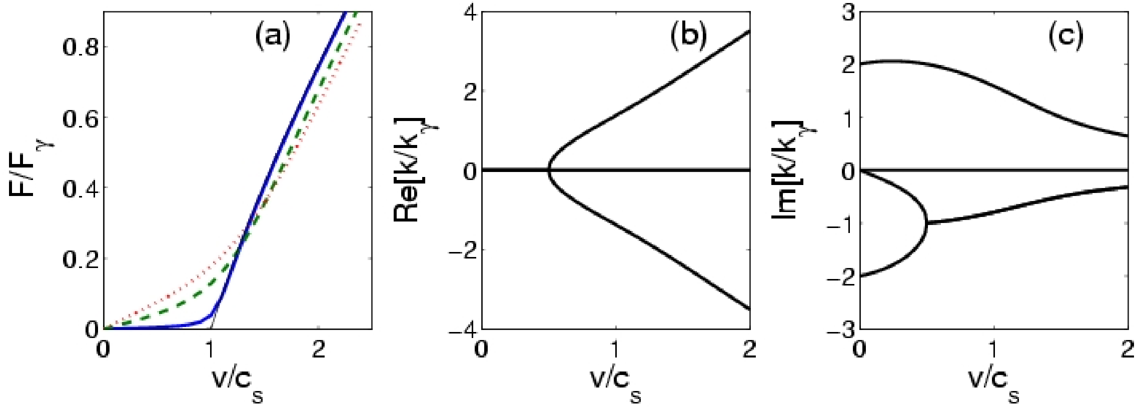


Low v :

- emitted $k_{\parallel}$ purely imaginary
- no real propagating phonons
- perturbation localized around defect

Critical velocity $v_c < c$:

- corresponds to bifurcation point
- decreases with Γ / μ



High v:

- emitted propagating phonons:
  → Cerenkov cone
  → parabolic precursors
- spatial damping of Cerenkov cone

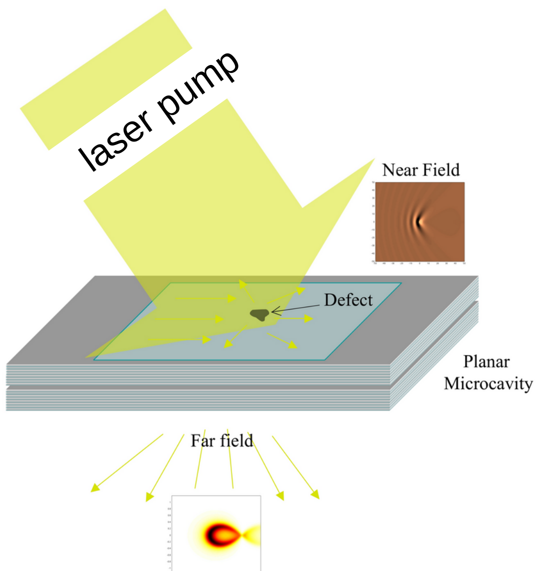
M. Wouters, IC, *Superfluidity and Critical Velocities in Nonequilibrium BECs*, PRL **105**, 020602 (2010).

Part 3:

Quantum fluids of light with a unitary dynamics

Field equation of motion

Planar microcavities & cavity arrays



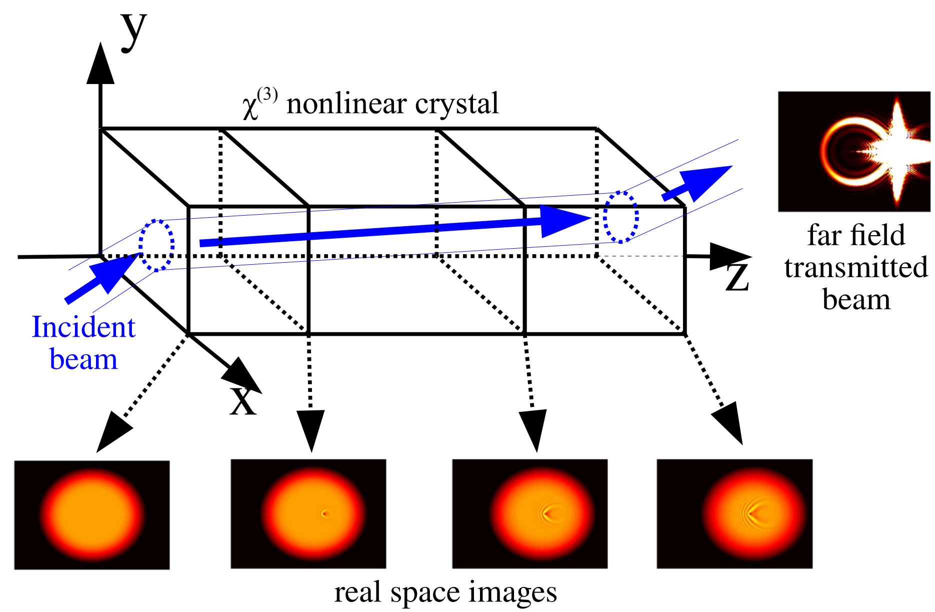
Pump needed to compensate losses:
driven-dissipative dynamics in real time
stationary state ≠ thermodyn. equilibrium

Driven-dissipative CGLE evolution

$$i\frac{dE}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|E|^2}-\gamma\right)\right\}E+F_{ext}$$

Quantum correl. sensitive to dissipation

Propagating geometry



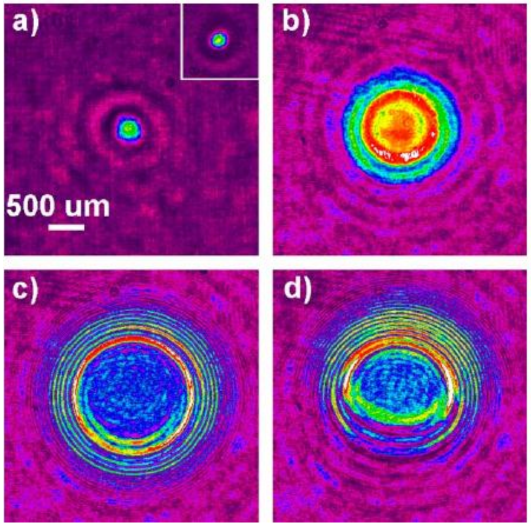
Monochromatic beam
Incident beam sets initial condition @ z=0
MF → Conserv. paraxial propag. → GPE

$$i\frac{dE}{dz}=\left\{-\frac{\hbar\nabla^2_{xy}}{2\beta}+V_{ext}+g|E|^2 E\right\}E$$

- V_{ext} , g proportional to -(ε(r)-1) and $\chi^{(3)}$
- Mass → diffraction (xy)

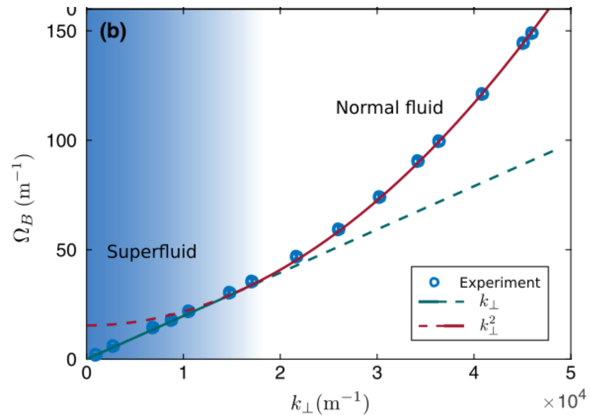
A few landmark experiments (among many)

Dispersive superfluid-like shock waves



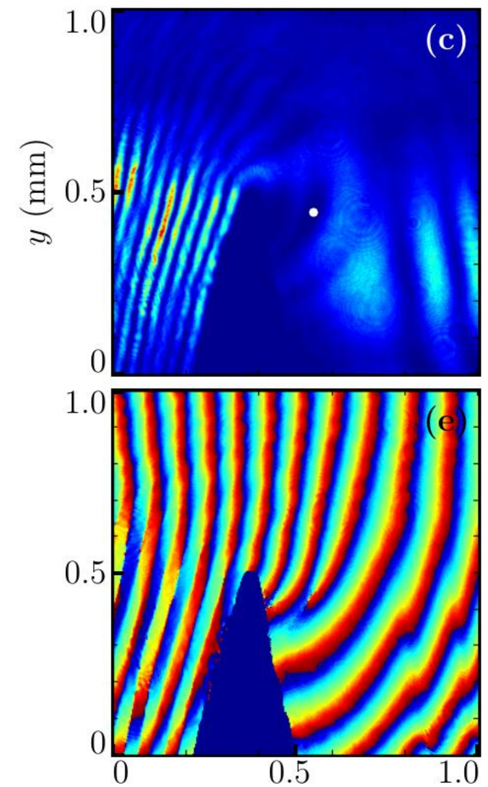
Wan et al., Nat. Phys. 3, 46 (2007)

Bogoliubov dispersion of collective excitations



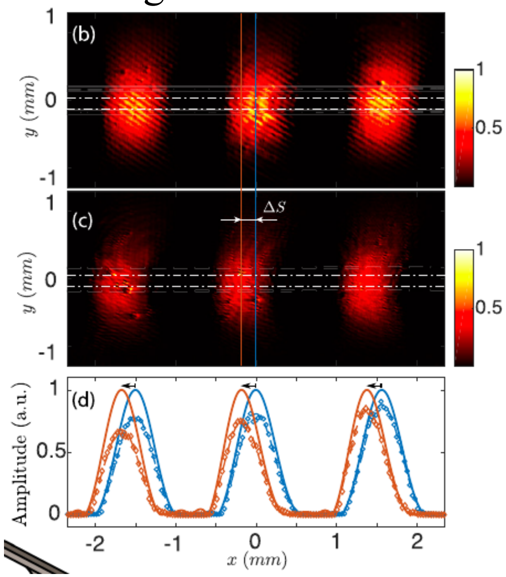
Q. Fontaine et al., PRL 2018

Hydrodynamic nucleation of quantized vortices



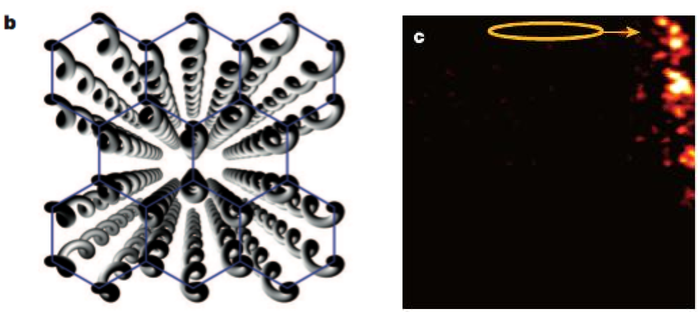
D. Vocke et al., PRA (2016)

Microscopic structure of Bogoliubov modes



Q. Fontaine et al., Phys. Rev. Res. (2020)

Chiral edge states in (photonic) Floquet topological insulator



Rechtsman, et al., Nature 496, 196 (2013)

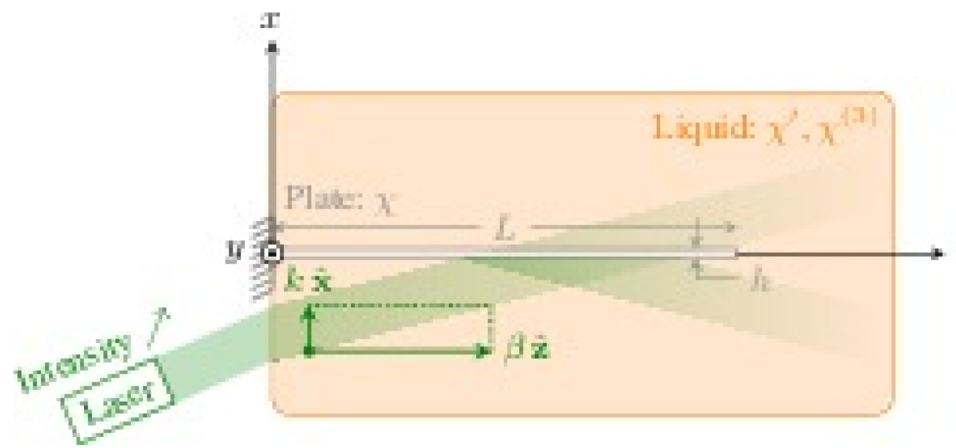
Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the fluid density/momentum pattern
- Obstacle typically is defect embedded in semiconductor material
- Impossible to measure mechanical friction force exerted onto obstacle

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in liquid nonlinear medium, so can move and deform
- Mechanical force measurable from magnitude of slab deformation



P.-E. Larré, IC, *Optomechanical Signature of a Frictionless Flow of Superfluid Light*, Phys. Rev. A 91, 053809 (2015).

Frictionless flow of superfluid light (II)

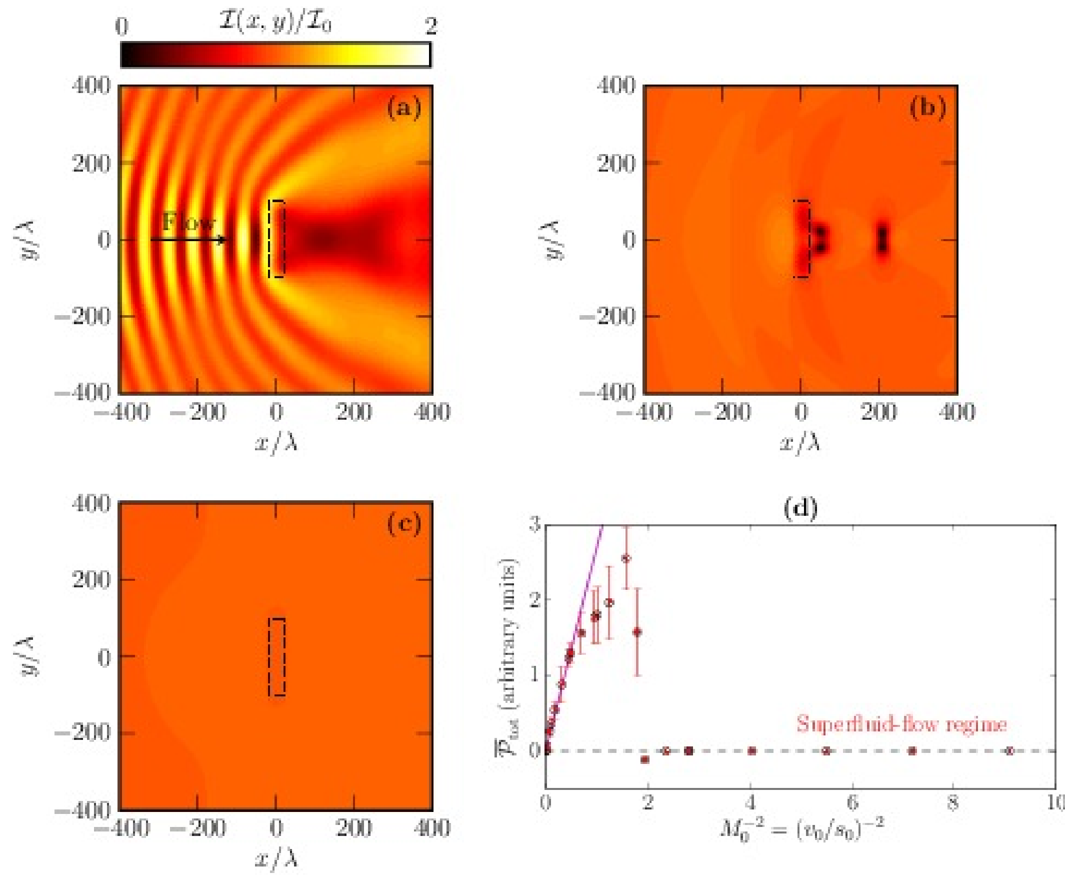
Numerics for propagation GPE of monochromatic laser:

$$i\,\partial_z E = -\frac{1}{2\beta}\left(\partial_{xx}+\partial_{yy}\right)E + V(r)\,E + g\,|E|^2 E$$

with $V(r)=-\beta\,\Delta\varepsilon(r)/(2\varepsilon)$ with rectangular cross section and $g=-\beta\,\chi^{(3)}/(2\varepsilon)$

For growing light power, superfluidity visible:

- Intensity modulation disappears
- Suppression of opto-mechanical force



Fused silica slab:

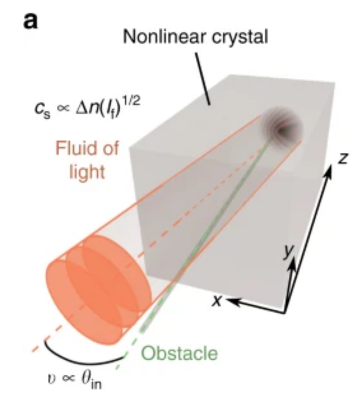
deformation almost in the μm range

P.-E. Larré, IC, *Optomechanical Signature of a Frictionless Flow of Superfluid Light*, Phys. Rev. A 91, 053809 (2015).

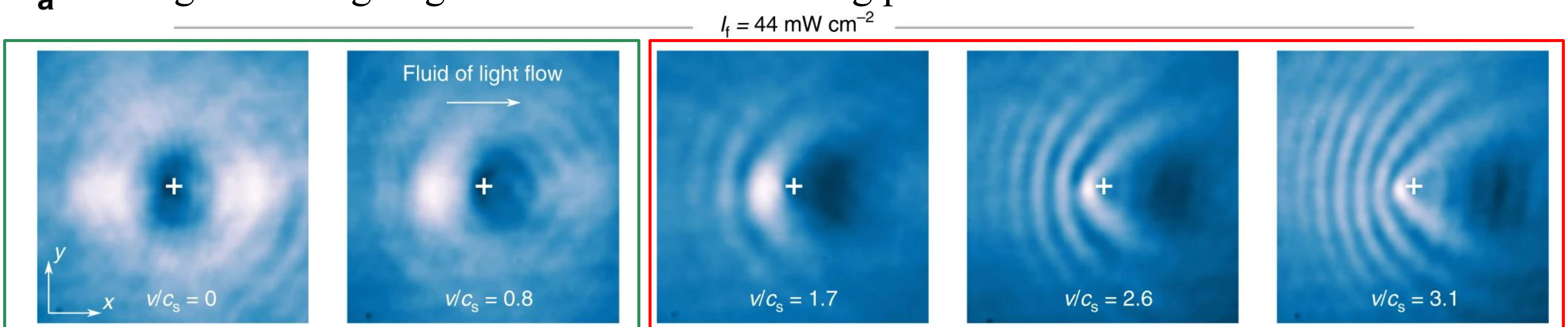
Frictionless flow of superfluid light (III)

Smarter all-optical design:

- defect → static Δn by additional beam
- friction force measured by displacement of defect beam

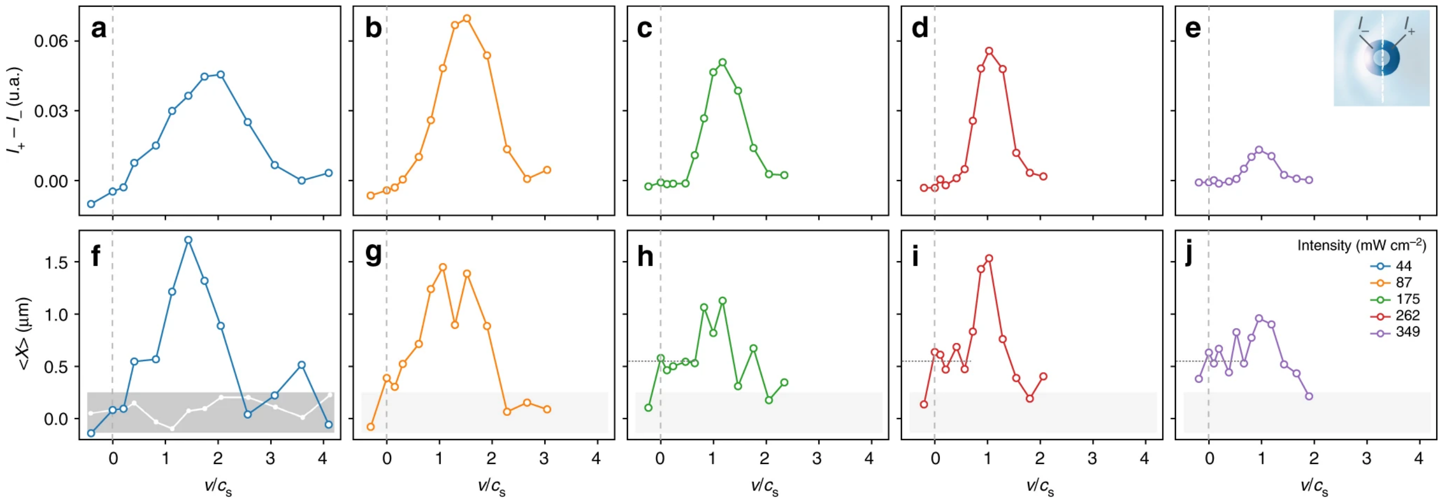


Images of out-going fluid beam for increasing power



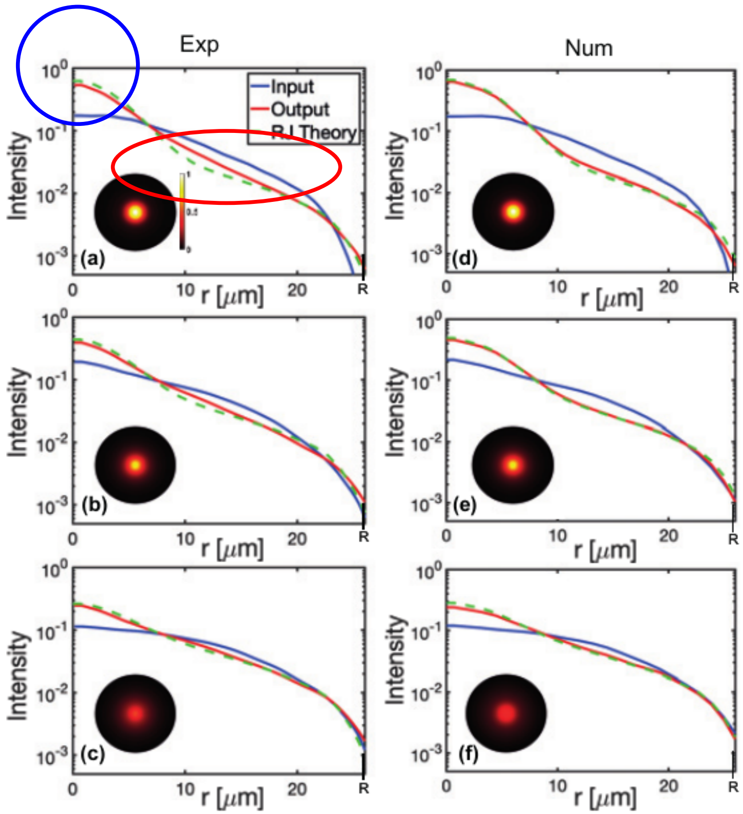
superfluid regime → no fringes

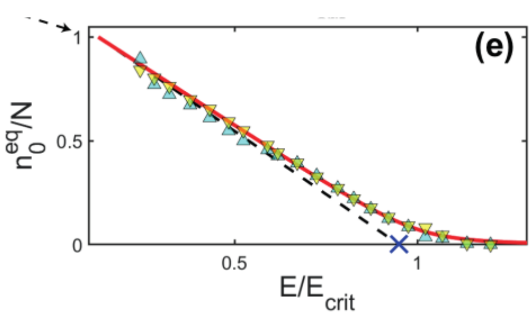
fringes → friction

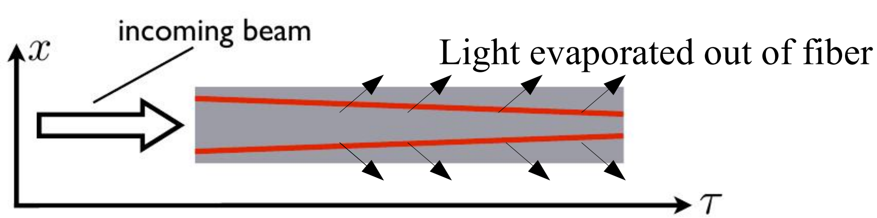


Figures from:
C. Michel et al.,
Nat. Comm. 2018

Condensation of classical waves







Expts in progress in Picozzi's lab @ Dijon

Monochromatic beam
with noisy spatial profile
injected into multi-mode fiber

Classical nonlinearity → stays monochromatic
GPE evolution during propagation

Thermalizes to:

- classical condensate
- thermal cloud with
  Rayleigh-Jeans *1/k²* high-*k* tail

- What about quantum effects?
- How to recover Planckian?

- Can be used for evaporative cooling of light?
  New source of coherent light

Images from Baudin PRL (2020) + EPL (2020)
Earlier expts in Sun et al., Nature Physics 8, 470 (2012)
Other works by Krupa, Wabnitz, etc.

More sophisticated theory: Chiocchetta et al., EPL 2016

How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

$$i\frac{\partial E}{\partial z} = -\frac{1}{2\beta_0}\left(\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}\right) - \frac{1}{2D_0}\frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2 E$$

Propagation coordinate z $\rightarrow$ time

Physical time $\rightarrow$ extra spatial variable, dispersion D_0 $\rightarrow$ temporal mass

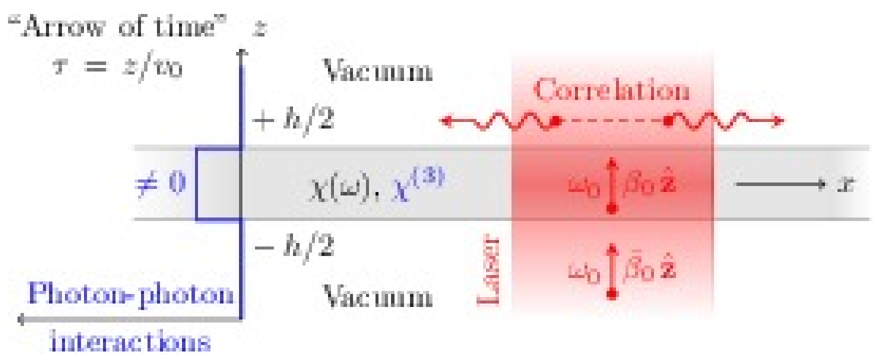
Upon quantization $\rightarrow$ conservative many-body evolution in z: $i\frac{d}{dz}|\psi\rangle = H|\psi\rangle$

with $$H = N\iiint dx\,dy\,dt\left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E} - \frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t} + V\,\hat{E}^\dagger\hat{E} + \hat{E}^\dagger\hat{E}^\dagger\hat{E}\,\hat{E}\right]$$

Same z commutator $$\left[\hat{E}(x.y,t,z),\hat{E}^+(x',y',t',z)\right] = \frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$$

P.-E. Larré, IC, *Propagation of a quantum fluid of light in a cavityless nonlinear optical medium: General theory and response to quantum quenches,* PRA **92**, 043802 (2015)
See also old work by Lai and Haus, PRA 1989

Dynamical Casimir emission at quantum quench (I)



Monochromatic wave @ normal incidence

Slab of weakly nonlinear medium

→ Weakly interacting Bose gas at rest

Air / nonlinear medium interface

→ sudden jump in interaction constant when moving along z

Mismatch of Bogoliubov ground state in air and in nonlinear medium

→ emission of phonon pairs at opposite k on top of fluid of light

Propagation along z

→ conservative quantum dynamics

Important question: what is quantum evolution at late times? Thermalization?

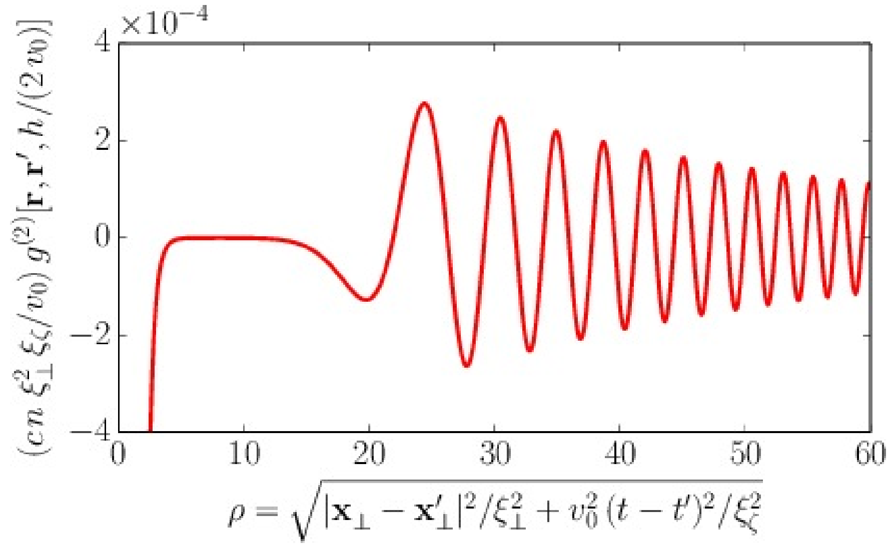
P.-E. Larré and IC, PRA **92**, 043802 (2015)

Dynamical Casimir emission at quantum quench (II)

Observables:

- Far-field → correlated pairs of photons at opposite angles
- Near-field → peculiar pattern of intensity noise correl.

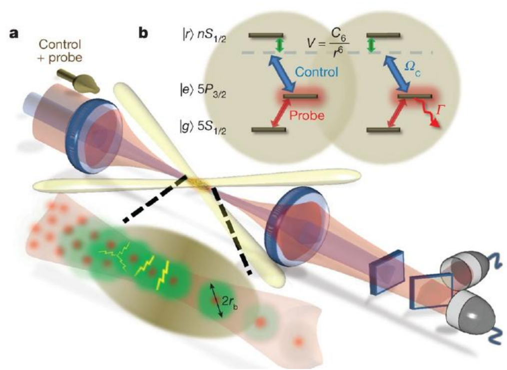
First peak propagates at the speed of sound c_s



quantum simulation of fluctuations in early universe

:

Quantum dynamics most interesting in strongly nonlinear media, e.g. Rydberg polaritons

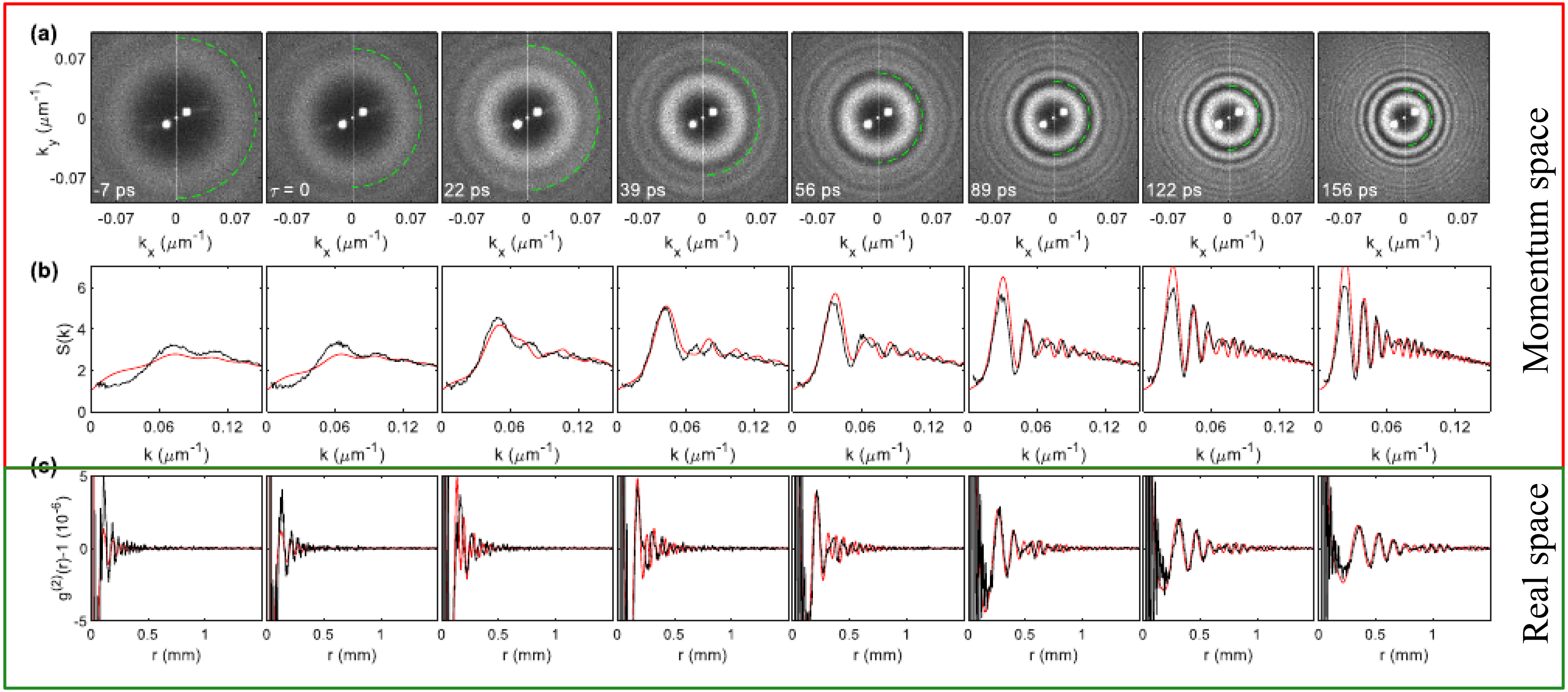


P.-E. Larré and IC, PRA **92**, 043802 (2015)

Dynamical Casimir emission at quantum quench (III)

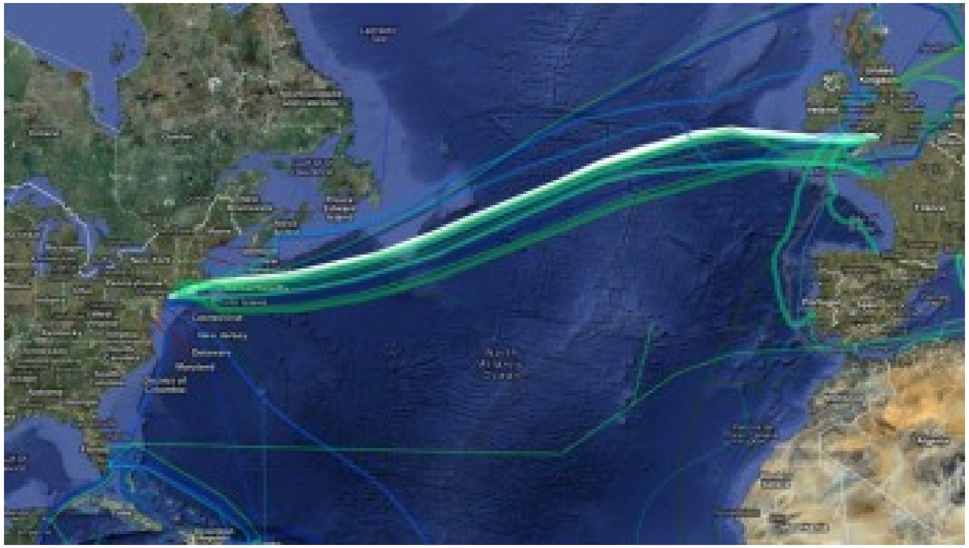
Experimental data from: Steinhauer, Abuzarli, Bienaimé, Piekarski, Liu, Aladjidi, Giacobino, Bramati, Glorieux (2021)
"Spontaneous and stimulated dynamical Casimir effects in a quantum fluid of light"

Experimental correlation pattern in outgoing beam



A potentially important technological issue...

Long-distance fiber-optic set-ups
→ telecom over distances $\sim 10^4$ km



Can optical coherence be preserved?

Several disturbing effects:

- (extrinsic) fluctuations of fiber temperature, length, etc.
- (intrinsic) Fiber material has some (typically weak) $\chi^{(3)}$
  Shot noise on photon number gives fluctuations of $n_{refr} \sim n_0 + \chi^{(3)} I$

  Statistical mechanics suggests that phase fluctuations destroy 1D BEC

  → light at the end of fiber has lost its (temporal) phase coherence

Is this intuitive picture correct? How to tame phase decoherence?

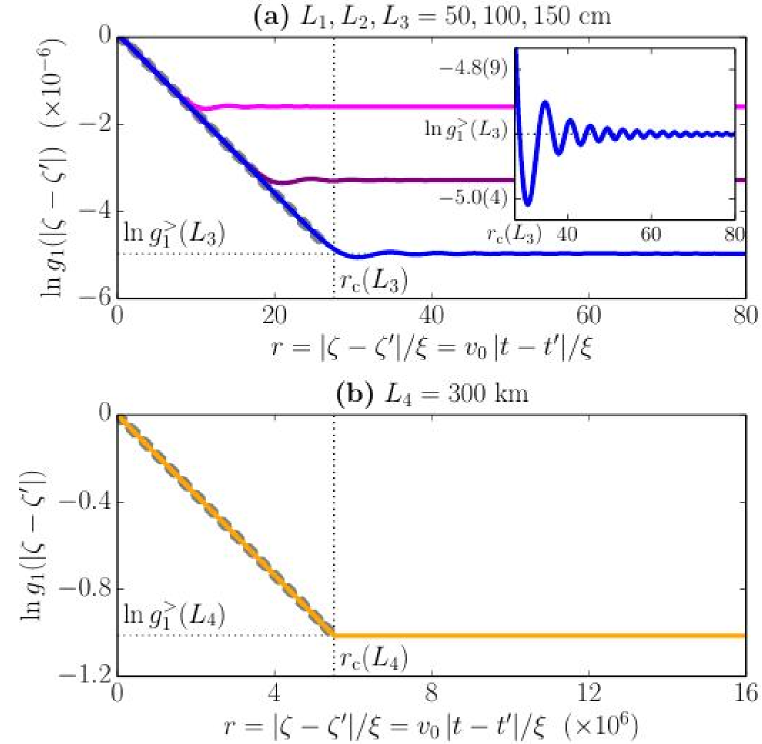
“Pre-thermalized” 1D photon gas

Perfectly coherent light injected into 1D optical fiber:

- quantum quench of interactions ~ $\chi^{(3)}$
- pairs of Bogoliubov excitations generated

Resulting phase decoherence in $g^{(1)}(t-t')$:

- Exponential decay at short $|t-t'| < 2z / c_s$
  (c_s = speed of Bogol. sound)
- Plateau at long $|t-t'| > 2z / c_s$
- Low-k modes eventually tends to thermal $T_{eff} = \mu / 2$
- Hohenberg-Mermin-Wagner theorem prevents long-range order in 1D quasi-condensates at finite T

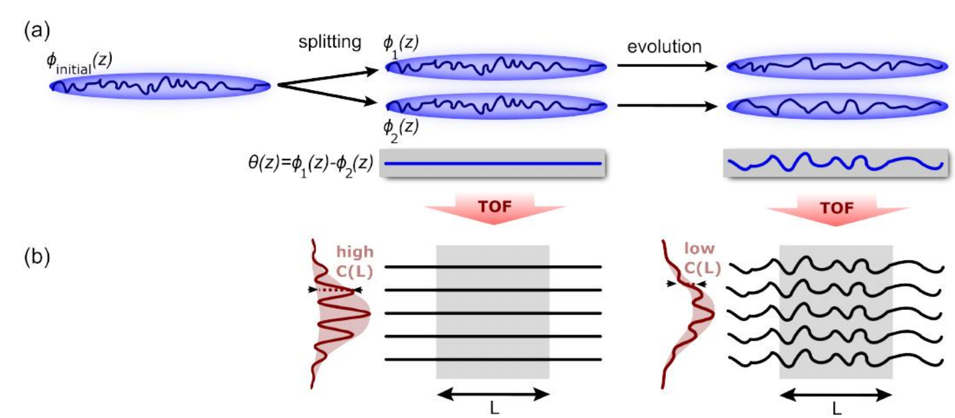


Effect small for typical Si fibers, still potentially harmful on long distances

Decoherence slower if tapering used to “adiabatically” inject light into fiber

Related cold atom expts by J. Schmiedmayer when 1D quasi-BEC suddently split in two

Nature Physics 9, 640–643 (2013)



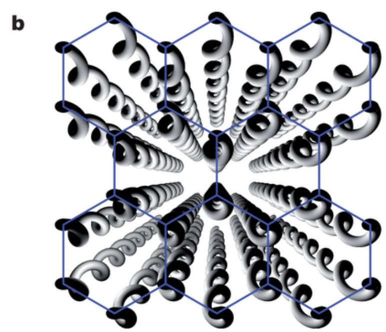
P.-E. Larré and IC, *Prethermalization in a quenched one-dimensional quantum fluid of light*, arXiv:1510.05558

A quite generic *quantum simulator*

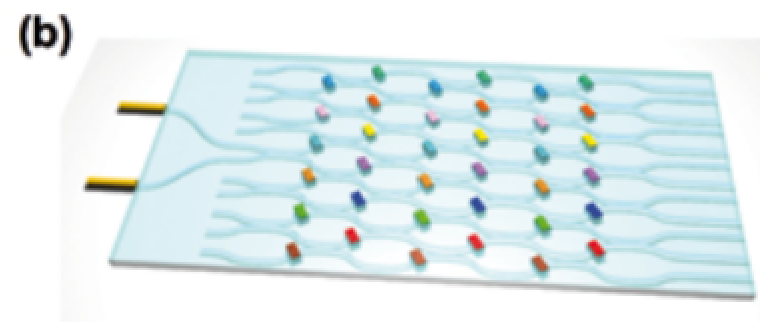
Quantum many-body evolution in *z:*

$$i\frac{d}{dz}|\psi\rangle = H|\psi\rangle \quad \text{with:} \quad H = N\iiint dx\,dy\,dt \left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E} - \frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t} + V\hat{E}^\dagger\hat{E} + \hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$$

- Physical time *t* plays role of extra spatial coordinate
- Same *z* commutator: $\left[\hat{E}(x.y,t,z),\hat{E}^{+}(x',y',t',z)\right] = \frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$



Rechtsman et al., Nature 2012



Clever design of V(x,y,z) → simulate wide variety of physical systems:

- Arbitrary splitting/recombination of waveguides → quench of tunneling
- Modulation along *z* → Floquet topological insulators
- In addition to photonic circuit → many-body due to photon-photon interactions
- On top of moving fluid of light → simulate general relativistic QFT

P.-E. Larré, IC, PRA **92**, 043802 (2015)

Part 4:

a brief journey through the early days of topological photonics

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Topological photonics

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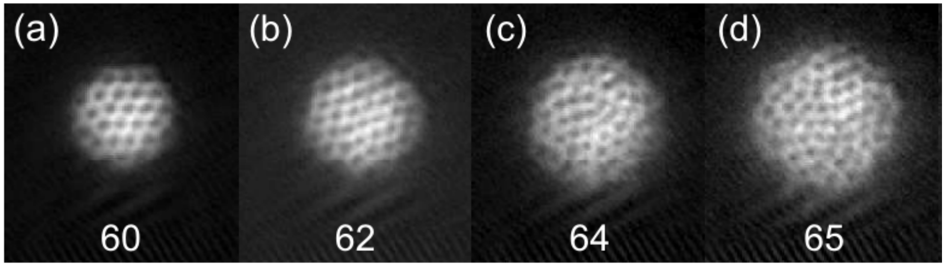
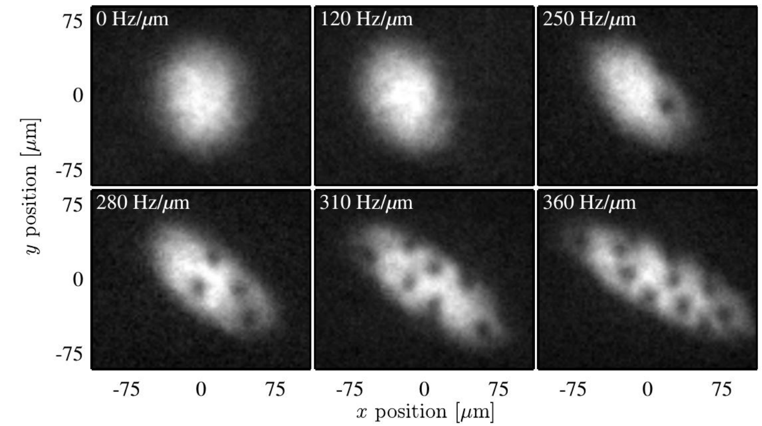
(published 25 March 2019)

0034-6861/2019/91(1)/015006(76)

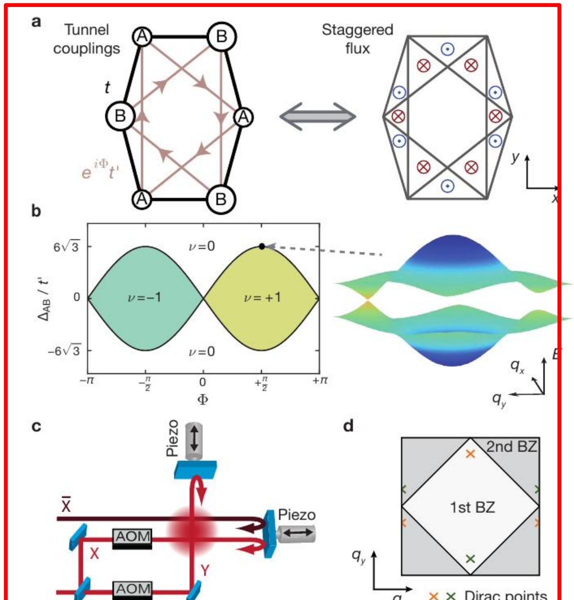
Prehistory: Synthetic gauge fields for atoms



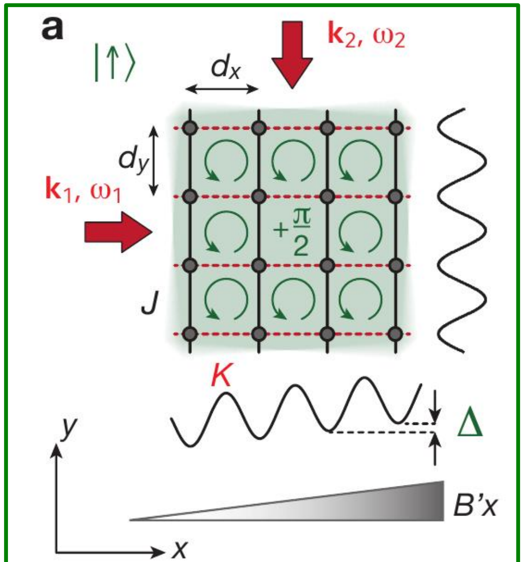
Coriolis force in rotating frame: vortices in atomic gas
Bretin et al., PRL 92, 050403 (2004)



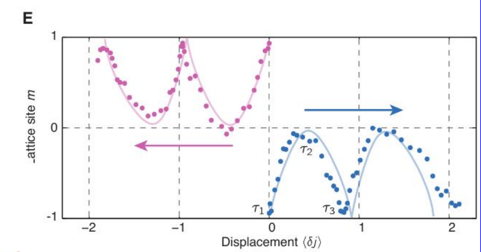
Synthetic gauge field by Raman + magnetic field
Lin et al. Nature 471, 83 (2011)



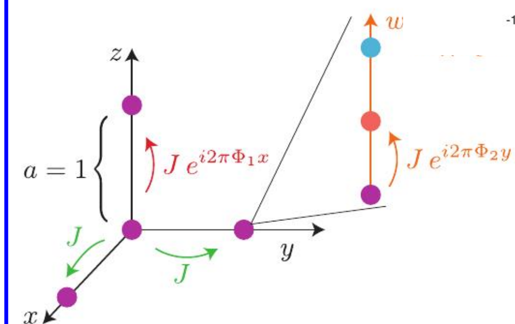
Shaken lattice
Haldane model for atoms
Jotzu et al., Nature 515, 237 (2014)

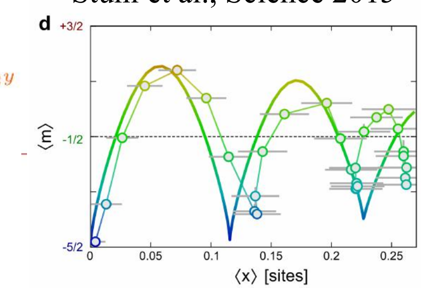


Laser-assisted hopping
2D Harper-Hofstadter model
Aidelsburger et al., PRL 111, 185301 (2013)



Stuhl et al., Science 2015





Mancini et al., Science 2015.

Synthetic dimensions (so far 1+1D = 2D)
Chiral edge states in synthetic Hall ribbons

2008-9 – The birth of *topological photonics* (th)

PRL **100**, 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

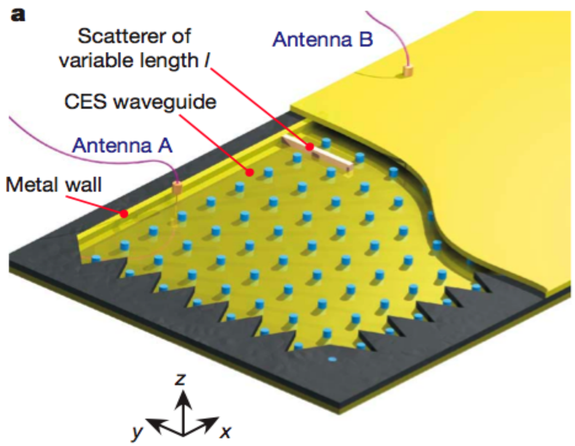
We show how, in principle, to construct analogs of quantum Hall edge states in "photonic crystals" made with nonreciprocal (Faraday-effect) media. These form "one-way waveguides" that allow electromagnetic energy to flow in one direction only.

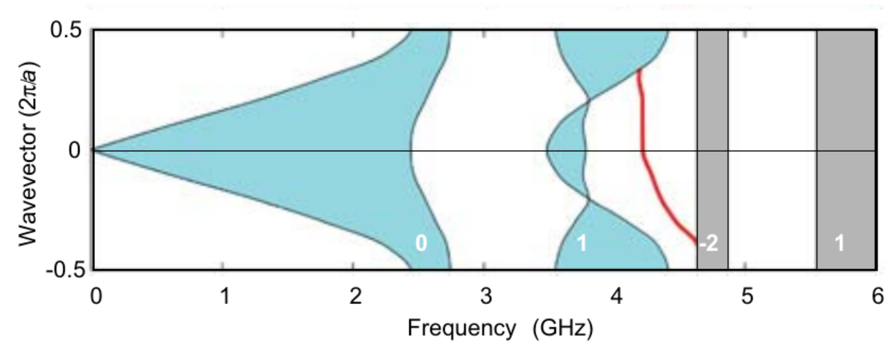
DOI: 10.1103/PhysRevLett.100.013904 PACS numbers: 42.70.Qs, 03.65.Vf

- IQH depends on geometrical properties of Bloch band states ("TKNN" Thouless et al. PRL 1982):
  - Berry connection $\mathcal{A}_n^a = \dfrac{\langle u_n|\boldsymbol{B}_0(\omega_n)|\nabla_k^a u_n\rangle - \langle \nabla_k^a u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}{2i\langle u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}$,
  - Berry curvature $\mathcal{F}_n^{ab}(\boldsymbol{k}) = \nabla_k^a \mathcal{A}_n^b - \nabla_k^b \mathcal{A}_n^a$
  - Integer-valued Chern number $C_n^{(1)}(\Sigma) = \dfrac{1}{2\pi}\iint_\Sigma dk_a \wedge dk_b \mathcal{F}_n^{ab}.$
  - C_n fixes transverse conductivity σ_H and number of edge states (bulk-boundary correspondence)

- Haldane-Raghu → IQH not specific to fermionic electrons
  - ✓ Complex band structures can be realized for photons in periodic structures, aka photonic crystals
  - ✓ Need to break T-reversal to have $C_n \neq 0$ → include magnetic elements
  - ✓ σ_H not directly defined, but chiral edge states give one-way waveguide on the edge

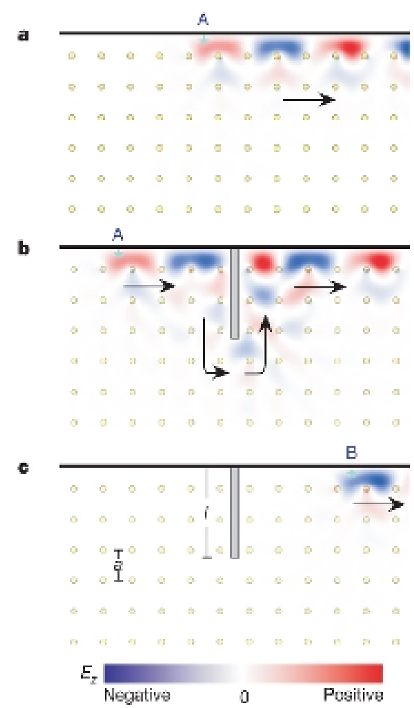
2008-9 – The birth of *topological photonics* (expt)

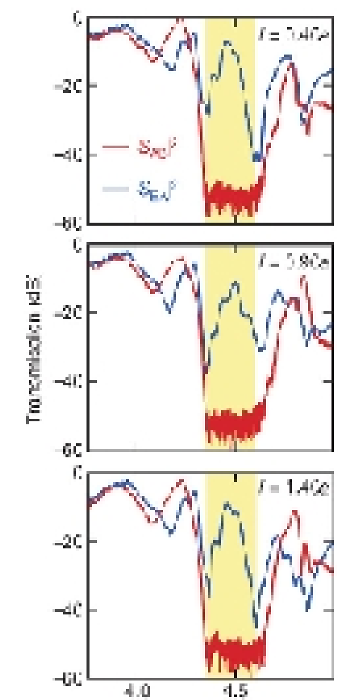




Magneto-optical photonic crystals for µ-waves

- T-reversal broken by magnetic elements
- Band wih non-trivial Chern number:
  → chiral edge states within gaps





Experiment:

- measure transmission from antenna to receiver
- only in one direction → unidirectional propagation
- immune to back-scattering by defects → topologically protected

Z Wang, Y Chong, JD Joannopoulos, M Soljačić, *Observation of unidirectional backscattering-immune topological electromagnetic states*, Nature 461, 772 (2009)

2013 - Harper-Hofstadter & Haldane models for visible photons

Goal:

- avoid the need of magnetic materials
- scale up to visible light where optical nonlinearities and quantum emitters easily accessible

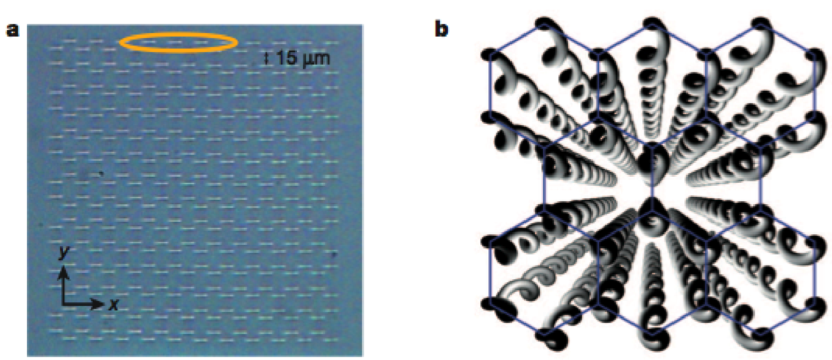
Many proposals: geometrical phases (Umucalilar), opto-mechanics (Rabl), ...

2D lattice of coupled cavities with tunneling phase

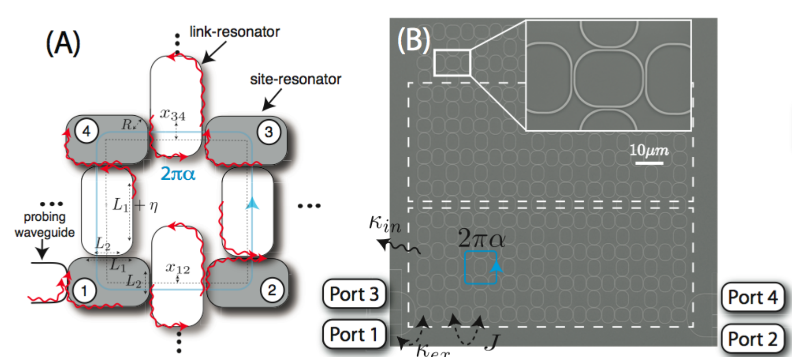
$$H = \sum_i \hbar\omega_o \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \left[\hbar F_i(t)\, \hat{a}_i^\dagger + \text{h.c.} \right]$$

Experiments along these lines:

- Floquet bands in helically deformed honeycomb waveguide lattices → Rechtsman/Szameit/Segev
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)
- strained honeycomb lattice for photons/polaritons → A. Amo/J.Bloch (C2N); Bellec et al. (Nice)



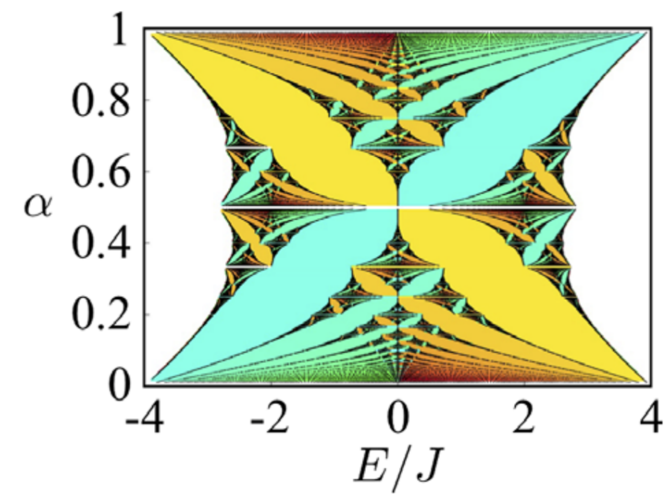
Rechtsman, Plotnik, et al., Nature 496, 196 (2013)



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

2013 - Imaging chiral edge states

2D square lattice of coupled resonators
at large magnetic flux



Eigenstates organize in bulk Hofstadter bands

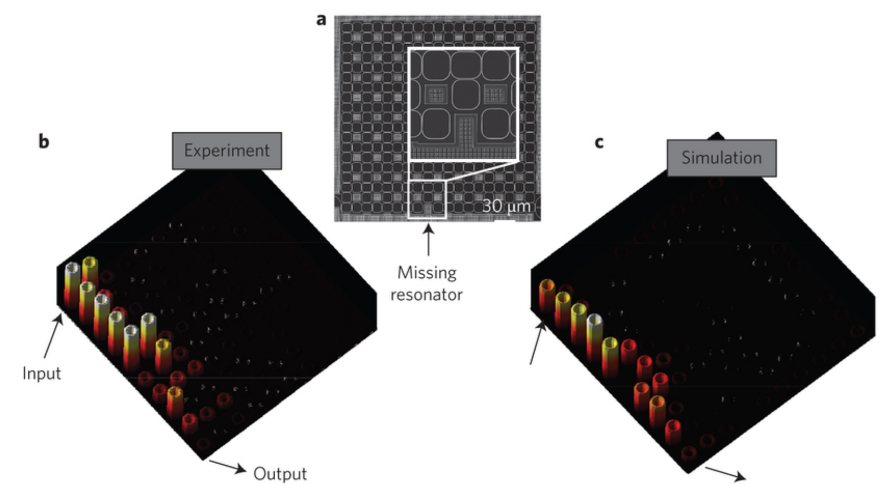
- Berry connection in k-space: $A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k} \rangle$
- Berry curvature $\Omega_n(\mathbf{k}) = i(\langle \partial_{k_x} u_{n,\mathbf{k}} | \partial_{k_y} u_{n,\mathbf{k}} \rangle - \langle \partial_{k_y} u_{n,\mathbf{k}} | \partial_{k_x} u_{n,\mathbf{k}} \rangle)$
- Chern number $C_n = \frac{1}{2\pi} \int_{\mathrm{BZ}} d^2k \Omega_n(k_x, k_y),$

Bulk-edge correspondance:

$A_{n,k}$ has non-trivial Chern number $C_n \neq 0$

$\rightarrow$ chiral edge states within gaps

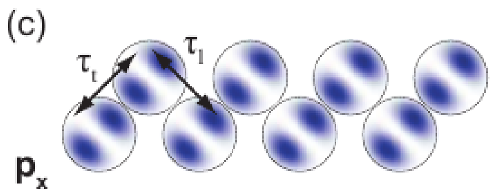
- unidirectional propagation
- (almost) immune to scattering by defects

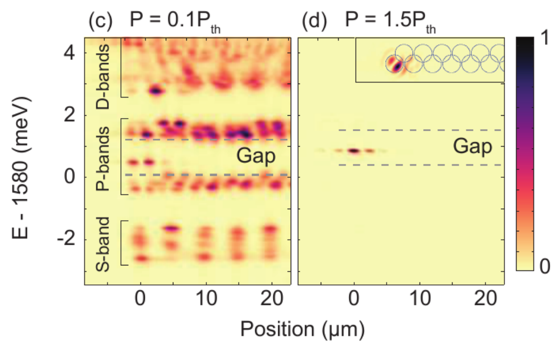


Hafezi et al.,Nat. Phot. **7**, 1001 (2013)
Similar images for Haifa expt

2017 – Topological lasing

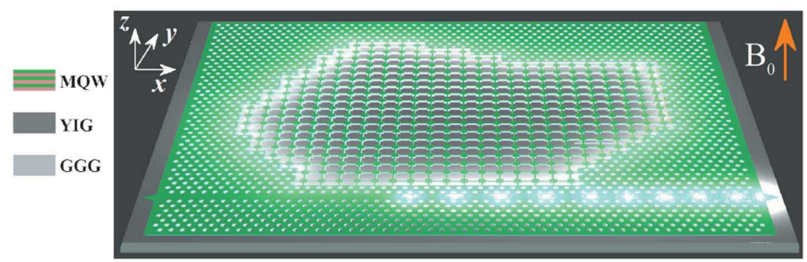
What happens if one adds gain to a topological model ?

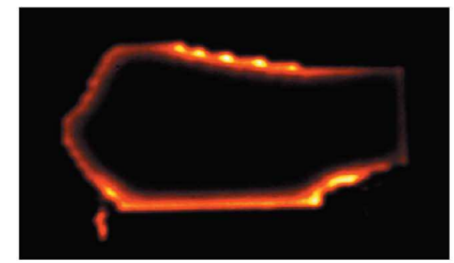


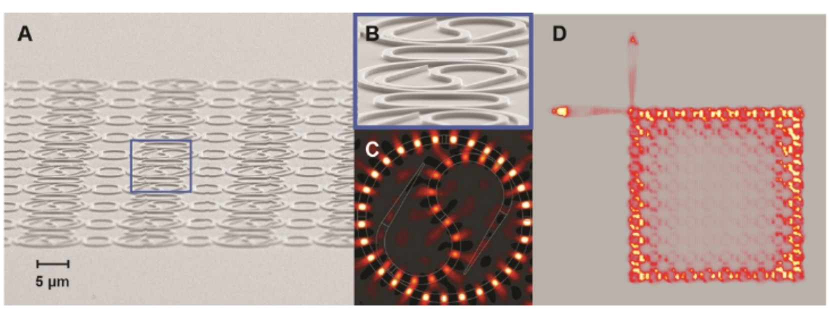


St. Jean, et al., Nat. Phot. ‘17
System: 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017
System: 2D photonic crystal slab, amplification by QWs, magnetic field to break T







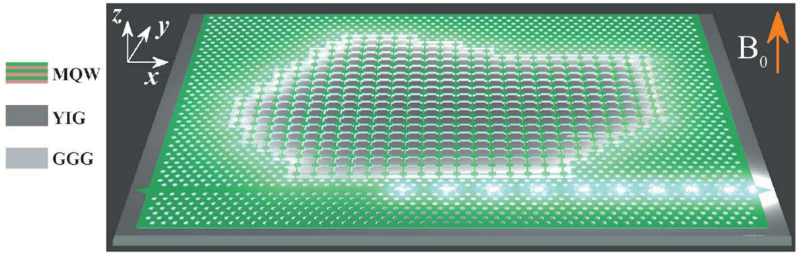
Bandres et al., Science 2018
System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Early theoretical work by Conti & Pilozzi, Solnyshkov, Nalitov & Malpuech.
Other expts: Khajavikhan’s group, PRL 2018...

Coherence of topolaser emission

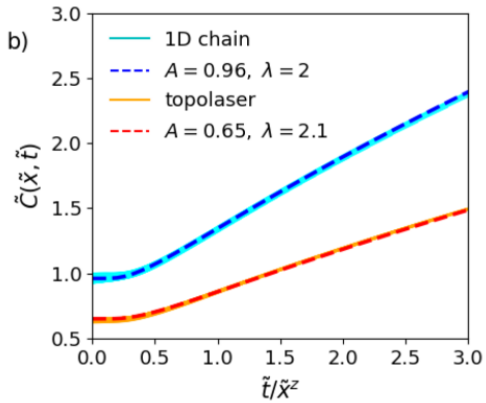
Technologically important in laser technology:

- Chiral motion → phase lock many individual sites
- Strong emission intensity and high coherence



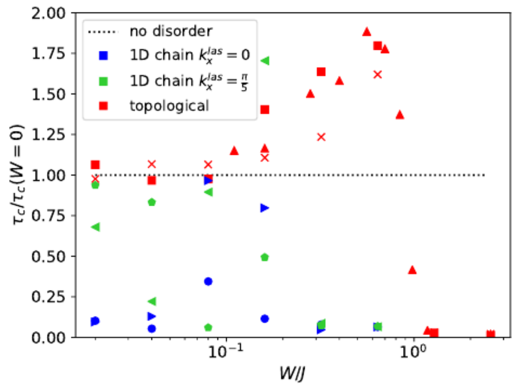
Important fundamental & applied questions:

- What are ultimate limitations of coherence?
- How robust is coherence to disorder?
- What advantage over standard lasers?



Laser operation in spatially extended system:

- Kardar-Parisi-Zhang model of non-equilibrium statistical mechanics (Altman/Diehl, Wouters, Canet/Minguzzi)
- spatio-temporal scaling properties of phase-coherence



In the presence of static disorder:

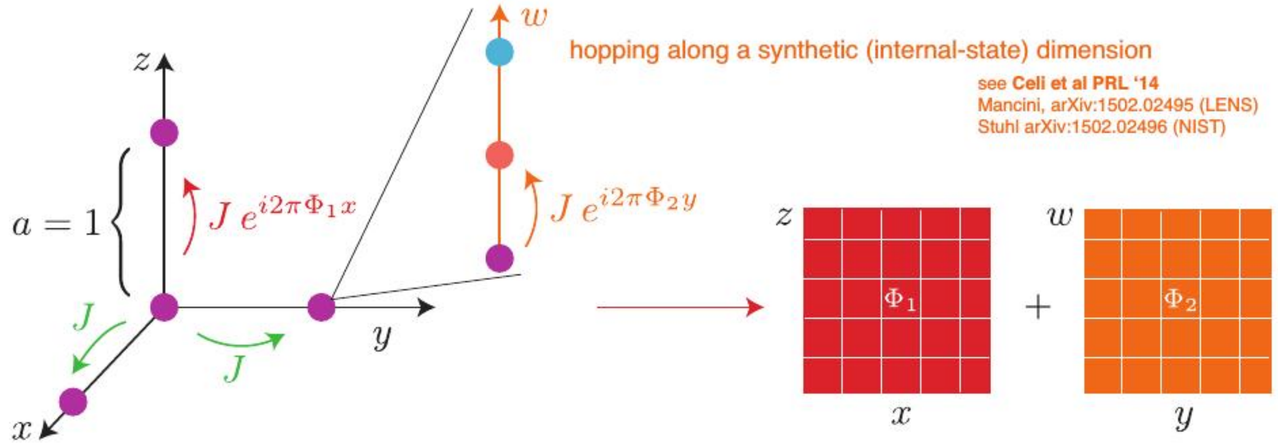
- Non-Topological: weak disorder suppresses temporal coherence (mode fragmentation, multimode emission, localization, etc.)
- Topological: robust spatio-temporal coherence, chiral propagation travels through/around defects without backscattering.

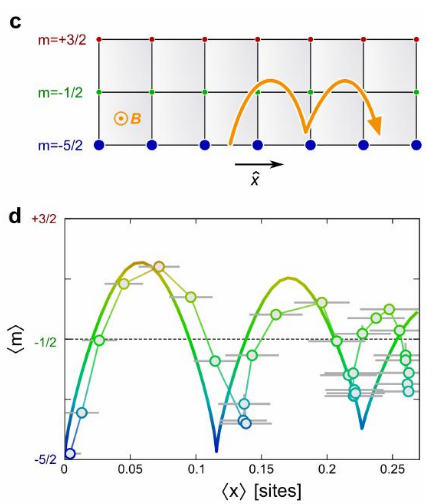
I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 2020

Part 4bis:

Towards higher dimensions

How to create 4D system with atoms?





Mancini et al. & Stuhl et al., Science 2015

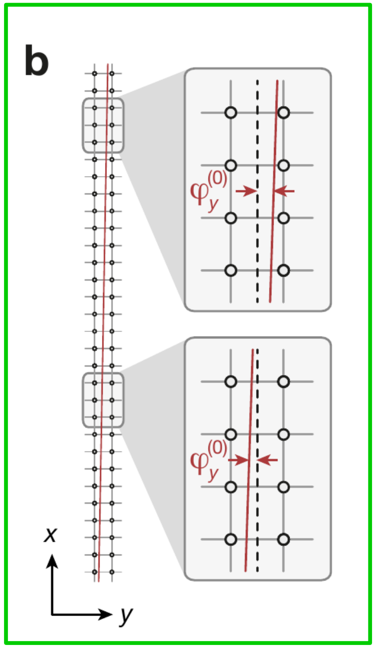
Internal state → Synthetic dimension w

Raman processes give tunneling along *w*

Spatial phase of Raman beams give Peierls phase in *xw*, *yw*, *zw*
Standard synthetic-B in *xy* and/or *yz* and/or *zx*

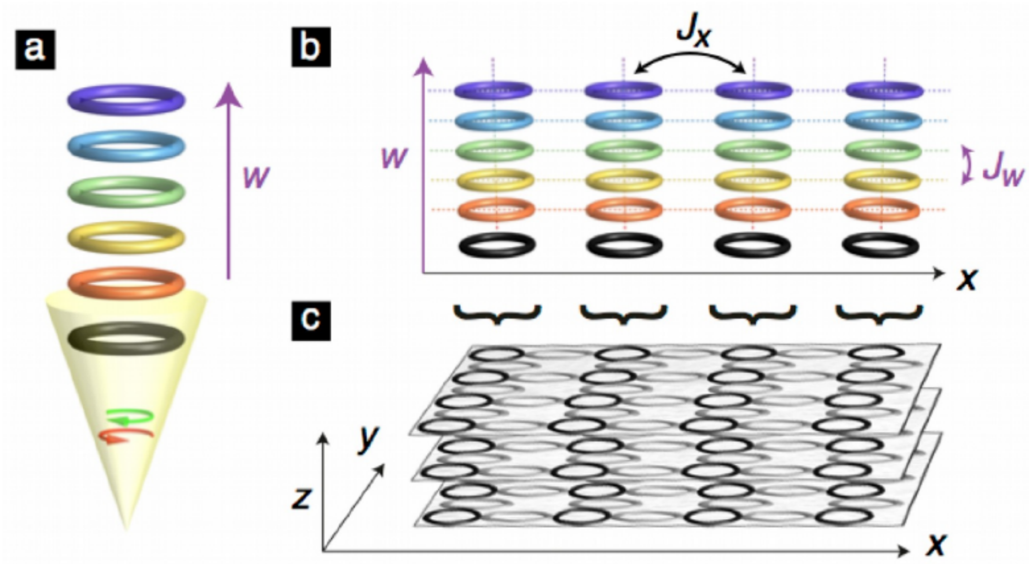
First experimental realization:

- 1+1 dimens. using 3 spin states
- Cyclotron + Reflection on edges



Recently: 2D topological pumping in 2D system → analogous to 4D IQHE
(Lohse, Price, et al., Nature 2018; similar results in photonics in Zilberberg et al., Nature 2018)

How to create synthetic dimensions for photons?



Different modes of ring resonators → synthetic dimension *w*

Tunneling along synthetic *w*:

- strong beam modulates resonator ε_{ij} at ω_{FSR} via optical $\chi^{(3)}$
- neighboring modes get linearly coupled
- phase of modulation → Peierls phase along synthetic *w*

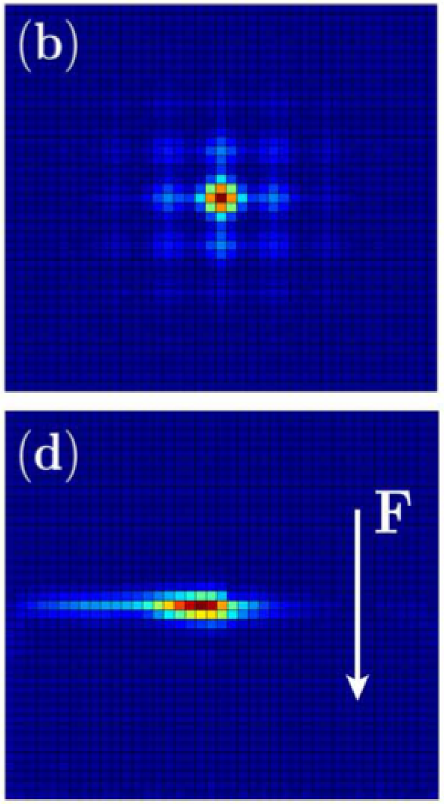
Extends Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008)

Peierls phase along *x,y,z* → Hafezi's ancilla resonators

Differently from atoms: can work with long synthetic dimension *w* with uniform tunneling

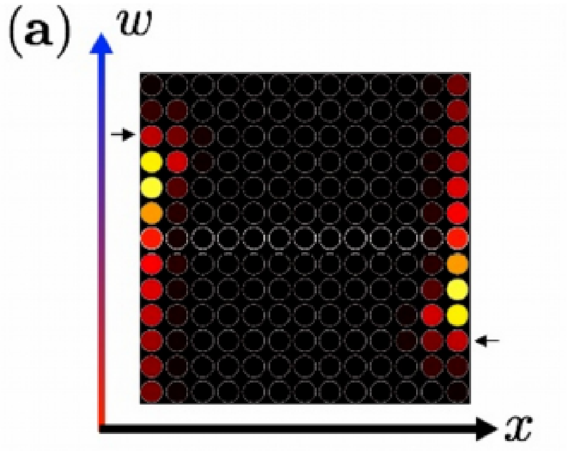
T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA (2016)

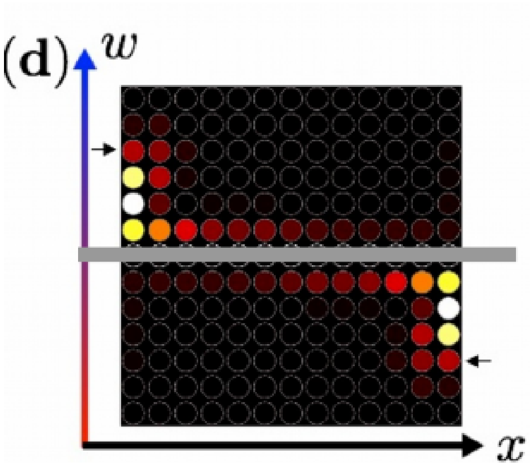
1+1 array: chiral edge states & optical isolation



1 (physical) + 1 (synthetic) dimensions: Hofstadter model

- Bulk topological invariant → Chern number
  - measured via Integer Quantum Hall effect
- Chiral states on edges:
  - Physical edges along x
  - Synthetic edges via design of ε(ω)
    (e.g. inserting absorbing impurities in chosen sites)
    → topologically protected optical isolator





Absorbing row of sites

T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016)

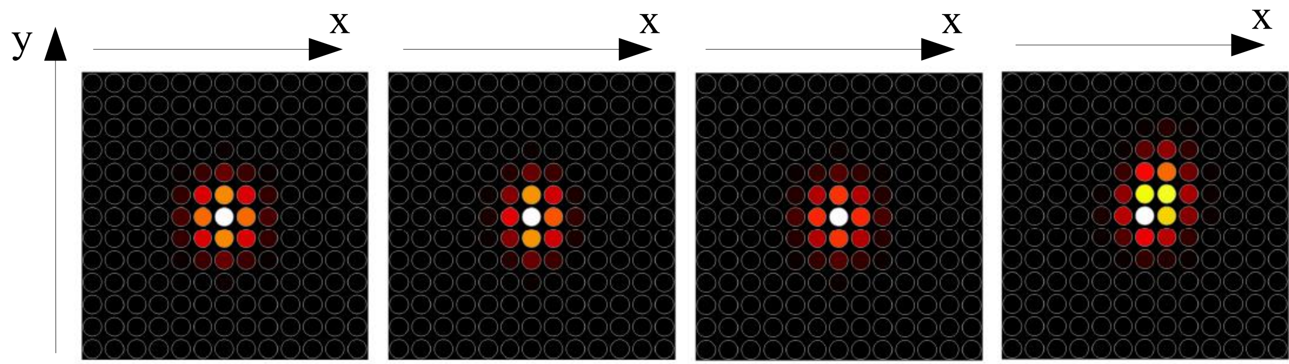
3+1 array: 4D Quantum Hall physics

4D magneto-electric response

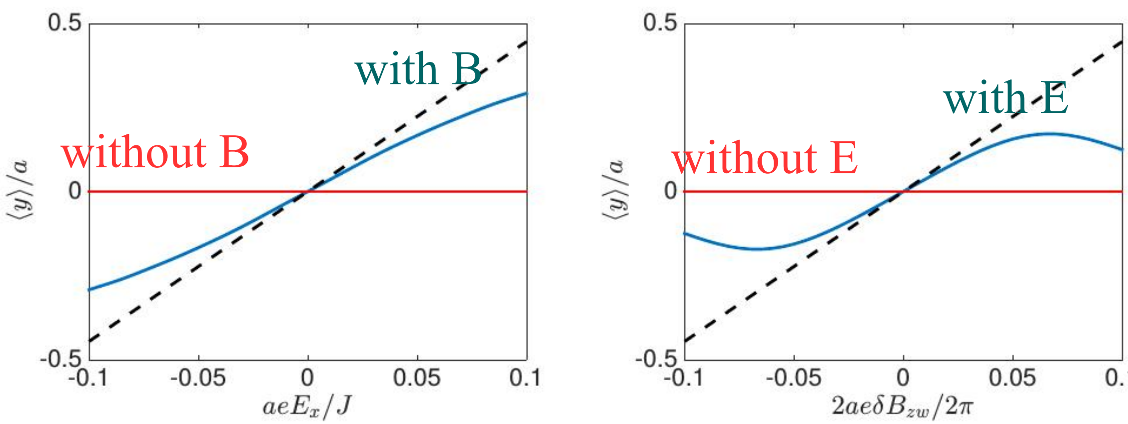
Nonlinear integer QH effect

Lateral shift of photon intensity distribution in response to external synth-E and synth-B:

- only present with both E & B
  - proportional to 2nd Chern



(a) $eE_x a, = 0$, $ea^2\delta B_{zw} = 0$ (b) $eE_x a = \frac{J}{10}$, $ea^2\delta B_{zw} = 0$ (c) $eE_x a = 0$, $ea^2\delta B_{zw} = \frac{2\pi}{10}$ (d) $eE_x a = \frac{J}{10}$, $ea^2\delta B_{zw} = \frac{2\pi}{10}$



$$j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathrm{d}^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} \mathrm{d}^4 k$$

T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016)

See also recent charge pumping experiments with atoms (Lohse et al. Nature ‘18) and light (Zilberberg et al. Nature ‘18)

Part 5:

The future:

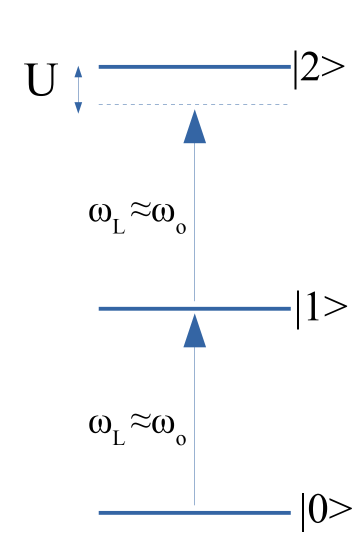
Strongly interacting fluids of light

Mott insulators &
Fractional Quantum Hall liquids

Photon blockade

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i F_i(t)\, \hat{b}_i + h.c.$$

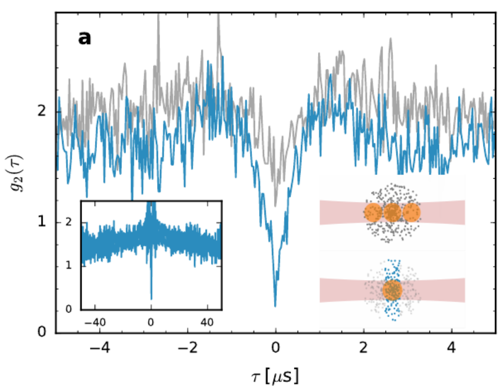


- Array of single-mode cavities at ω_o , tunnel coupling J , losses γ
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If U >> γ & J, coherent pump resonant with 0→1, but not with 1→2.

Photon blockade → Effectively impenetrable photons

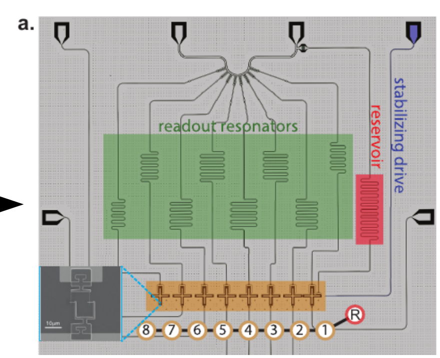
Opposite regime than non-interacting photons of Maxwell's eqs.

Single-cavity blockade observed in many platforms since the 2000s,
present challenge → scale up to many-cavity geometry

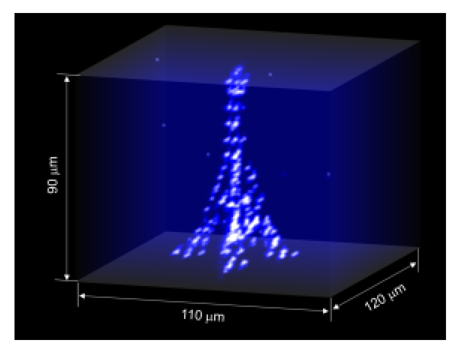


Polariton blockade via Rydberg-EIT

Circuit QED device



Fluid of spin excitations in lattice of Rydberg atoms. (Broways, Lukin,...)

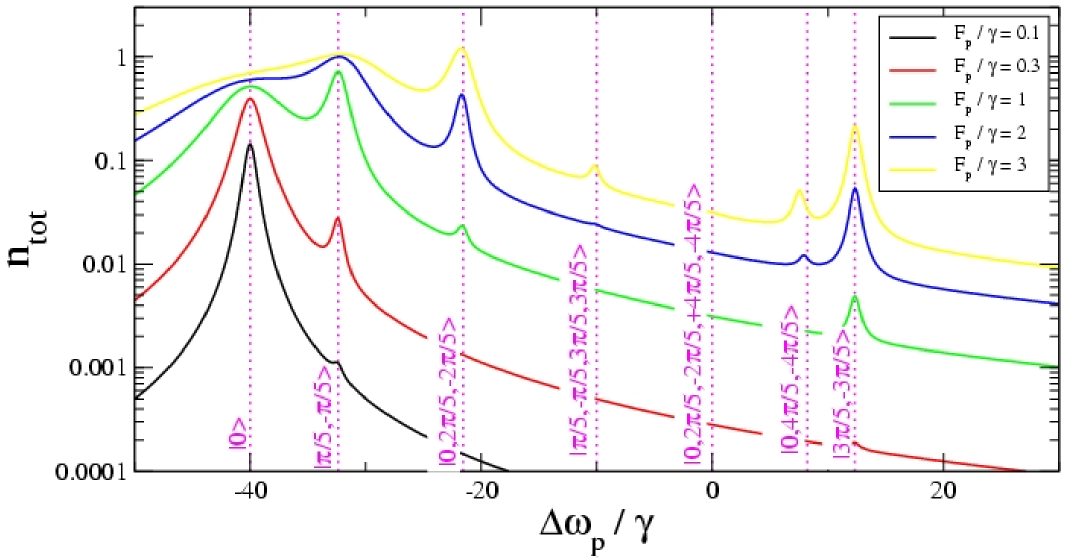


IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

Impenetrable "fermionized" photons in 1D necklaces

Many-body eigenstates of
Tonks-Girardeau gas
of impenetrable photons

Coherent pump
selectively addresses
specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

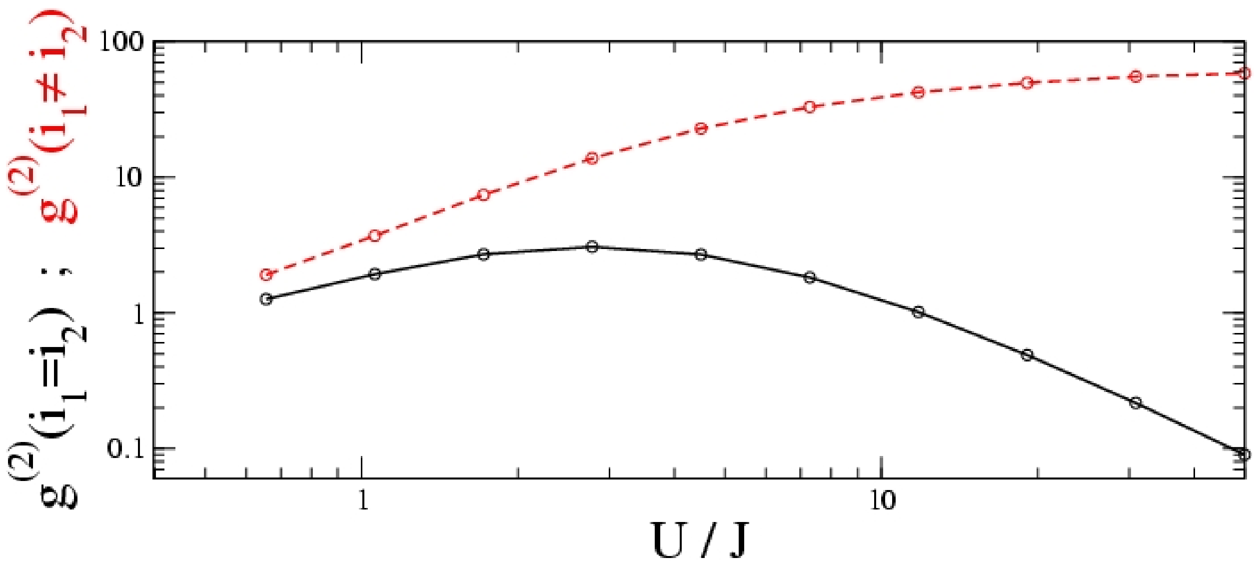
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- U/J >> 1: efficient photon blockade, impenetrable photons.

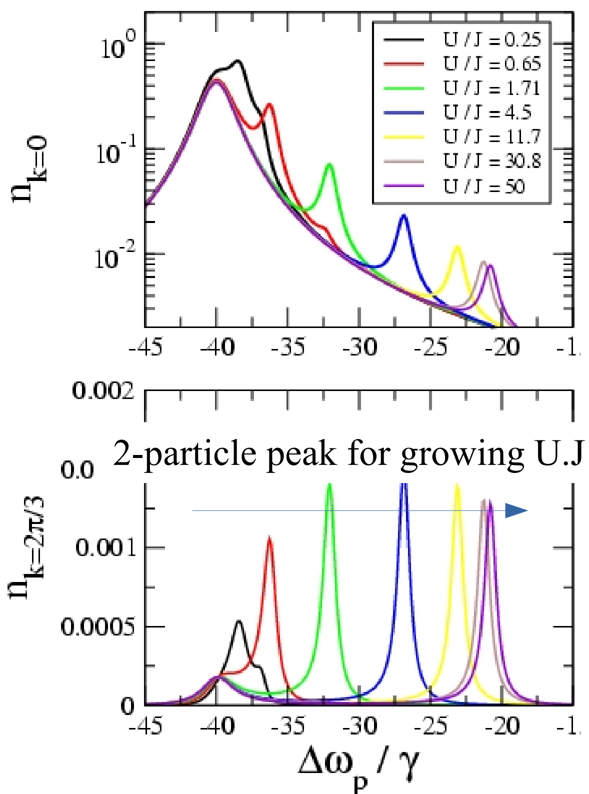
N-particle state excited by N photon transition:

- Plane wave pump with k_p=0: selects states of total momentum P=0
- Monochromatic pump at ω_p: resonantly excites states of many-body energy E such that $\omega_p = E / N$

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009)
See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics





Finite U/J, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ, larger probability of having N=0 or 2 photons than N=1
  - low U<<J: bunched emission for all pairs of i_1, i_2
  - large U>>J: antibunched emission from a single site; positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

Photon blockade + synthetic gauge field = FQHE for light

Bose-Hubbard model: $H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$

gauge field gives phase in hopping terms

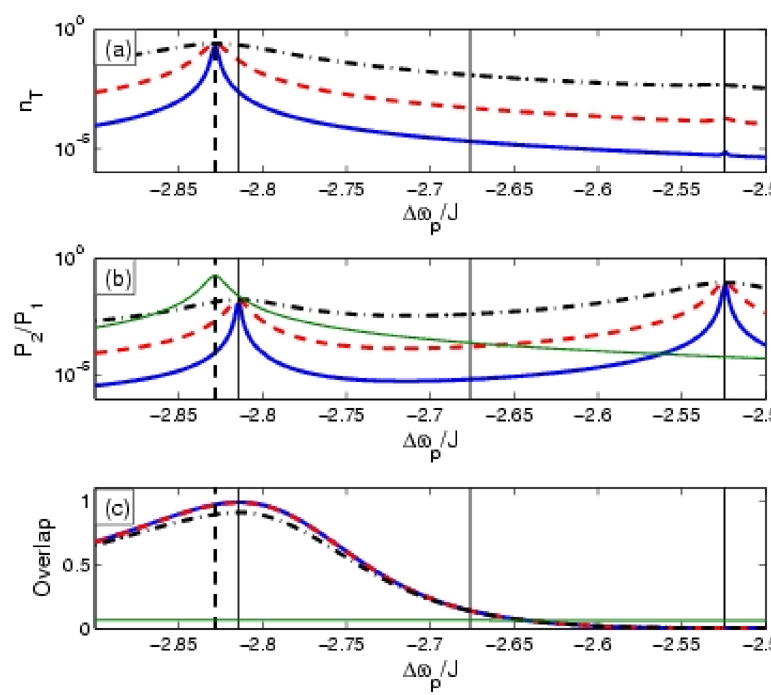
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, ..., z_N) = \mathcal{N}_L F_{\rm CM}^{(l)}(Z) e^{-\pi\alpha \sum_i y_i^2} \times \prod_{i<j}^{N} \left(\vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_i - z_j}{L} \middle| i \right) \right)^2$$

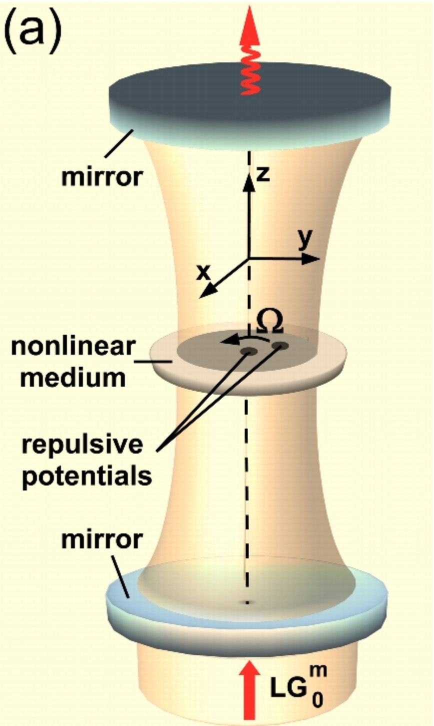
- no need for adiabatic following, etc....



R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

Continuous space FQH physics

Single cylindrical cavity. No need for cavity array



same form → Coriolis $F_c = -2m\Omega \times v$

→ Lorentz $F_L = e\, v \times B$

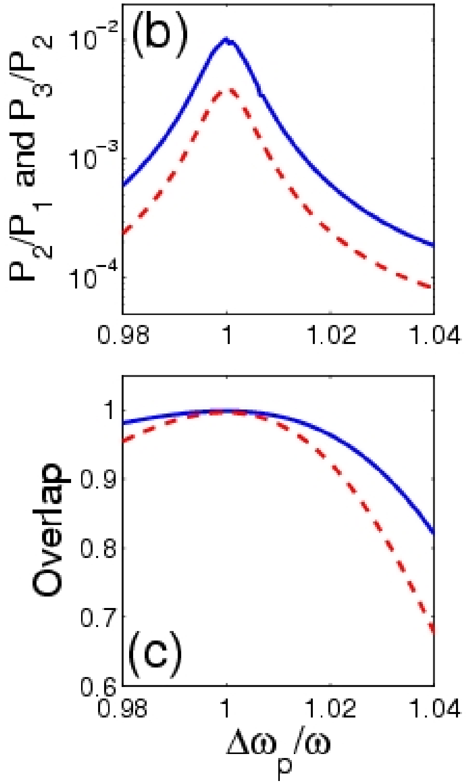
Photon gas injected by Laguerre-Gauss pump

with finite orbital angular momentum

Strong repuls. interact., e.g. layer of Rydberg atoms

Resonant peak in transmission due to Laughlin state:

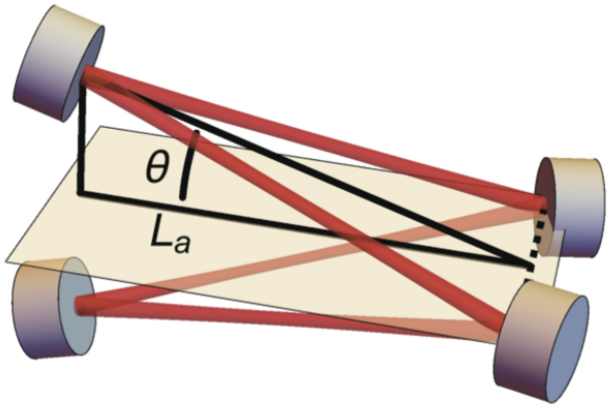
$$\psi(z_1,\ldots,z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

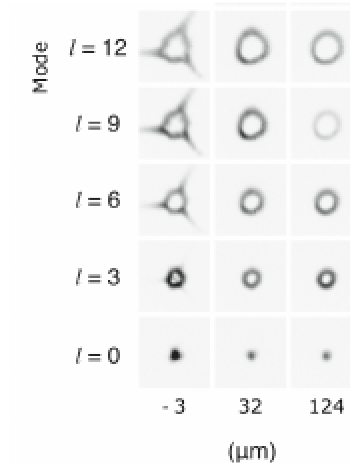


R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Experiment @ Chicago

A far smarter design



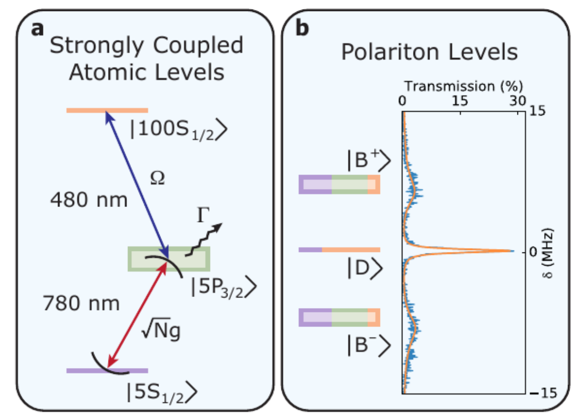


Non-planar ring cavity:

- Parallel transport→ synthetic B
- Landau levels for photons observed

Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities



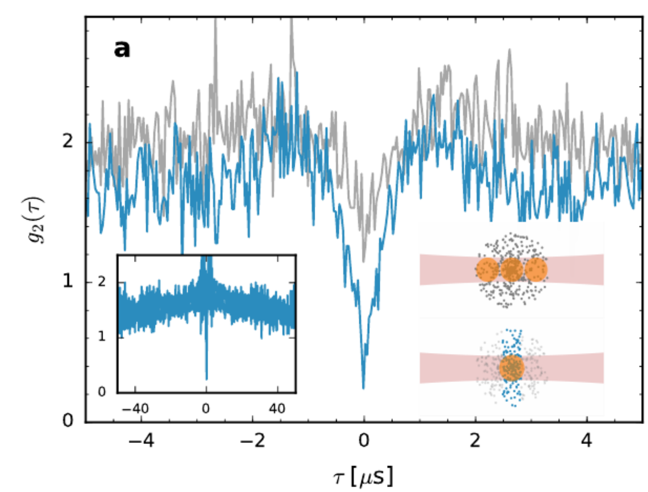
Polariton blockade on lowest (0,0) mode

- Equivalent to $\Delta_{\text{Laughlin}} > \gamma$

Easiest strategy for Laughlin

- Coherent pumping→ multi-photon peaks to few-body states
- Laughlin state → quantum correlations between orbital modes

(Umucalilar-Wouters-IC, PRA 2014)



Breaking news: 2-photon Laughlin state realized
(Clark et al., Nature 2020)

Figures from J. Simon's group @ U. Chicago
Schine et al., Nature 2016; Jia et al. 1705.07475

Experiment @ Chicago (II)

PHYSICAL REVIEW A **89**, 023803 (2014)

Probing few-particle Laughlin states of photons via correlation measurements

R. O. Umucalılar* and M. Wouters
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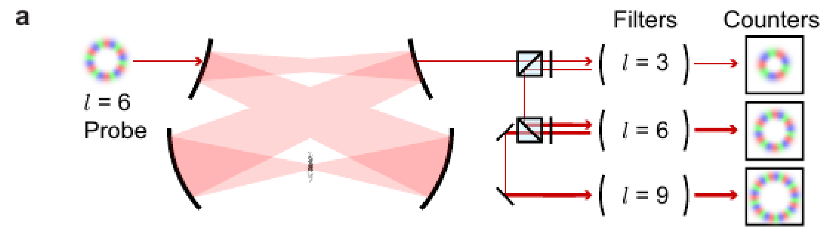
(Received 29 November 2013; published 5 February 2014)

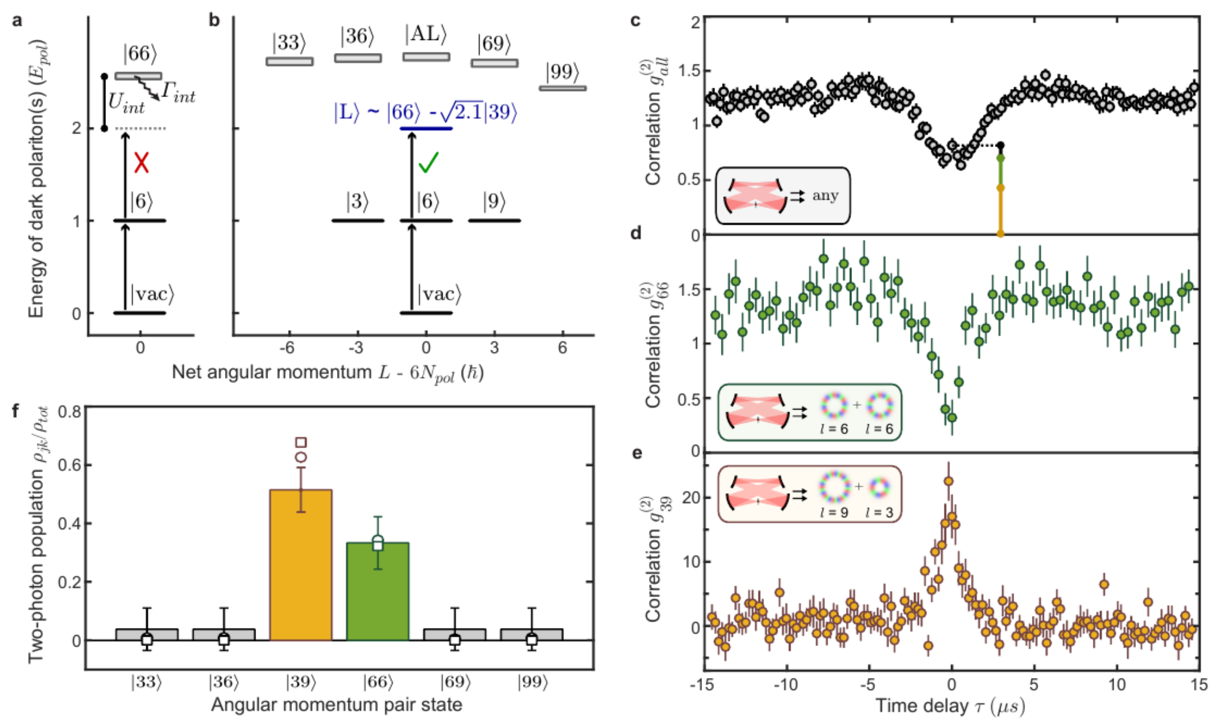
We propose methods to create and observe Laughlin-like states of photons in a strongly nonlinear optical cavity. Such states of strongly interacting photons can be prepared by pumping the cavity with a Laguerre-Gauss beam, which has a well-defined orbital angular momentum per photon. The Laughlin-like states appear as sharp resonances in the particle-number-resolved transmission spectrum. Power spectrum and second-order correlation function measurements yield unambiguous signatures of these few-particle strongly correlated states.



Quantum optical tricks to highlight generation of two-photon Laughlin state

Challenge: scale up to larger number of particles

Coherent pump scheme scales very bad with N for topological states

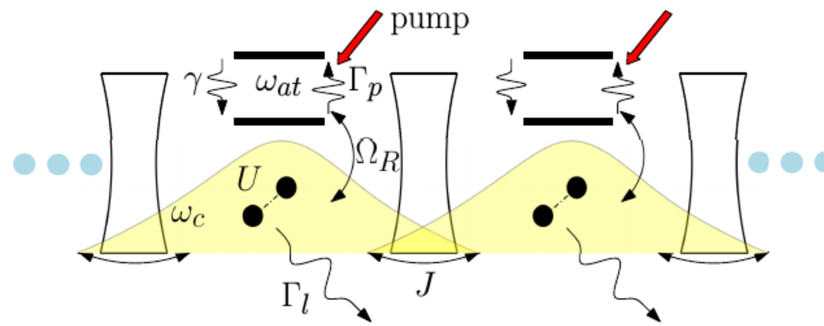


L. W. Clark, N. Schine, C. Baum, N. Jia, J. Simon, *Observation of Laughlin states made of light*, Nature 2020

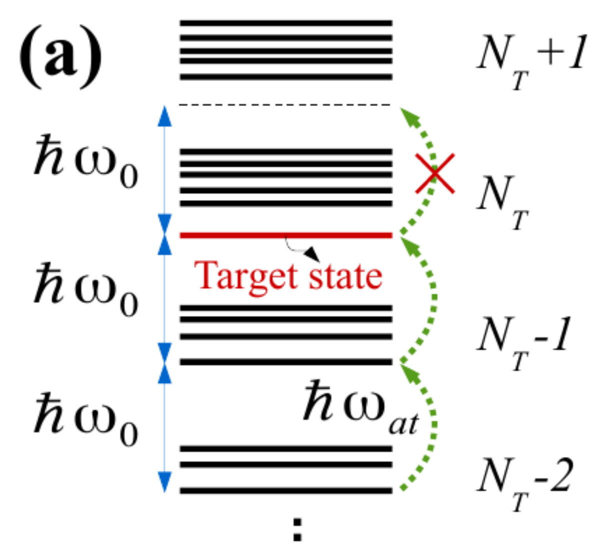
How to access larger particle numbers

Coherent pump only able to selectively excite few-photon states

→ Frequency-dependent incoherent pumping, e.g. collection of inverted emitters



- Lorentzian emission line around ω_{at}
  sophisticated schemes → other spectral shapes
- Emission only active if many-body transition
  is near resonance
- Injects photons until band is full (MI) or
  many-body gap develops (FQH)
- Many-body gap blocks excitation of higher states



Umucalilar-IC, PRA 2017
Lebreuilly, Biella et al., PRA 2017

General idea:
Kapit, Hafezi, Simon, PRX 2014
Lebreuilly et al. CRAS (2016)

Mott insulators of light

- non-Markovian master equation:
  frequency-dependent emission
  → rescaled jump operators

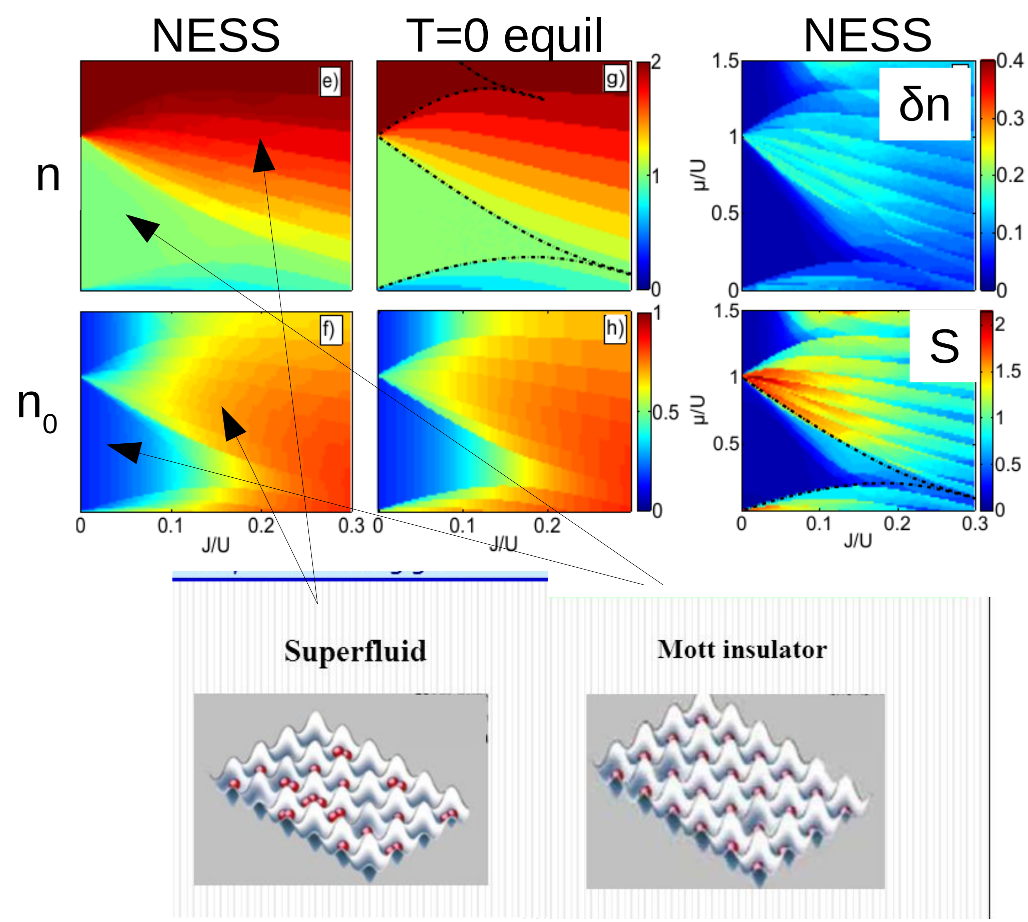
$$\bar{\mathcal{L}}_{\mathrm{em}}(\rho_{\mathrm{ph}}) = \frac{\Gamma_{\mathrm{em}}}{2} \sum_{i=1}^{k} \left[2\bar{a}_i^\dagger \rho_{\mathrm{ph}} \bar{a}_i - \bar{a}_i \bar{a}_i^\dagger \rho_{\mathrm{ph}} - \rho_{\mathrm{ph}} \bar{a}_i \bar{a}_i^\dagger \right]$$

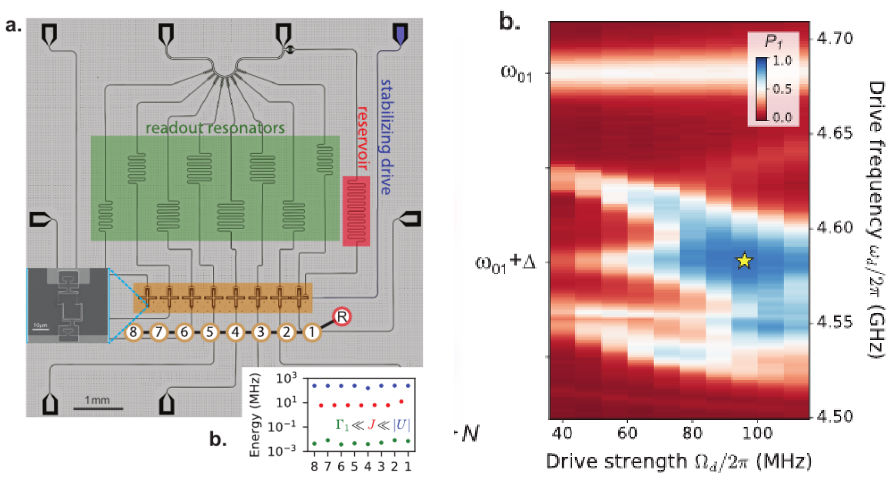
$$\langle f' | \bar{a}_i^\dagger | f \rangle = \frac{\Gamma_{\mathrm{pump}}/2}{\sqrt{(\omega_{\mathrm{at}} - \omega_{f',f})^2 + (\Gamma_{\mathrm{pump}}/2)^2}} \langle f' | a_i^\dagger | f \rangle$$

- driven-dissipative steady state stabilizes
  strongly correlated many-body states
  e.g. Mott-insulator, FQH…

- resembles low-T equilibrium
  (but interesting deviations in some cases)

- (in principle) no restriction to small N_{ph}
  only requirement → many-body energy gap





First expt: Ma *et al.* Nature 2019

Lebreuilly, Biella et al., 1704.01106 & 1704.08978
(published on PRA, 2017)

Related work in Kapit, Hafezi, Simon, PRX 2014

What about large FQH fluids?

Coherent pump:

- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

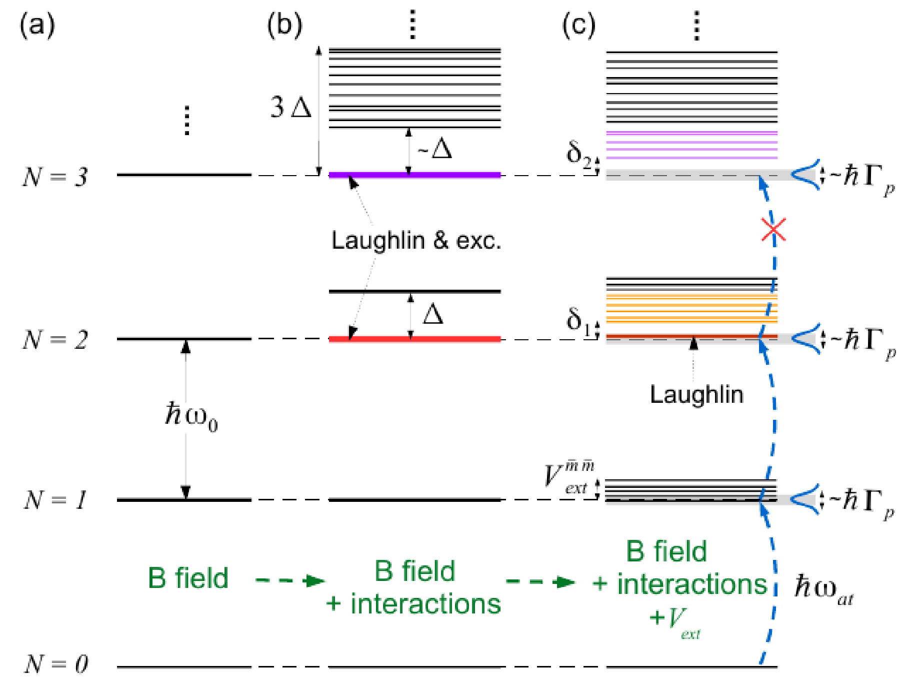
Frequency-dependent incoherent pump:

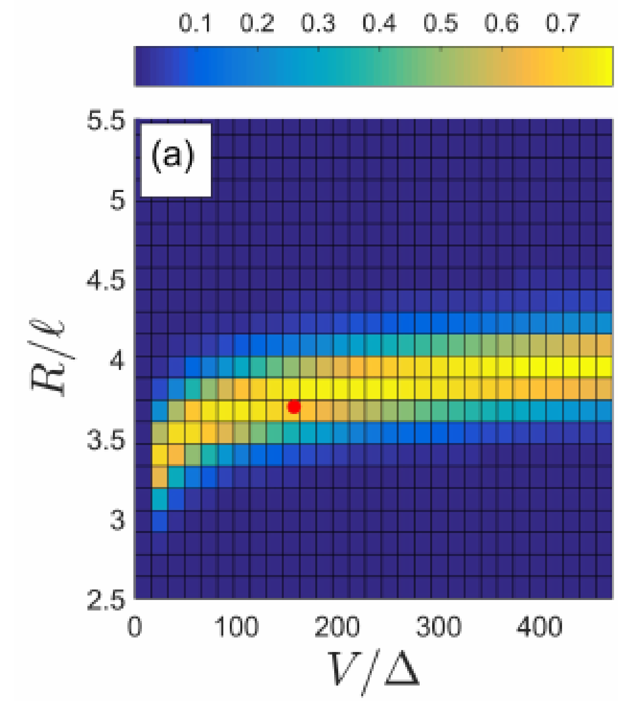
- Interactions → many-body gap Δ
- Edge excitations not gapped. Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

Calculations only possible for small systems:

- Large overlap with Laughlin states
- Excitations localized mostly on edge

Open question: what are ultimate limitations of this pumping method?





R. O. Umucalilar and IC, *Generation and spectroscopic signatures of a fractional quantum Hall liquid of photons in an incoherently pumped optical cavity*, PRA 2017.

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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I. Carusotto, C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013)

BEC

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nature physics FOCUS | REVIEW ARTICLE
https://doi.org/10.1038/s41567-020-0815-y

Check for updates

Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7]

Review article on Nature Physics (2020)

REVIEWS OF MODERN PHYSICS, VOLUME 91

Topological photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC,
RMP **91**, 015006 (2019)

We acknowledge generous financial support from:

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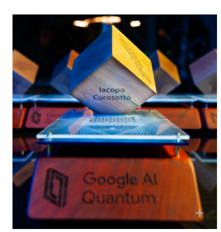

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for Research & Innovation

PhoQuS
Photons for Quantum Simulation

MIR-BOSE

Spontaneous coherence in spatially extended photonic systems: Non-Equilibrium BEC,
J. Bloch, IC, M. Wouters
arXiv:2106.11137

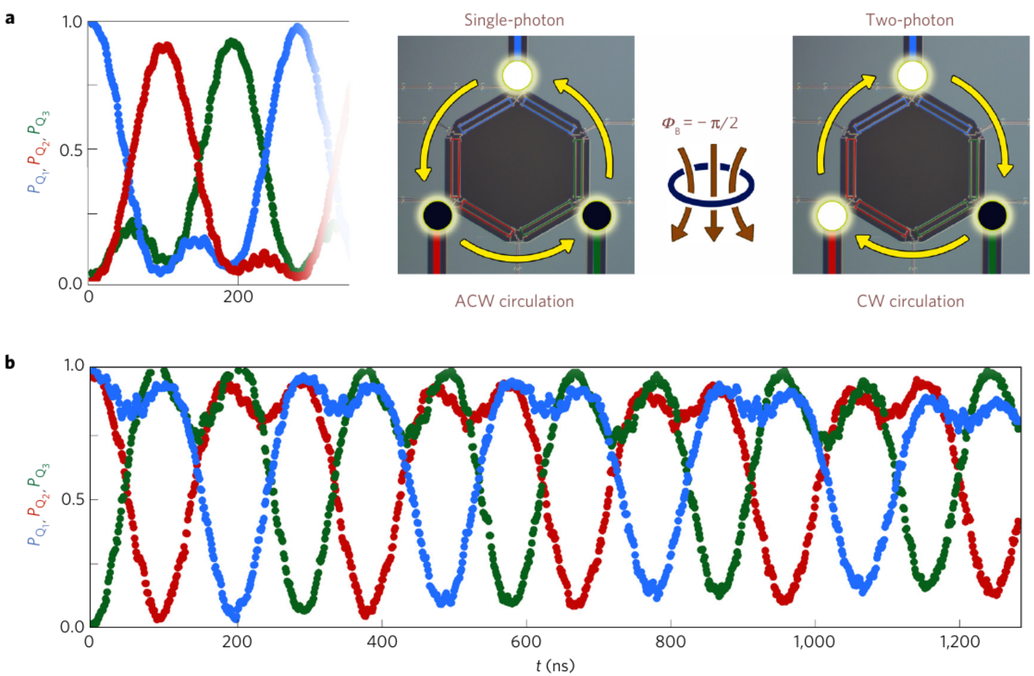
Part 6:

How to observe anyonic statistics of quasi-hole excitations?

Conservative dynamics in circuit-QED experiment:
interplay of strong interactions & synthetic magnetic field

Ring-shaped array of qubits in a superconductor-based circuit-QED platform

- Transmon qubit: two-level system
  → Impenetrable microwave photons
- Time-modulation of couplings
  → synthetic gauge field



➢ Independently initialize sites

➢ Follow unitary evolution until bosons lost
  (microwave photons → long lifetime)

➢ Monitor site occupation in time

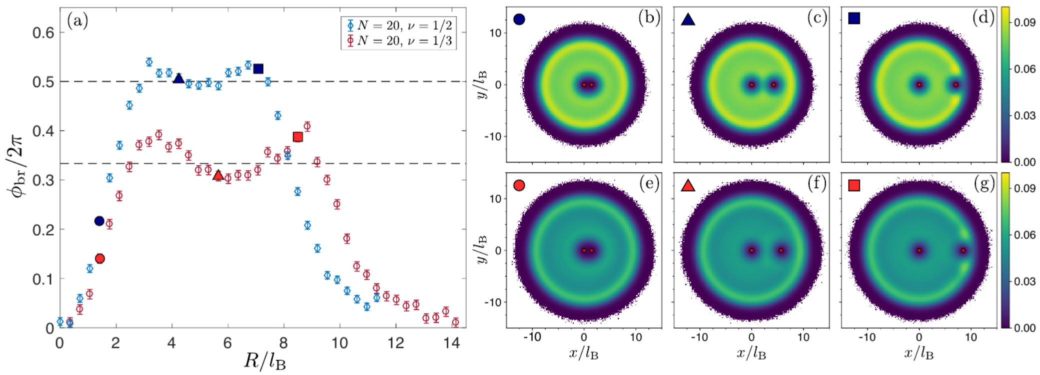
Roushan et al., Nat. Phys. 2016

“Many”-body effect:

two-photon state → opposite rotation compared to one-photon state

(similar to cold-atom experiment in Greiner’s lab: Tai et al., Nature 2017)

Observing anyonic statistics via time-of-flight measurements



Braiding phase → Berry phase when two quasi-holes are moved around each other

$$\varphi_{\rm B}(R) = i \oint_R \langle \Psi(\theta) | \partial_\theta | \Psi(\theta) \rangle d\theta.$$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_{\rm B}(R) = \frac{1}{\hbar} \oint_R \langle \Psi(\theta) | L_z | \Psi(\theta) \rangle d\theta = \frac{2\pi}{\hbar} \langle L_z \rangle$$

Self-similar expansion of lowest-Landau-levels → L_z can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2 \rangle_{\rm tof} = \frac{1}{N} \left(\frac{\hbar t}{\sqrt{2} M l_B} \right)^2 \left(\frac{\langle L_z \rangle}{\hbar} + N \right) = \left(\frac{\hbar t}{2 M l_B^2} \right)^2 \langle r^2 \rangle$$

Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern
Difficulty → small angular momentum difference of QH compared to total L_z

Umucalilar, Macaluso et al., *Observing anyonic statistics via time-of-flight measurements*, PRL (2018)

Quasi-Hole structure vs. anyon statistics (I)

- Compare (two) single quasi-holes and
  overlapping pair of quasi-holes:

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{1}{\hbar}\left[\langle \hat{L}_z\rangle_{|\eta_1|=|\eta_2|} - \langle \hat{L}_z\rangle_{\eta_1=\eta_2}\right].$$

- Relates to difference of density profiles:

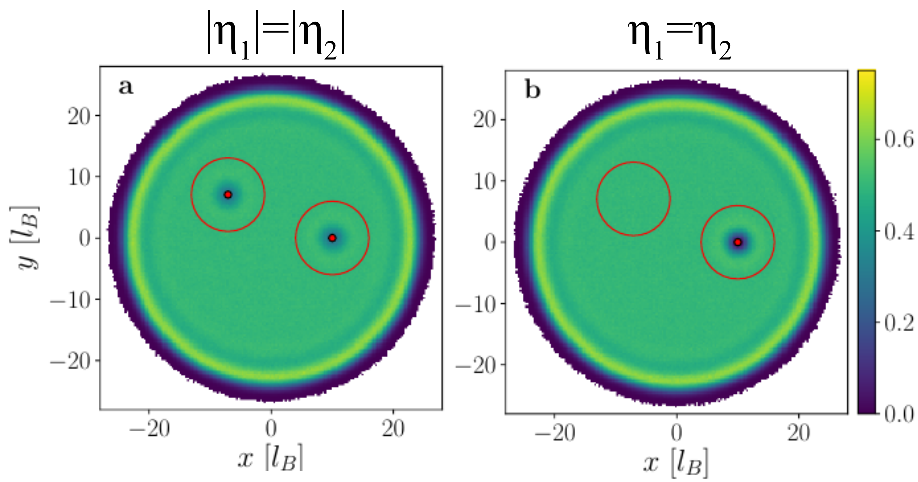
$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

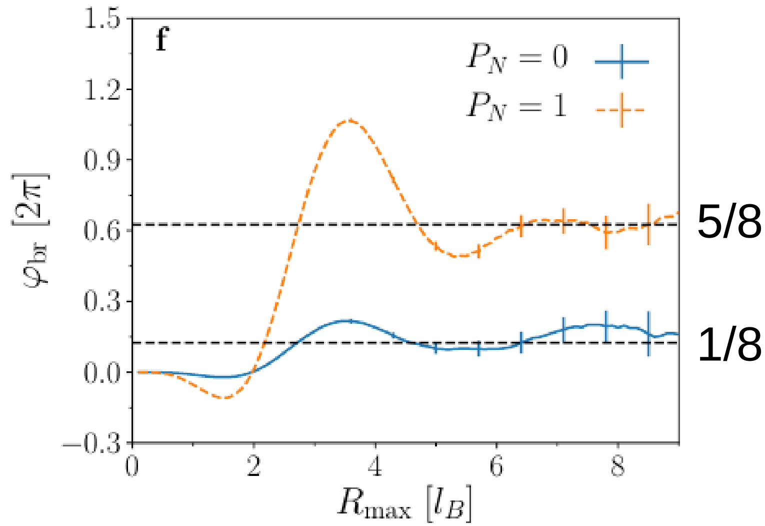
- Incompressibility → external region unaffected

- Statistics inferred from local density difference around QH core, i.e. variance of density depletion

- Insensitive to spurious excitation of
  (ungapped) edge states

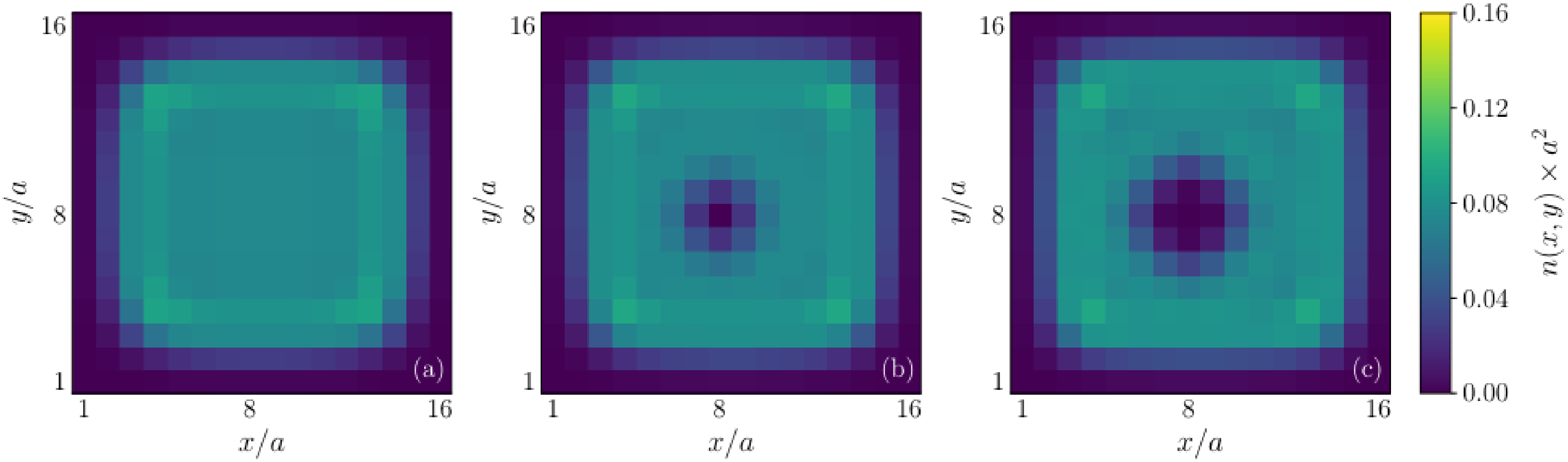
- Numerical calculation using Moore-Read wavefunction allows to distinguish fusion channels of even/odd
  total particle number





E. Macaluso, T. Comparin, L. Mazza, IC, *Fusion channels of non-Abelian anyons from angular-momentum and density-profile measurements*, PRL 2019

Quasi-Hole structure vs. anyon statistics (II)



N	L	α	Ω/t
12	16	0.15	1×10^{-3}

Discrete lattice model → Harper-Hofstadter-Bose-Hubbard

Ground state using Tree-Tensor-Network ansatz

- experimentally realistic "large" system
- open boundary conditions with harmonic trap
- repulsive potentials to pin quasi-holes

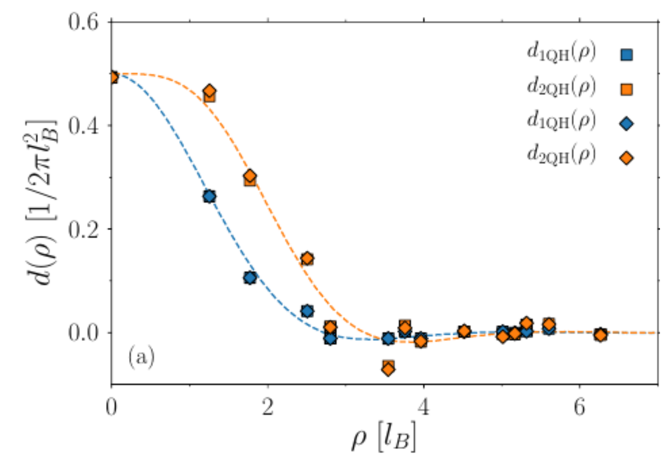
Apply discretized version of braiding phase formula

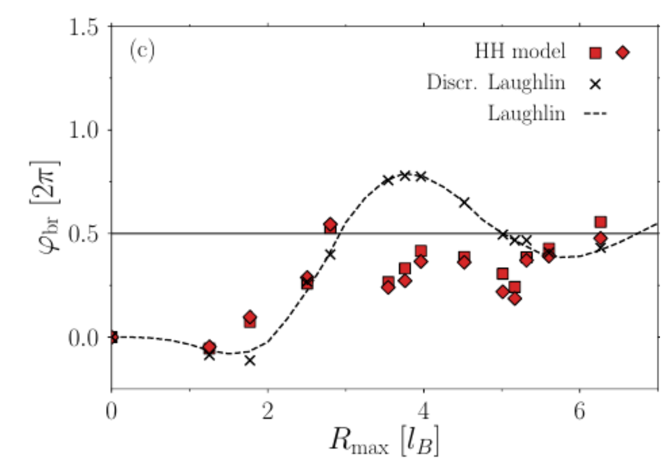
$$\frac{\varphi_{\mathrm{br}}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

to physical ground state wavefunction

→ Accurate reconstruction of anyonic statistics

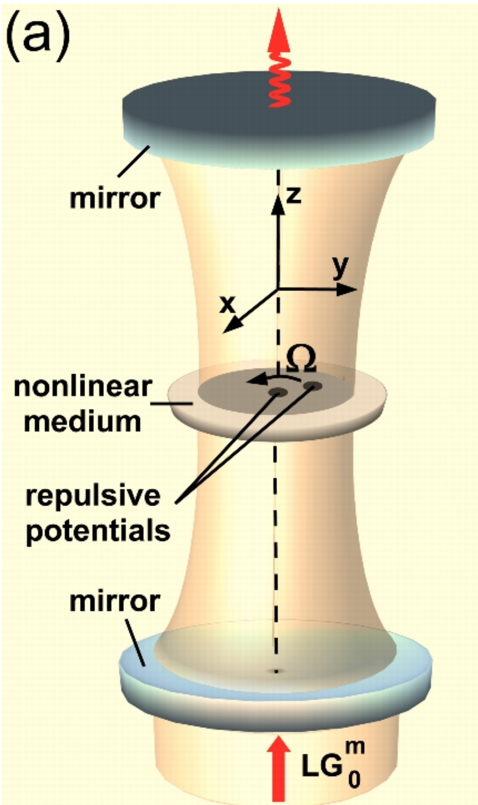
→ Experiment accessible in state-of-the-art circuit-QED systems





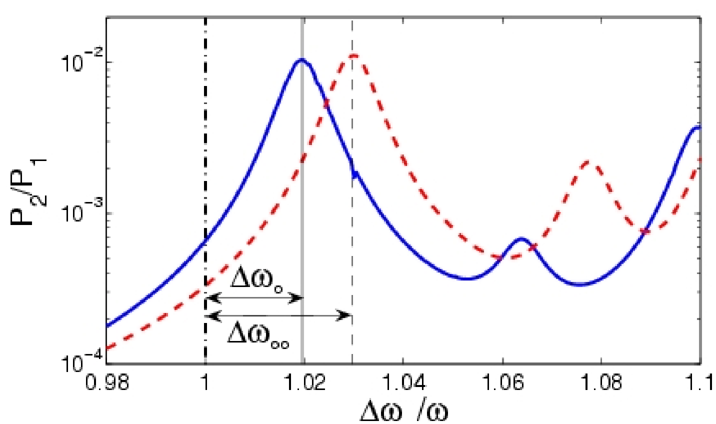
E. Macaluso *et al.*, *Charge and statistics of lattice quasiholes from density measurements: a Tree Tensor Network study,* Phys. Rev. Research (2020)

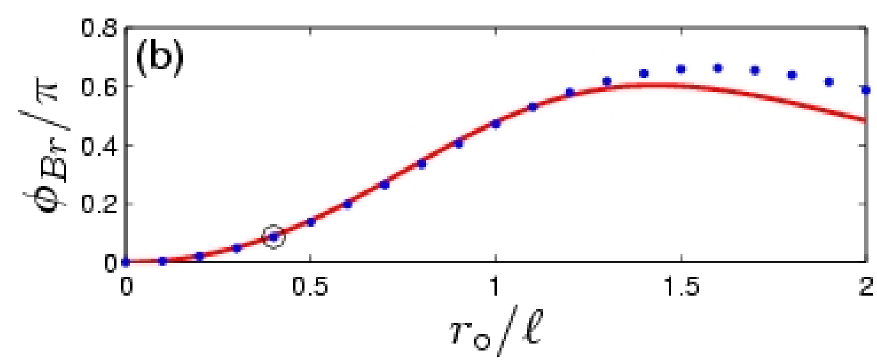
Optical signatures of the anyonic braiding phase



- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br} when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_{o})\, T_{rot} \quad [2\,\pi]$$



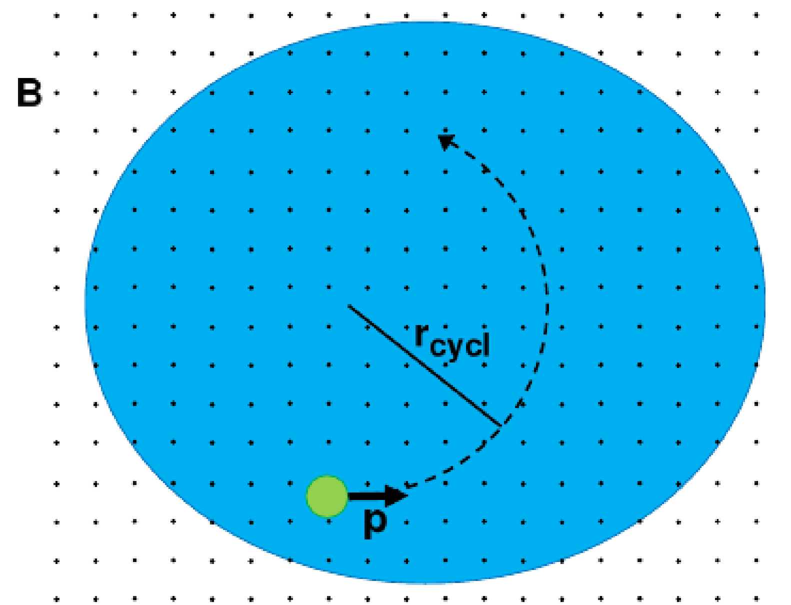


R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Quantum mechanics of anyons (I) – single particle

Laughlin wavefunction of Fractional Quantum Hall:

- quasi-holes → no E_{kin}, no independent life
- dressed by heavy impurity → anyonic molecule
- full-fledged mechanical degree of freedom



Born-Oppenheimer approx:

- Heavy impurity→ slow Degree of Freedom
- Light FQH particles → fast DoF

$$H_{\text{eff}} = \frac{\left[-i\nabla_{\mathbf{R}} - (Q - \nu q)\,\mathbf{A}(\mathbf{R})\right]^2}{2\mathcal{M}}$$

- Mass $\mathcal{M}$ → M (impurity) + QH dragging effect
- Impurity & FQH particles feel (Synth-)B,
  so synth-Charge → Q (impurity) – ν q (QH)

Cyclotron orbit → fractional charge and BO mass correction

A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020

Quantum mechanics of anyons (II) – two particles

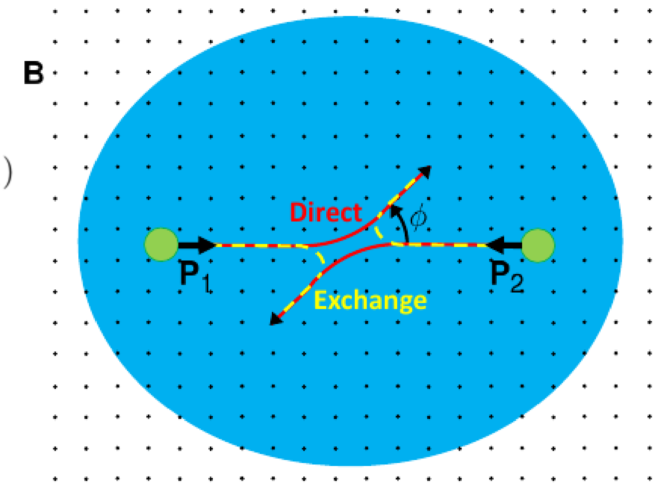
Each particle → attached flux

$$\mathcal{A}_j(\mathbf{R}) = \mathcal{A}_q(\mathbf{R}_j) + \mathcal{A}_{\text{stat},j}(\mathbf{R})$$
$$= \frac{\mathcal{B}_q}{2}\mathbf{u}_z \times \mathbf{R}_j + (-1)^j \frac{\nu}{R_{\text{rel}}^2}\mathbf{u}_z \times \mathbf{R}_{\text{rel}}$$

Relative motion:

- inter-particle potential
- statistical A_{rel} due to attached flux

$$H_{\text{rel}} = \frac{[\mathbf{P}_{\text{rel}} + \mathbf{A}_{\text{rel}}(\mathbf{R}_{\text{rel}})]^2}{2\mathcal{M}_{\text{rel}}} + V_{\text{ii}}(R_{\text{rel}})$$

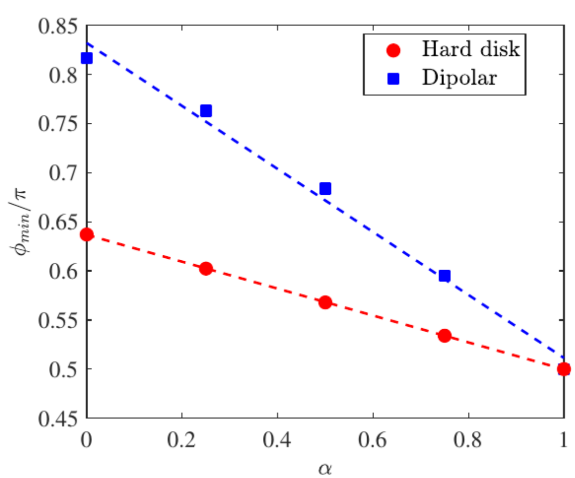


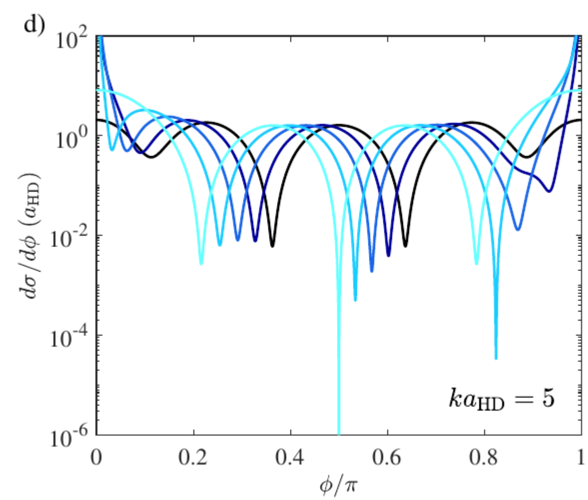
2-body scattering: interference of direct & exchange

- fringes in differential cross section
- fringe position depends on attached flux, i.e. fractional statistics

Measures fractional statistics

Scheme works best with polar molecules (heavy + long-range interactions) in atoms (light FQH gas)





A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020