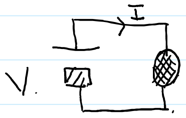
Quantum Transport.

I.] Basics of transport.

1.) Why look at transport?



$I(V)$. informations on the properties of the system

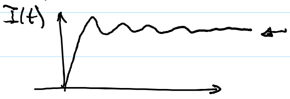
- Quite complicated: system is out of equilibrium. [dissipation? Isolated systems]
- Linear response.: small V. → small current.

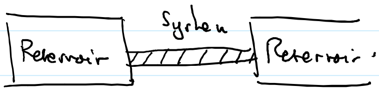
$$I = G.V.$$

↑ conductance of the system

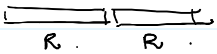
Much simpler to compute G.

2.) Various transport quantities, basic vocabulary.

a.)  steady state 

Modelization. 

$V = R.I.$ ↑ resistance. $I = G V.$ ↑ conductance.

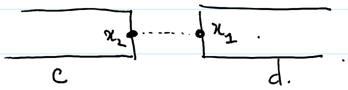


$R \propto L$ of the system

$R = \rho L.$ conductivity: $\sigma = 1/\rho$

In general ohm's law means. $R = \rho L^{2-d}.$

b.) Quantum point contact.

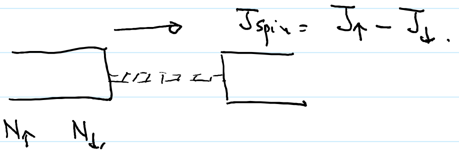


$$H_{tun} = -\tau \left[d^{\dagger}(x_1)\, c(x_2) + c^{\dagger}(x_2)\, d(x_1) \right]$$

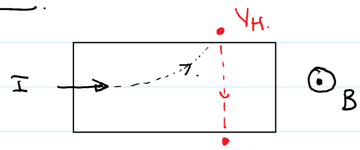
$$H = H_{Left} + H_{Right} + H_{tun}.$$

Note: $I(V)$ not necessarily linear.

c.) Other transport quantities.

heat, momentum, ... Spin. 

Hall.



$$q E_y = q \vec{V} \wedge \vec{B} \qquad J = n \vec{V}$$

$$= q \frac{J}{n} B$$

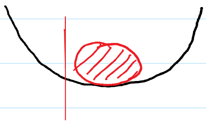
$$E_y \propto 1/n$$

Finite frequency transport.

$$\vec{j}(t) = \int dt' \, \sigma(t-t') \, E(t) \quad \to \quad j(\omega) = \sigma(\omega) \, E(\omega).$$

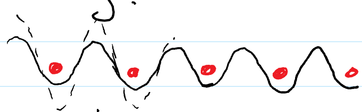
3) Cold atoms.

Super Complicated !



- kick the system

- Shaking.

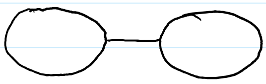


Does not measure conductivity

Amplitude: T. Stöferle et al. PRL 92 130403 (04).
(related)

A. Tokuno + TG, PRL 106 205301 (2011). phase shaking

↳ $\sigma(\omega)$.



II] Two important derivations.

1) Kubo formula.

Linear response: G.D. Mahan Many particle physics.

$$H = H_0 + \int dx \, h(x,t) \, \mathcal{O}(x).$$

number ↑ ← operator.

$$\langle A(x) \rangle_t = \frac{1}{Z} \mathrm{Tr}\left[\rho(t) \, A(x) \right]$$

at time $-\infty$ $\quad \rho(t=-\infty) \propto e^{-\beta H_0}$

Perturbation is applied adiabatically from $t=-\infty$. $\quad h(x,t) \, e^{\delta t}$, $\delta \to 0^+$.

$$\langle A(x) \rangle_{H_0} = 0$$

$$\langle A(x)\rangle_{H_0} = 0$$

$$\langle A(x)\rangle_{t,\,H_0+\int\lambda O} = \langle A(x)\rangle_{t,\,H_0} + \int_{x't'} \chi_{xx',tt'}\, \lambda(x',t') + \lambda^2$$

Fourier transform. $\quad A_{q\omega} = \int dx\,dt\; e^{i(\omega t - qr)} \langle A(x)\rangle_t$.

$\chi(x-x', t-t')$

$$A_{q\omega} = \chi(q,\omega)\,\lambda(q,\omega).$$

response $\quad$ perturbation.

$$\chi(x_1-x_2, t_1-t_2) = -i\,\theta(t_1-t_2)\,\langle [\hat{A}(x_1,t_1), \hat{O}(x_2,t_2)]\rangle_{H_0}$$

$$\hat{A}(t) = e^{iH_0 t} A\, e^{-iH_0 t}.$$

Fluctuation - Dissipation theorem: $\rightarrow$ thermal equilibrium.
$\rightarrow$ Response of the system to a perturbation.
$\hookrightarrow$ fluctuations of certain observables in equilibrium.

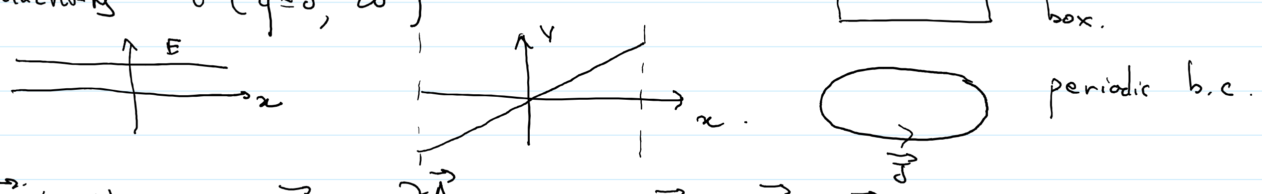
Exercise: $\quad \overline{\dfrac{dE(t)}{dt}} = -2\omega\, \mathrm{Im}\,\chi_{OO^+}(\omega).$ $\qquad \int dx\,[\lambda(x,t)\,O(x) + \lambda^*(x,t)\,O^+(x)]$

$$E(t) = \mathrm{Tr}[\rho(t)\,H(t)]$$

\# Conductivity:

$$\vec{j}(x,t) = \int dx'\,dt'\; \sigma(x-x', t-t')\, E(x',t')$$

Conductivity $\sigma(q=0, \omega)$



$\vec{A}(x,t) \qquad \vec{E} = \dfrac{\partial\vec{A}}{\partial t}. \qquad \vec{P} \to \vec{P} - q\vec{A}.$

$$H = \sum_k \frac{(\hbar k - qA)^2}{2m}\, c_k^\dagger c_k.$$

$$= H_0 \underbrace{- \sum_k 2\frac{qA\,\hbar k}{2m}\, c_k^\dagger c_k}_{\vec{A}\cdot\vec{J}} + \sum_k \frac{q^2A^2}{2m}\, c_k^\dagger c_k.$$

$$\left.\frac{\partial H}{\partial A}\right| = J.$$

tigth binding current $2t\sum_k \sin(k)\, c_k^\dagger c_k$.

$A \to 0$

$$\langle J \rangle_{q\omega} = \chi_{q\omega} \cdot A_{q\omega} .$$

$\chi_{q\omega}$ is the FT of $\chi = -i\,\theta(t_1 - t_2)\, \langle [\hat{J}(x_1, t_1), \hat{J}(x_2, t_2)] \rangle_{H_0}$.

$$E = \frac{\partial A}{\partial t} \to E_\omega = i\omega A_\omega .$$

$\sigma(q,\omega)$ is the FT $\quad \dfrac{-i}{i\omega} \displaystyle\int_0^{+\infty} dt\, e^{i(\omega + i\delta)t} \langle [\hat{J}(q,t), \hat{J}(q, t=0)] \rangle_{H_0}$.

Free particles $\quad [J, H_0] \quad \hat{J} = e^{iH_0 t} J e^{-iH_0 t} = J$

$$[\hat{J}(q=0,t), \hat{J}(q=0,0)] = [J(q=0), J(q=0)] = 0 .$$

$$\cancel{J = \sum_k \frac{\hbar k}{m} c_k^\dagger c_k} \to J = \sum_k \frac{(\hbar k - qA)}{m} c_k^\dagger c_k .$$

$$\langle J \rangle = \langle \tilde{J} \rangle - \frac{qA}{m} \sum_k \langle c_k^\dagger c_k \rangle \leftarrow \text{"Diamagnetic term"}$$

$$\sigma = \tilde{\sigma} - \frac{q^2 N}{i(\omega + i\delta) m} . \qquad \frac{1}{x + i\delta} = \mathcal{P}\left(\frac{1}{x}\right) - i\pi\delta(x).$$

$$\sigma_{\text{free}} = 0 + \text{imaginary} + \frac{q^2 N \pi}{m} \delta(\omega).$$

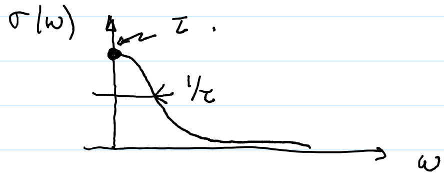
Drude weight.

- Question:

Modification of σ by scattering, interactions, etc

$$\sigma \sim \frac{-i}{\underset{\uparrow 0^+}{\omega + i\delta}} \to \frac{-i}{\omega + i/\tau} \leftarrow \text{lifetime.} \qquad \mathrm{Re}\,\sigma \simeq \frac{1/\tau}{\omega^2 + (1/\tau)^2} .$$

Drüde form for conductivity



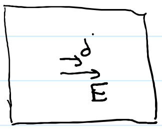
Exercise:

Free particles $\dfrac{(\hbar k)^2}{2m}$. $\quad U \displaystyle\int dx\, (\rho(x))^2$

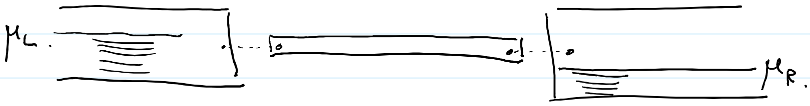
Hubbard model. $\quad U \sum_i n_{\uparrow i} n_{\downarrow i}$

free particles. $\quad G = \dfrac{\sigma}{L}$.

2.) Landauer - Bütiker formula for conductivity

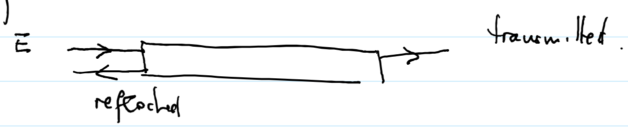


Kubo.



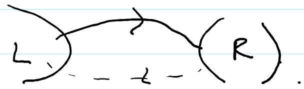
$$\mu_L - \mu_R = qV.$$

free particles: E is conserved.



$\mathcal{T}(E)$ transmission coefficient.

$$J_k^{L\to R} = v_k\, \mathcal{T}(E_k)\, f_L(E_k)\left[1 - f_R(E_k)\right]$$



$$J_k^{R\to L} = v_k\, \mathcal{T}(E_k)\, f_R(E_k)\left[1 - f_L(E_k)\right]$$

$$J^{tot} = q \sum_k v_k\, \mathcal{T}(E_k)\left[f_L(E_k) - f_R(E_k)\right].$$

$$v_k = \frac{\partial \mathcal{E}_k}{\partial p} = \frac{\partial E_k}{\hbar\, \partial k}.$$

$$= q \int \frac{dk}{(2\pi)} \frac{\partial E_k}{\hbar\, \partial k}\left[f_L(E_k) - f_R(E_k)\right] \mathcal{T}(E_k).$$

$$= \frac{q}{h} \int dE\, \mathcal{T}(E)\left[f(E-\mu_L) - f(E-\mu_R)\right]$$

$$\mu_L = \frac{qV}{2} \qquad \mu_R = \frac{-qV}{2}$$

$$J = \frac{q}{h}\int dE\, \mathcal{T}(E)\left[f\left(E - \frac{qV}{2}\right) - f\left(E + \frac{qV}{2}\right)\right]$$

Linear response.

$$f\left(E - \frac{qV}{2}\right) - f\left(E + \frac{qV}{2}\right) = \left(\frac{qV}{2} + \frac{qV}{2}\right)\frac{\partial f}{\partial E}.$$

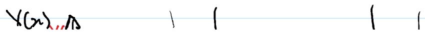
$$J = \frac{q^2}{h} V\, \mathcal{T}(E_F).$$

$$T=0 \quad \frac{\partial f}{\partial E} = -\delta(E-E_F)$$

$$G = \frac{q^2}{h}\mathcal{T}(E_F)$$

↳ Conductance quantization

\# L.B. vs Kubo. ?

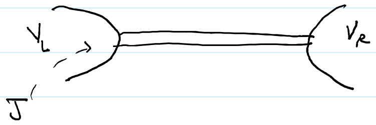


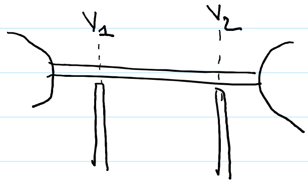
L.B. vs Kubo ?

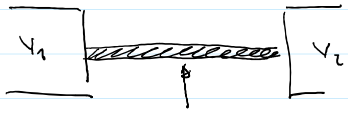
voltage drop → contact resistance. $R = R_L^{cont} + R_{system} + R_R^{cont}$.

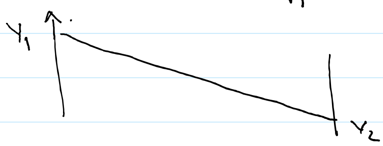
L.B: two point contact measurement. $V_L - V_R$. J → $G = \frac{q^2}{h}$.

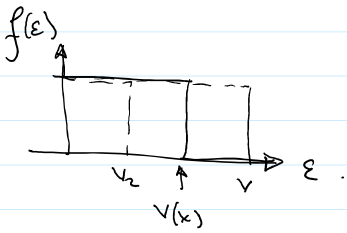
Kubo: four point contact ''

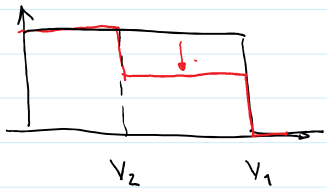








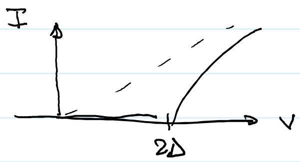


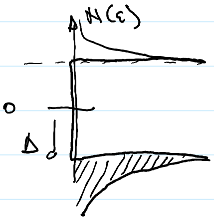


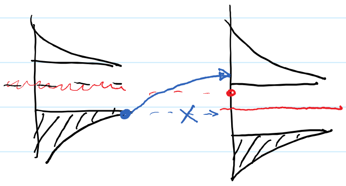
III.) Non linear transport.

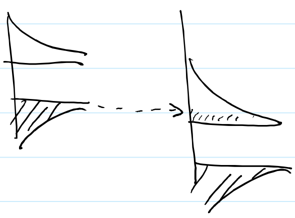
1.) QPC:

Blonder Tinkham Klapwijk PRB 25 4515 (1982).









Multiple Andreev Reflexions.

1 Quasiparticle + N pairs. $N \Delta\mu = 2\Delta$.

proba : $(t)^{2N}$.